AKT-090910, Hours 1-2: 3-Colourings, Kauffman and Jones

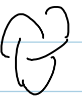
September-09-09
12:00 PM

1. Web: "Dror Bar-Natan" → classes → AKT

2. A bit of content:

Show http://www.math.toronto.edu/~drorbn/classes/0304/KnotTheory/NonObvious/NonObvious.html

$I(D) = \left| \begin{array}{l}\text{The arcs of } D \text{ can be coloured with 3 colours} \\ \text{(RGB) so that each xing is either mono- or} \\ \text{tri-chromatic, and so that all colours appear}\end{array} \right| \in \{True, False\}$

$I(O)$, $I($$)$, invariance under R-moves.

<u>HW</u> 1. Is there an efficient way to compute $I(D)$? How hard is it?

2. How many of the 250 knots on the "Rolfsen Table" are "3-colourable"?

3. Go over the front side of the "about" handout.

4. Another bit of content:

Show http://www.math.toronto.edu/~drorbn/Gallery/KnottedObjects/Candies/

Do the Jones/Kauffman starting from

$\times = A\,)(+ B \asymp$

5. The minimal Jones program following

http://katlas.org/wiki/The_Jones_Polynomial#How_is_the_Jones_polynomial_computed.3F or following Pensieve/2009-09/AKT-090910-ComputingJones.nb:

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In[1]:= K = X[1, 4, 2, 5] ** X[3, 6, 4, 1] ** X[5, 2, 6, 3]

Out[1]= X[1, 4, 2, 5] X[3, 6, 4, 1] X[5, 2, 6, 3]

In[2]:= t1 = K /. X[a_, b_, c_, d_] :> A P[a, d] P[b, c] + B P[a, b] P[c, d]

Out[2]= (B P[1, 4] P[2, 5] + A P[1, 5] P[4, 2])
         (A P[2, 6] P[5, 3] + B P[5, 2] P[6, 3]) (B P[3, 6] P[4, 1] + A P[3, 1] P[6, 4])

In[3]:= t2 = Expand[t1]

Out[3]= A B^2 P[1, 4] P[2, 5] P[2, 6] P[3, 6] P[4, 1] P[5, 3] +
         A^2 B P[1, 5] P[2, 6] P[3, 6] P[4, 1] P[4, 2] P[5, 3] +
         B^3 P[1, 4] P[2, 5] P[3, 6] P[4, 1] P[5, 2] P[6, 3] +
         A B^2 P[1, 5] P[3, 6] P[4, 1] P[4, 2] P[5, 2] P[6, 3] +
         A^2 B P[1, 4] P[2, 5] P[2, 6] P[3, 1] P[5, 3] P[6, 4] +
         A^3 P[1, 5] P[2, 6] P[3, 1] P[4, 2] P[5, 3] P[6, 4] +
         A B^2 P[1, 4] P[2, 5] P[3, 1] P[5, 2] P[6, 3] P[6, 4] +
         A^2 B P[1, 5] P[3, 1] P[4, 2] P[5, 2] P[6, 3] P[6, 4]

In[4]:= t3 = t2 //. {P[a_, b_] P[b_, c_] :> P[a, c], P[a_, b_]^2 :> P[a, a]}

Out[4]= A^2 B P[2, 2] + A B^2 P[1, 1] P[2, 2] + B^3 P[1, 1] P[2, 2] P[3, 3] + A^2 B P[4, 4] +
         A^3 P[3, 3] P[4, 4] + A B^2 P[3, 3] P[4, 4] + A^2 B P[6, 6] + A B^2 P[2, 2] P[6, 6]

In[5]:= t4 = Expand[t3 /. P[a_, a_] -> -A^2 - B^2 /. B -> 1 / A]
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Out[5]= $-\frac{1}{A^9} + \frac{1}{A} + A^3 + A^7$