Do not turn this page until instructed.

Math 240 Algebra I — Term Test

University of Toronto, October 24, 2006

Solve the 5 problems on the other side of this page.
Each of the problems is worth 20 points.
You have an hour and 45 minutes.

Notes.

- No outside material other than stationary and a basic calculator is allowed.
- We will have an extra hour of class time in our regular class room on Thursday, replacing the first tutorial hour.
- The final exam date was posted by the faculty — it will take place on Wednesday October 13 from 2PM until 5PM at room 3 of the Clara Benson Building, 320 Huron Street (south west of Harbord cross Huron, home of the Faculty of Physical Education and Health).

Good Luck!

Solve the following 5 problems. Each of the problems is worth 20 points. You have an hour and 45 minutes.

Problem 1. Let F be a field with zero element 0_F, let V be a vector space with zero element 0_V and let $v \in V$ be some vector. Using only the axioms of fields and vector spaces, prove that $0_F \cdot v = 0_V$.

Problem 2.

1. In the field $\mathbb{C}$ of complex numbers, compute

$$\frac{1}{2+3i} + \frac{1}{2-3i} \qquad \text{and} \qquad \frac{1}{2+3i} - \frac{1}{2-3i}.$$

2. Working in the field $\mathbb{Z}/7$ of integers modulo 7, make a table showing the values of a^{-1} for every $a \neq 0$.

Problem 3. Let V be a vector space and let W_1 and W_2 be subspaces of V. Prove that $W_1 \cup W_2$ is a subspace of V iff $W_1 \subset W_2$ or $W_2 \subset W_1$.

Problem 4. In the vector space $M_{2\times 2}(\mathbb{Q})$, decide if the matrix $\begin{pmatrix} 1 & 2 \\ -3 & 4 \end{pmatrix}$ is a linear combination of the elements of $S = \left\{ \begin{pmatrix} 1 & 0 \\ -1 & 0 \end{pmatrix}, \begin{pmatrix} 0 & 1 \\ 0 & 1 \end{pmatrix}, \begin{pmatrix} 1 & 1 \\ 0 & 0 \end{pmatrix} \right\}$.

Problem 5. Let V be a finite dimensional vector space and let W_1 and W_2 be subspaces of V for which $W_1 \cap W_2 = \{0\}$. Denote the linear span of $W_1 \cup W_2$ by $W_1 + W_2$. Prove that $\dim(W_1 + W_2) = \dim W_1 + \dim W_2$.

Good Luck!