*"TBA" @MP 203.

*Tell TAs which probs to grade.

Math 240 Algebra I, Tuesday Sep 12 2006, hours 1-2.

* About this class

Hidden Agenda: Lurd how math is done; abstraction & generalization, definition, Theorem, proof.

* The real numbers: a set $\mathbb{R}$ with two binary operations $+$ and $\cdot$ and two special elements $0, 1$ so that

R1 $a+b=b+a$ & $ab=ba$

R2 Assoc.

R3 $0, 1$

R4 negatives & inverses. $\forall a \exists c\ a+c=0$; $\forall b \neq 0 \exists d\ b \cdot d = 1$

R5 distributivity.

Much of algebra (though not all?) follows—

Follows: $(a+b)(a-b)=a^2-b^2$

Doesn't follow: For every a there is an x s.t. either $a=x \cdot x$ or $-a=x \cdot x$, (or both)

Def a Field

Examples 1. The reals $\mathbb{R}$

2. The rationals $\mathbb{Q}$

3. The complex numbers $\mathbb{C}=\{a+bi\}$

4. The numbers $0, 1, 2, 3, 4, 5, 6$ with a funny def. of $+$ and $\cdot$

useful in

Proof of $(a+b)(a-b)$

Example for $\mathbb{Z}/7$

Def of $\mathbb{C}$

Def of $\mathbb{Z}/p$

Math 240 Algebra I, Thu sep 14 2006, hour 3.

* "Odd group" at MP 118

on board.

F1 $a+b=b+a$ $ab=ba$

F2 $(a+b)+c=a+(b+c)$ $a(bc)=(ab)c$

F3 $a+0=a$ $a\cdot 1=a$

F4 $\forall a \exists b\ a+b=0$, $\forall a\neq 0 \exists b\ ab=1$

F5 $a(b+c)=ab+ac$ Thm $(a+b)(a-b)=a^2-b^2$

Lemma 1 $a\cdot 0=0$ (in fact $a\cdot b=0$ iff $a=0$ or $b=0$)

Lemma 2 $(-a)\cdot b=-(a\cdot b)$

Precise definition of $\mathbb{C}$.

Math 240 Pages **MUST** begin with "06-240"! (e)

06-240/Homework Assignment 1

Read appendices A through D in our textbook (with higher attention to C and D), and solve the following problems:

1. Suppose a and b are nonzero elements of a field F. Using only the field axioms, prove that $a^{-1}b^{-1}$ is a multiplicative inverse of ab. State which axioms are used in your proof.
2. Write the following complex numbers in the form $a+ib$, with $a,b\in\mathbb{R}$:
   1. $\frac{1}{2i}+\frac{-2i}{5-i}$.
   2. $(1+i)^5$.
3. 1. Prove that the set $F_1=\{a+b\sqrt{3}:a,b\in\mathbb{Q}\}$ (endowed with the addition and multiplication inherited from $\mathbb{R}$) is a field.
   2. Is the set $F_2=\{a+b\sqrt{3}:a,b\in\mathbb{Z}\}$ (with the same addition and multiplication) also a field?
4. Let $F_4=\{0,1,a,b\}$ be a field containing 4 elements. Assume that $1+1=0$. Prove that $b=a^{-1}=a^2=a+1$. (*Hint:* For example, for the first equality, show that $a\cdot b$ cannot equal 0, a, or b.)

This assignment is due at the tutorials on Thursday September 21. Here and everywhere, **neatness counts!!** You may be brilliant and you may mean just the right things, but if the teaching assistants will be having hard time deciphering your work they will give up and assume it is wrong.

06-240/Navigation Panel [Hide]

#	Week of...	Notes and Links
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9	Nov 6	HW7
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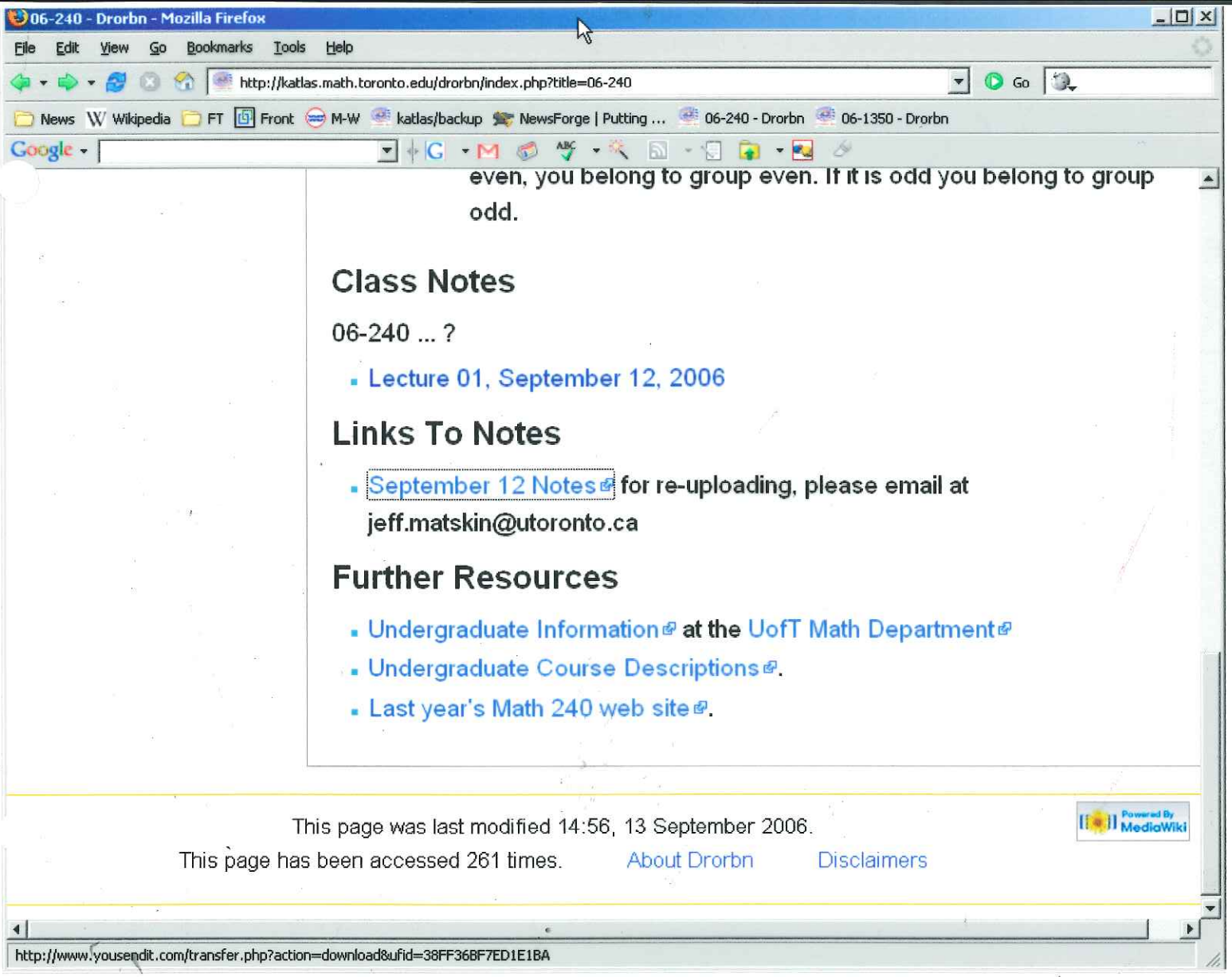
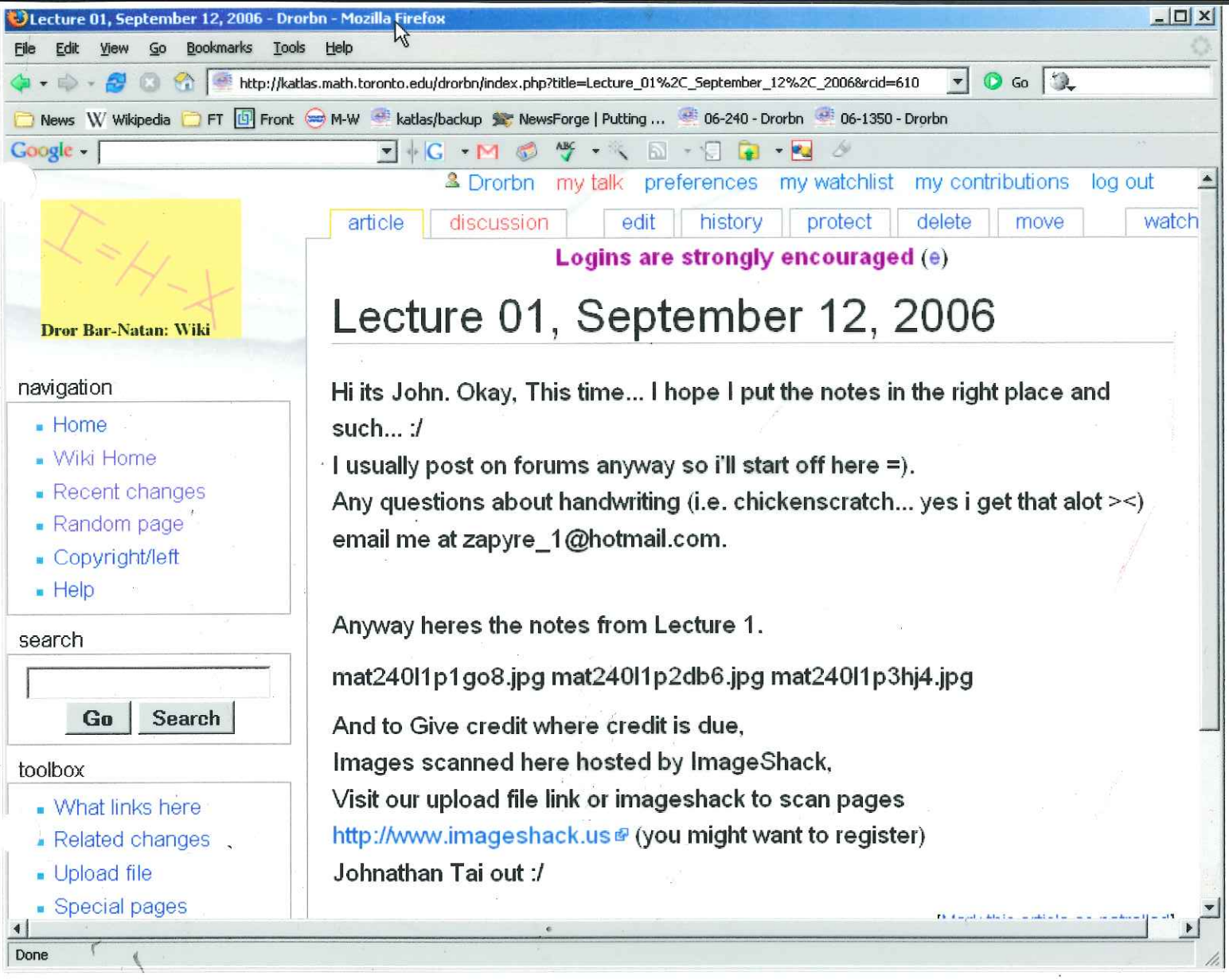
Neatness Counts!

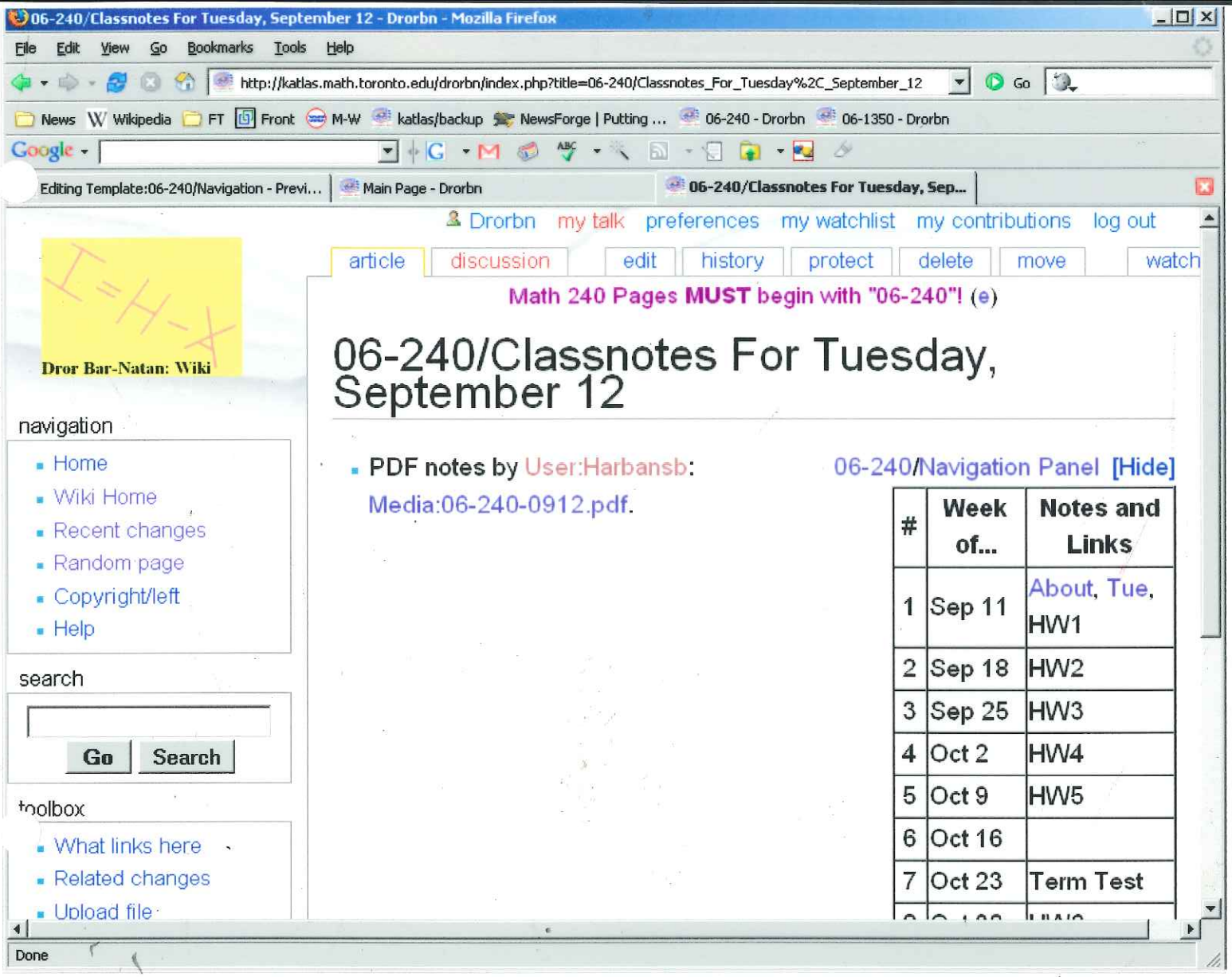


Notes should be linked from the navigation panel.



* Chat goes under "discussion" and/or User Pages.

* Avoid external links, avoid Microsoft.



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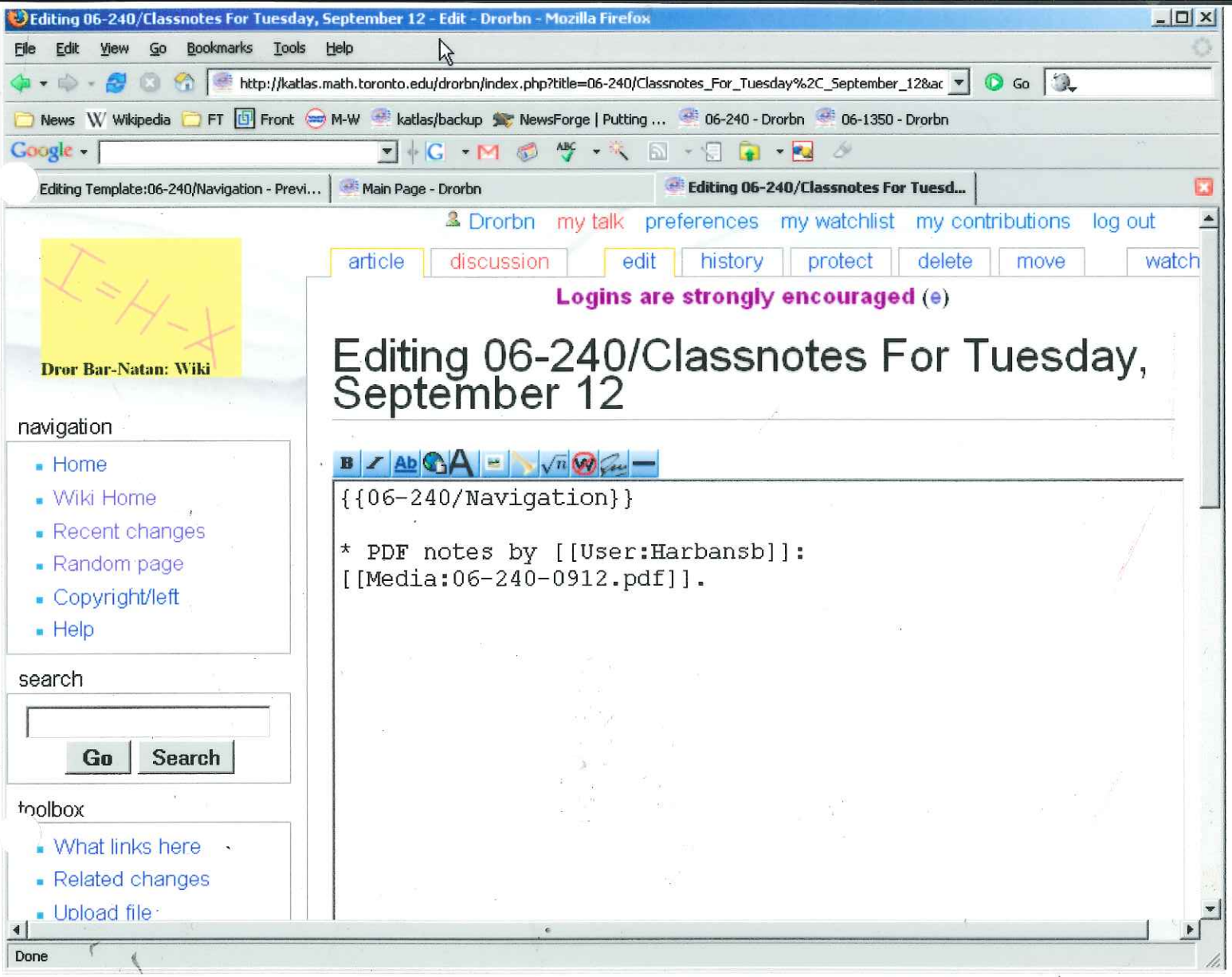
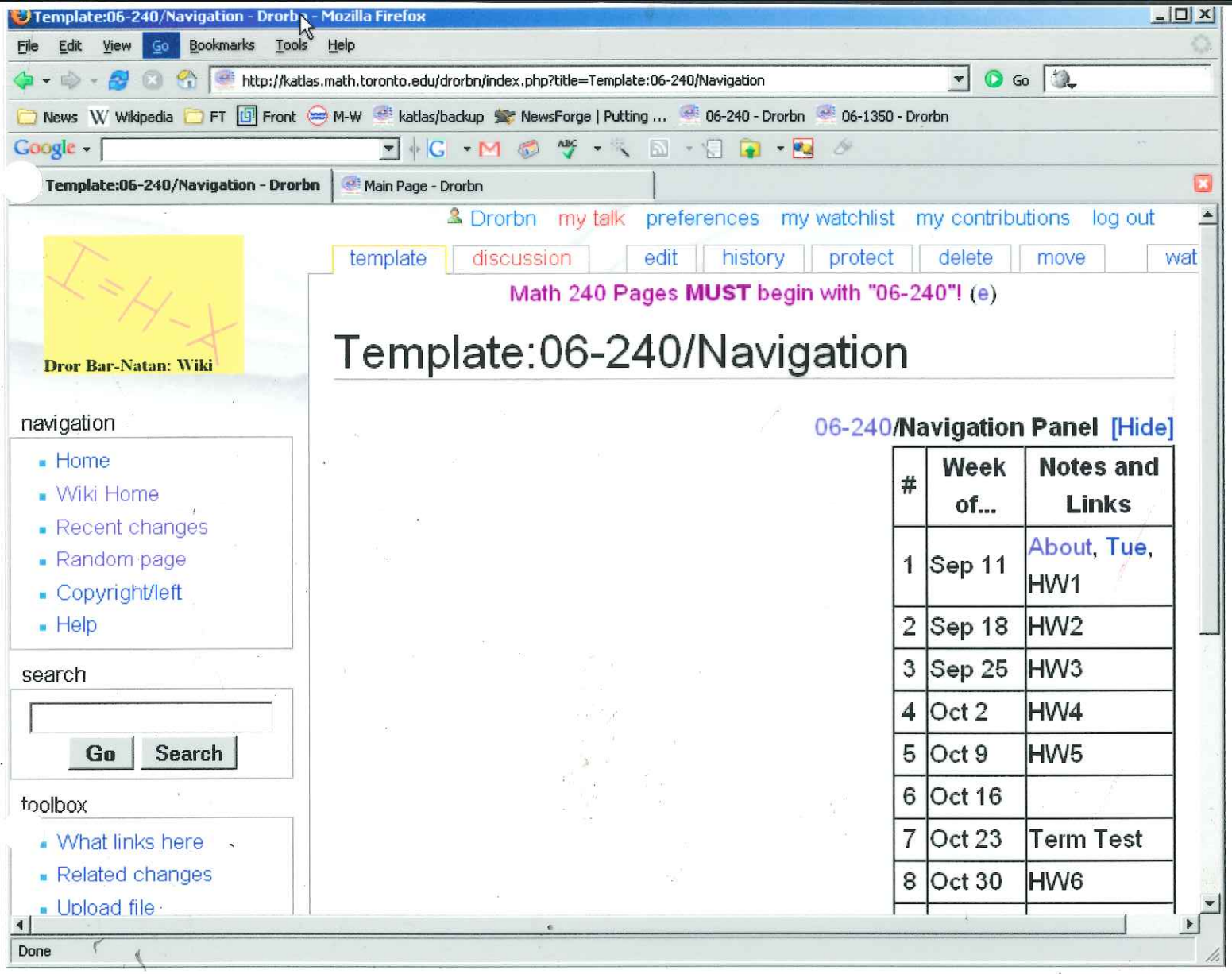


- Navigation inserted.
- Room for notes by others.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Math 240 Pages MUST begin with "06-240"! (e)
Template:06-240/Navigation
navigation
Home
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What links here
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Upload file
06-240/Navigation Panel [Hide]
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Done

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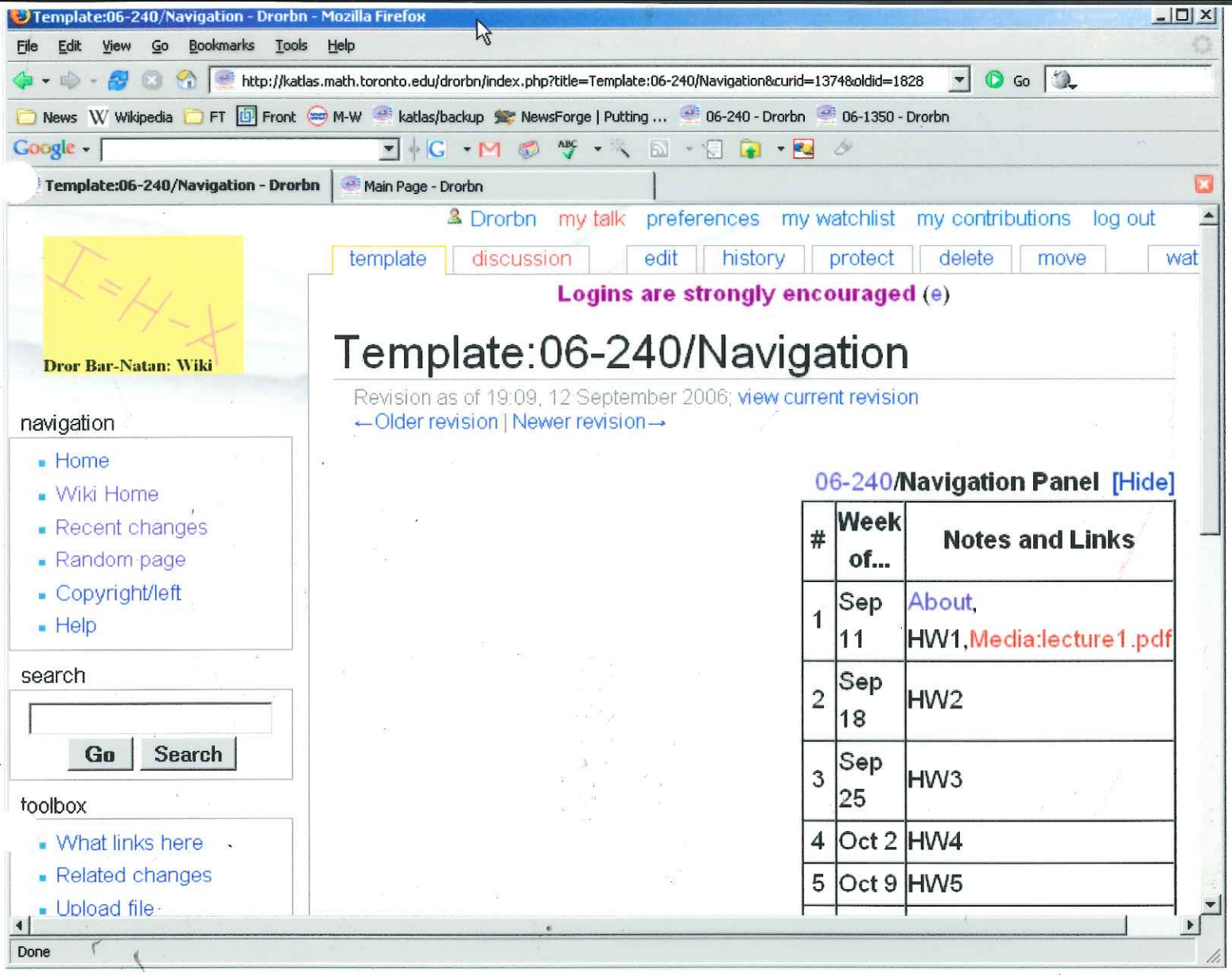
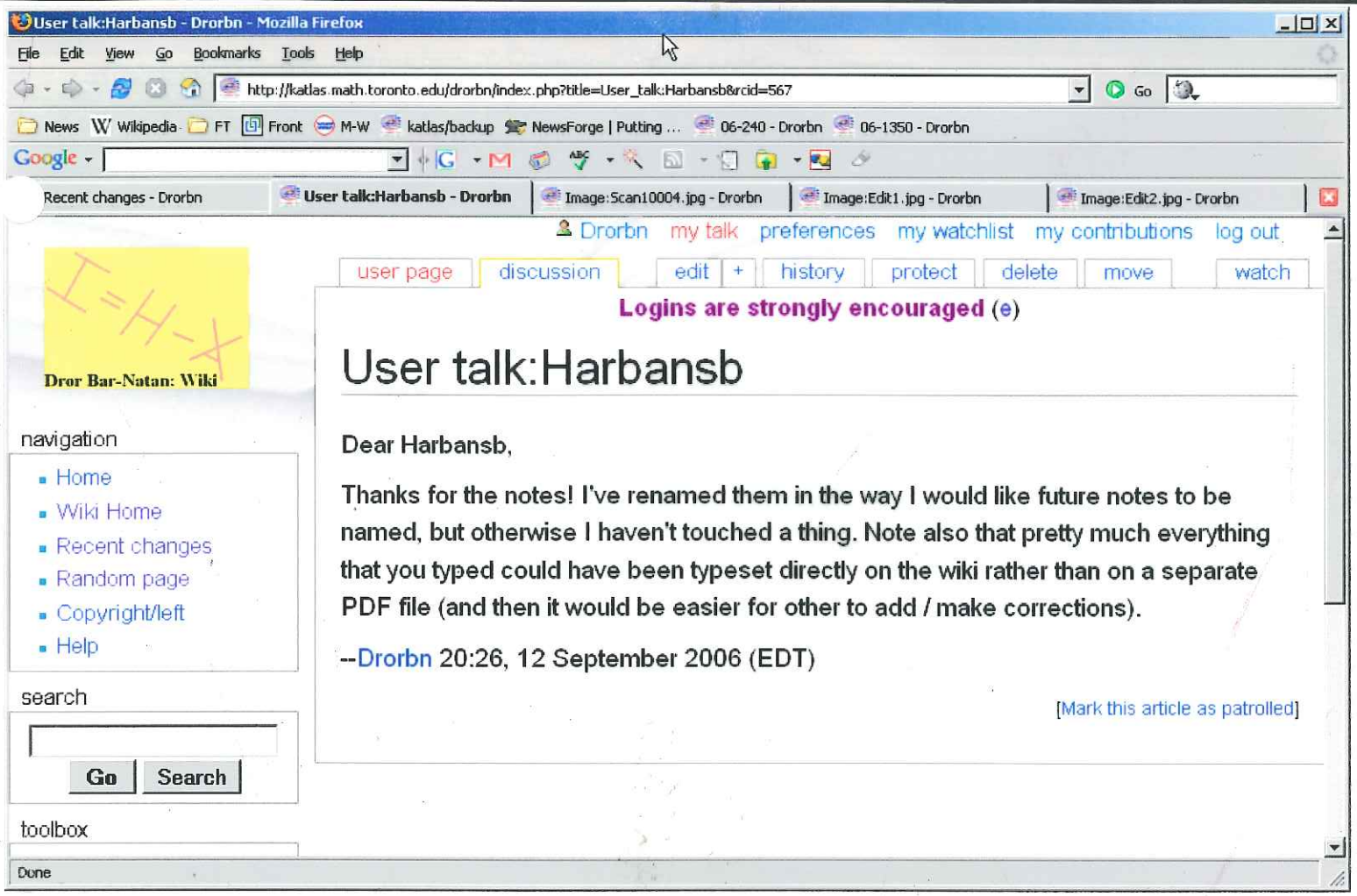


The Navigation Panel should remain clean and neat...



All page titles must start with 06-240 !

Use the wiki for typesetting!

Math 240 Algebra I, Tue Sep 19 2006, hours 4-5

on board:
- F1 Commutativity
- F2 Associativity
- F3 Units
- F4 Inverses
- F5 Distributivity

$\mathbb{C} = \{(a,b) : a,b \in \mathbb{R}\}$

$0_{\mathbb{C}} = (0,0)$ $1_{\mathbb{C}} = (1,0)$

$(a,b)+(c,d) := (a+c, b+d)$

$(a,b)\cdot(c,d) := (ac-bd, ad+bc)$

F1 F2 F3 F5 are mechanical. F4: $-(a,b) := (-a,-b)$

$(a,b)^{-1} := \left(\frac{a}{a^2+b^2}, \frac{-b}{a^2+b^2}\right)$

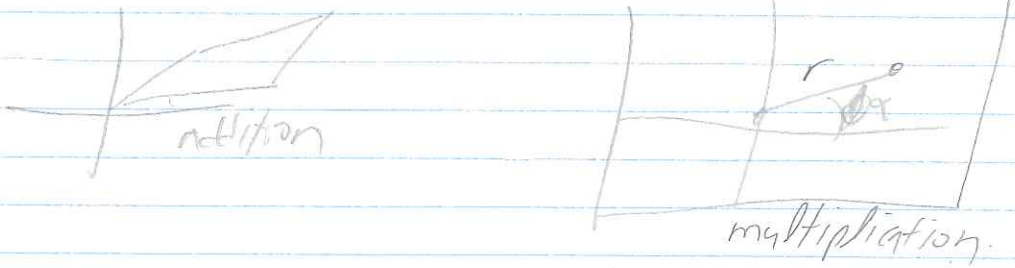
$(0,1) \sim i$ $(a,0) \sim a$ $(a,b) = a+bi$

If $z = a+bi$, define "the conjugate of z" to be $\bar{z} := a-bi$

$\overline{(3+7i)} \cdot \overline{3-7i} = 3+7i$

$z \cdot \bar{z} = |z|^2$ $z^{-1} = \frac{\bar{z}}{|z|^2}$

The geometrical picture



Claim Let $n > 1$ and x by integers. Then there are unique integer q (quotient) and r (remainder) s.t. $x = n \cdot q + r$ and $0 \le r \le n-1$

Denote $r = x \bmod n$ (rem)

The Field $\mathbb{Z}_p$

Example: $\mathbb{Z}/5$. Cont....

Thu sep 21, hour 6.

cont. { forces & areas
a motivation for vector spaces

VS1 $x+y=y+x$
VS2 Ass
VS3 $x+0=x$
VS4 $\forall x \exists y\ x+y=0$
VS5 $1x=x$
VS6 $a(bx)=(ab)x$
VS7 $a(x+y)=\ldots$
VS8 $(a+b)x=\ldots$

Examples: 1. F^n

2. $M_{m\times n}(F)$

3. ~~$\mathcal{F}(S,F)$~~ $\mathcal{F}(S,F)$ on S a set.

4. polynomials.

5. $\mathbb{C}/\mathbb{R}$

Thm Cancellation law

Thm 0 is unique

Thm negatives are unique

Thm 1. $0x=0$

2. $(-a)x=-ax=a(-x)$

3 $a\cdot 0=0$.

* APUS
* Announce Picture Day!

Math 240 Algebra I, Tue Sep 26 2006, hours 7-8.

on board

$a, b, c \in F$
$x, y, z \in V$

VS1 $x+y=y+x$
VS2 Assoc
VS3 $x+0=0$
VS4 $\forall x \exists y \; x+y=0$
VS5 $1x=x$
VS6 $a(bx)=(ab)x$
VS7 $a(x+y)$
VS8 $(a+b)x$

Examples:

1. F^n
2. $M_{m\times n}(F)$
3. $\mathbb{C}/\mathbb{R}$, $\mathbb{R}/\mathbb{Q}$
4. $\mathcal{F}(S, F)$

one further example: Polynomials

Thm Cancellation law Thm 0 is unique

Thm negatives are unique Thm $0x=0$
$(-a)x=-(ax)=a(-x)$
$a\cdot 0=0$.

Def $W \subset V$ a subspace if it is a v.s. with the ops it inherits from V.

Thm $W \subseteq V$ is a subspace iff it is "closed under addition & multiplication by scalars"; i.e., if ...

Example 1. $\{A \in M_{n\times n}(F) : A^t = A\}$
2. $\{A \in M_{n\times n}(F) : \operatorname{tr} A = 0\}$

Thm The intersection of two subspaces is a subspace.

Example The union isn't necessarily a subspace.

Thu ~~Sep 28~~, hour 9

Def Let $u_1 \ldots u_n$ be in V, and let $v \in V$. We say that v is a "linear combination" of the $\{u_i\}$ if $v = \sum a_i u_i$ for some $a_i \in F$.

.../cont.

Thm Let $u_i \in V$ be some vectors Then

$$W = \text{span}(u_i) = \{\text{all linear combinations of the } u_i\}$$

is a subspace of V.

Example Vitamins from the book.

Example In $P_3(\mathbb{R})$,

$$2x^3 - 2x^2 + 12x - 6$$

is a l.c. of

$$x^3 - 2x^2 - 5x - 3 \text{ and } 3x^3 - 5x^2 + 4x - 9$$

but $3x^3 - 2x^2 + 7x + 9$ isn't.

06-240/Class Photo

From Drorbn

Our class on September 28, 2006:

Class Photo: click to enlarge

Please identify yourself in this photo! There are two ways to do that:

- Log in to this Wiki and edit this page. Put your name, userid, email address and location in the picture in the alphabetical list below.
- Send Dror an email message with this information.

The first option is more fun but less private.

06-240/Navigation Panel [Hide]

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Register of Good Deeds

Add your name / see who's in!

Who We Are

First name	Last name	UserID	Email	In the photo	Comments
Dror	Bar-Natan	Drorbn	drorbn@ math.toronto.edu	facing everybody, as the photographer	Take this entry as a model and leave it first. Otherwise alphabetize by last name. Feel free to leave some fields blank
Mick	Carberry	MC	Mick.Carberry@utoronto.ca	long haired, bearded old guy in back	
Richard	Cerezo	Cerezo	richard.cerezo@utoronto.ca	Guy in black jacket and black hat on far right, second from the bottom.	

*Class photo is online!

Math 240 Algebra I, Tue Oct 3 2006, hours 10-11

on board:

Goal for next ~~few~~ weeks:

All vector spaces and the "same" as F^n

1. "Same"?
2. So what?
3. So why bother with vector spaces?
4. How is it proved?

Is 86415 divisible by 7?

$$\begin{array}{r} 86415 \\ -70 \\ \hline 1641 \\ -14 \\ \hline 261 \\ -14 \\ \hline 121 \\ -7 \\ \hline 56 \\ -56 \\ \hline 0 \end{array}$$

Yes

1. How come it worked? (use $x, t, =$ in $\mathbb{Z}/7$)
2. Do it for 17! (use $\div$ in $\mathbb{Z}/17$)

$[e_i = \begin{pmatrix} 0 \\ i \\ 0 \end{pmatrix}$ form a "basis" of $F^n]$

---

on board:

Example In $P_3(\mathbb{R})$, $v_1 = 2x^3 - 2x^2 + 12x - 6$ is a linear combination of

$u_1 = x^3 - 2x^2 - 5x - 3$ and $u_2 = 3x^3 - 5x^2 - 4x - 9$

but $v_2 = 3x^3 - 2x^2 + 7x + 8$ isn't.

See harder examples in hw

Def $S \subset V$ "generates" or "spans" V.

Examples In $M_{2\times 2}(\mathbb{R}) = V$ $M_1 = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$ $M_2 = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$ $M_3 = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}$ $M_4 = \begin{pmatrix} 0 & 0 \\ 0 & 1 \end{pmatrix}$

$N_1 = \begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix}$ $N_2 = \begin{pmatrix} 1 & 0 \\ 1 & 1 \end{pmatrix}$ $N_3 = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix}$ $N_4 = \begin{pmatrix} 1 & 1 \\ 1 & 0 \end{pmatrix}$

Then $M_1 - M_4$ & $N_1 - N_4$ generate V, but $M_1 - M_3$ ($N_1 - N_3$) do not.

A subset S is "linearly dependent" if it is "wasteful" i.e., if $\exists\ a_i \in F$ not all 0, & $u_i \in S$ s.t. $\sum a_i u_i = 0$.

Otherwise S is "linearly independent".

Examples $\{e_i\}$, $\left\{\begin{pmatrix} 1 \\ 3 \end{pmatrix}, \begin{pmatrix} 2 \\ 6 \end{pmatrix}, \begin{pmatrix} 7 \\ 9 \end{pmatrix}\right\}$ ..../cont.

Comments 1. $\emptyset$ is lin. indep.

2. $\{u\}$ is lin. indep iff $u \neq 0$.

Thurs, hour 12

3. Suppose $S_1 \subset S_2 \subset V$
   a. if S_1 is dep., so is S_2
   b. if S_2 is indep., so is S_1.

4. If S is lin. indep. in V, and $v \in V$ is not in S. Then $S \cup \{v\}$ is lin. dep. iff. $v \in \text{span}(S)$.

Def Basis $\beta \subset V$

examples 1. $\emptyset$ for $\{0\}$

2. e_i for F^n

3 E^{ij} for $M_{m \times n}(F)$

4. $(1, x, \ldots, x^n)$ for $P_n(F)$

4. $\{1, x, x^2, \ldots\}$ for $P(F)$

Thm A subset $\beta \subset V$ is a basis if every $v \in V$ can be expressed in a unique way as a l.c. of elements in β.

TABLE 1.1 Vitamin Content of 100 Grams of Certain Foods

	A (units)	B_1 (mg)	B_2 (mg)	Niacin (mg)	C (mg)
Apple butter	0	0.01	0.02	0.2	2
Raw, unpared apples (freshly harvested)	90	0.03	0.02	0.1	4
Chocolate-coated candy with coconut enter	0	0.02	0.07	0.2	0
Clams (meat only)	100	0.10	0.18	1.3	10
Cupcake from mix (dry form)	0	0.05	0.06	0.3	0
Cooked farina (unenriched)	(0)[a]	0.01	0.01	0.1	(0)
Jams and preserves	10	0.01	0.03	0.2	2
Coconut custard pie (baked from mix)	0	0.02	0.02	0.4	0
Raw brown rice	(0)	0.34	0.05	4.7	(0)
Soy sauce	0	0.02	0.25	0.4	0
Cooked spaghetti (unenriched)	0	0.01	0.01	0.3	0
Raw wild rice	(0)	0.45	0.63	6.2	(0)

Source: Bernice K. Watt and Annabel L. Merrill, *Composition of Foods* (Agriculture Handbook Number 8), Consumer and Food Economics Research Division, U.S. Department of Agriculture, Washington, D.C., 1963.

[a]Zeros in parentheses indicate that the amount of a vitamin present is either none or too small to measure.

Wikipedia Image

The vitamin content of 100 grams of each food can be recorded as a column vector in R^5—for example, the vitamin vector for apple butter is

$$\begin{matrix} A \\ B_1 \\ B_2 \\ N \\ C \end{matrix}\begin{pmatrix} 0.00 \\ 0.01 \\ 0.02 \\ 0.20 \\ 2.00 \end{pmatrix}.$$

apple butter

Considering the vitamin vectors for cupcake, coconut custard pie, raw brown rice, soy sauce, and wild rice, we see that

$$\begin{matrix} A \\ B_1 \\ B_2 \\ N \\ C \end{matrix}\begin{pmatrix} 0.00 \\ 0.05 \\ 0.06 \\ 0.30 \\ 0.00 \end{pmatrix} + \begin{pmatrix} 0.00 \\ 0.02 \\ 0.02 \\ 0.40 \\ 0.00 \end{pmatrix} + \begin{pmatrix} 0.00 \\ 0.34 \\ 0.05 \\ 4.70 \\ 0.00 \end{pmatrix} + 2\begin{pmatrix} 0.00 \\ 0.02 \\ 0.25 \\ 0.40 \\ 0.00 \end{pmatrix} = \begin{pmatrix} 0.00 \\ 0.45 \\ 0.63 \\ 6.20 \\ 0.00 \end{pmatrix}.$$

cupcake, coc. pie, brown rice, soy sauce, wild rice

Thus the vitamin vector for wild rice is a linear combination of the vitamin vectors for cupcake, coconut custard pie, raw brown rice, and soy sauce. So 100 grams of cupcake, 100 grams of coconut custard pie, 100 grams of raw brown rice, and 200 grams of soy sauce provide exactly the same amounts of the five vitamins as 100 grams of raw wild rice. Similarly, since

$$2\begin{pmatrix} 0.00 \\ 0.01 \\ 0.02 \\ 0.20 \\ 2.00 \end{pmatrix} + \begin{pmatrix} 90.00 \\ 0.03 \\ 0.02 \\ 0.10 \\ 4.00 \end{pmatrix} + \begin{pmatrix} 0.00 \\ 0.02 \\ 0.07 \\ 0.20 \\ 0.00 \end{pmatrix} + \begin{pmatrix} 0.00 \\ 0.01 \\ 0.01 \\ 0.10 \\ 0.00 \end{pmatrix} + \begin{pmatrix} 10.00 \\ 0.01 \\ 0.03 \\ 0.20 \\ 2.00 \end{pmatrix} + \begin{pmatrix} 0.00 \\ 0.01 \\ 0.01 \\ 0.30 \\ 0.00 \end{pmatrix} = \begin{pmatrix} 100.00 \\ 0.10 \\ 0.18 \\ 1.30 \\ 10.00 \end{pmatrix},$$

apple butter, apples, choc., farina, jam, spag., clams

200 grams of apple butter, 100 grams of apples, 100 grams of chocolate candy, 100 grams of farina, 100 grams of jam, and 100 grams of spaghetti provide

HW: Page 34 ↦ pages 53-56

Math 240 Algebra I,
Thursday October 5 2006, hour 12.

on board.

From last time:

$M_1 = \begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$, ...

$N_1 = \begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix}$, ...

The M_i's generate $M_{2\times 2}$

$M_1 = (N_1 + N_2 + N_3 + N_4 - 3N_1)/3$

$\Rightarrow$ The M_i generate too.

A general fact: If $T \subset \text{span}\, S$ then $\text{span}\, T \subset \text{span}\, S$

"linear combs of linear combs are linear combs"

$S \subset V$ is lin indep $\Leftrightarrow$ whenever $u_i \in S$ are distinct, $\sum a_i u_i = 0 \Rightarrow a_i = 0$.

"no waste"

Comments:

1. $\emptyset$ is lin indep.
2. $\{u\}$ is lin indep iff $u \neq 0$.
3. Suppose $S_1 \subset S_2 \subset V$
   a. If S_1 is dep, so is S_2
   b. If S_2 is indep, so is S_1
4. If S is lin indep in V and $v \in V$ is not in S, then $S \cup \{v\}$ is lin dep iff $v \in \text{span}\, S$

Def A basis $\beta \subset V$

Examples
1. $\emptyset$ for $\{0\}$
2. $\{e_i\}$ for F^n
3. E^{ij} for $M_{m\times n}(F)$
4. $(x, \ldots x^n)$ for $P_n(F)$
5. $(1, x, x^2, \ldots)$ for $P(F)$.

Thm A subset $\beta \subset V$ is a basis iff every $v \in V$ can be expressed in a unique way as a l.c. of elements in β.

Thm If a finite set S generates a v.s. V, then there is a subset $\beta \subset S$ which is a basis of V.

Math 240 Algebra I, Tue Oct 10 2006, hours 13-14.

Our first non-language theorem:

Thm If a v.s. V has a finite basis, then every other basis of V has the same number of elements in it

Def If V has a finite basis, we say it is "finite-dimensional" and let $\dim V$ = the number of elements in (any) basis of V

Lemma (The replacement lemma)
$|G| = n$, $\operatorname{span} G = V$,
$|L| = m$, L lin indep
$\Rightarrow$ $\exists H \subset G$, $|H| = n-m$,
$\operatorname{span}(H \cup L) = V$

from before If a finite set S generates V, then there is a subset $B \subseteq S$ which is a basis of V.

Pf of thm from lemma

Pf of lemma

Thm Assume $\dim V = n$

a. If $\operatorname{span} G = V$, then $|G| \geq n$ and if $|G| = n$, then G is a basis.

b. If L is lin indep and $|L| = n$, then L is a basis

c. Every lin. indep subset of V can be extended to a basis.

Thm V f.d, $W \subseteq V$ a subspace $\Rightarrow$ W is f.d. & $\dim W \leq \dim V$

Thm In this case, every basis of W can be extended to a basis of V.

Math 240 Algebra I, Thu Oct 12 2006, hour 15

* Pictorial review of last class
* Subspaces etc.
* Lagrange Approx.
* every v.s. has a basis ??

} Agenda

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* on board { Last class involved many details add later but there was also some structure.

* claim Every lin. indep. set can be extended to a basis.

Thm V is f.d. $W \subset V$ a subspace $\Rightarrow$
1. W is f.d.
2. $\dim W \leq \dim V$
3. Every basis of W can be extended to a basis of V.

Thm Every v.s. has a basis
* depends on the "axiom of choice"
* A basis for $\mathbb{R}$ over $\mathbb{Q}$.

1. Who cares?
2. Do it.

Lagrange Let x_i, $i = 1, \ldots, n+1$ be distinct pts in $\mathbb{R}$,
Let y_i $i = 1, \ldots n+1$ be in $\mathbb{R}$.
can you find a polynomial $p \in P_n(\mathbb{R})$ s.t. $p(x_i) = y_i$?
is it unique?

Isomorphism

From Wikipedia, the free encyclopedia

For the term in sociology, see isomorphism (sociology).

In mathematics, an **isomorphism** (Greek:*isos* "equal", and *morphe* "shape") is a bijective map f such that both f and its inverse f^{-1} are homomorphisms, i.e. *structure-preserving* mappings.

Informally, an isomorphism is a kind of mapping between objects, which shows a relationship between two properties or operations. If there exists an isomorphism between two structures, we call the two structures **isomorphic**. In a certain sense, isomorphic sets are **structurally identical**, if you choose to ignore finer-grained differences that may arise from how they are defined.

According to Douglas Hofstadter:

> *"The word "isomorphism" applies when two complex structures can be mapped onto each other, in such a way that to each part of one structure there is a corresponding part in the other structure, where "corresponding" means that the two parts play similar roles in their respective structures."* (Gödel, Escher, Bach, p. 49)

Contents

Purpose

Isomorphisms are frequently used by mathematicians to save themselves work. If a good isomorphism can be found from a relatively unknown part of mathematics into some well studied division of mathematics, where many theorems are already proved, and many methods are already available to find answers, then the function can be used to map whole problems out of unfamiliar territory over to "solid ground" where the problem is easier to understand and work with.

Physical analogies

Here are some everyday examples of isomorphic structures:

- A solid cube made of wood and a solid cube made of lead are both solid cubes; although their matter differs, their

geometric structures are isomorphic.
- A standard deck of 52 playing cards with green backs and a standard deck of 52 playing cards with brown backs; although the colours on the backs of each deck differ, the decks are structurally isomorphic — if we wish to play cards, it doesn't matter which deck we choose to use.
- The Clock Tower in London (that contains Big Ben) and a wristwatch; although the clocks vary greatly in size, their mechanisms of reckoning time are isomorphic.
- A six-sided die and a bag from which a number 1 through 6 is chosen; although the method of obtaining a number is different, their random number generating abilities are isomorphic. This is an example of functional isomorphism, without the presumption of geometric isomorphism.

Practical example

The following are examples of isomorphisms from ordinary algebra.

- Consider the logarithm function: For any fixed base b, the logarithm function $\log_b$ maps from the positive real numbers $\mathbb{R}^+$ onto the real numbers $\mathbb{R}$, formally:
  $$\log_b : \mathbb{R}^+ \to \mathbb{R}$$
  This mapping is one-to-one and onto, that is, it is a bijection from the domain to the codomain of the logarithm function. In addition to being an isomorphism of sets, the logarithm function also preserves certain operations. Specifically, consider the group $(\mathbb{R}^+, \times)$ of positive real numbers under ordinary multiplication. The logarithm function obeys the following identity:
  $$\log_b(x \times y) = \log_b(x) + \log_b(y)$$
  But the real numbers under addition also form a group. So the logarithm function is in fact a group isomorphism from the group $(\mathbb{R}^+, \times)$ to the group $(\mathbb{R}, +)$.
- Consider the group $\mathbf{Z}_6$, the numbers from 0 to 5 with addition modulo 6. Also consider the group $\mathbf{Z}_2 \times \mathbf{Z}_3$, the ordered pairs where the x coordinates can be 0 or 1, and the y coordinates can be 0, 1, or 2, where addition in the x-coordinate is modulo 2 and addition in the y-coordinate is modulo 3. These structures are isomorphic under addition, if you identify them using the following scheme:
  (0,0) -> 0
  (1,1) -> 1
  (0,2) -> 2
  (1,0) -> 3
  (0,1) -> 4
  (1,2) -> 5
  or in general $(a,b) \to (3a + 4b) \bmod 6$. For example note that (1,1) + (1,0) = (0,1) which translates in the other system as 1 + 3 = 4. Even though these two sets "look" different, they are indeed **isomorphic**. More generally, the direct product of two cyclic groups $\mathbf{Z}_n$ and $\mathbf{Z}_m$ is cyclic if and only if n and m are coprime.

Abstract examples

A relation-preserving isomorphism

For example, if one object consists of a set X with an ordering $\leqslant$ and the other object consists of a set Y with an ordering

$\sqsubseteq$ then an isomorphism from X to Y is a bijective function $f : X \to Y$ such that

$$f(u) \sqsubseteq f(v) \text{ if and only if } u \leqslant v.$$

Such an isomorphism is called an *order isomorphism*.

An operation-preserving isomorphism

Suppose that on these sets X and Y, there are two binary operations $\star$ and $\Diamond$ which happen to constitute the groups $(X, \star)$ and $(Y, \Diamond)$. Note that the operators operate on elements from the domain and range, respectively, of the "one-to-one" and "onto" function f. There is an isomorphism from X to Y if the bijective function $f : X \to Y$ happens to produce results, that sets up a correspondence between the operator $\star$ and the operator $\Diamond$.

$$f(u) \Diamond f(v) = f(u \star v)$$

for all u, v in X.

Applications

In abstract algebra, two basic isomorphisms are defined:

- Group isomorphism, an isomorphism between groups
- Ring isomorphism, an isomorphism between rings. (Note that isomorphisms between fields are actually ring isomorphisms)

In Mathematical analysis, the Legendre transform maps hard differential equations into easier algebraic equations.

In universal algebra, one can provide a general definition of isomorphism that covers these and many other cases. For a more general definition, see category theory.

In graph theory, an isomorphism between two graphs G and H is a bijective map f from the vertices of G to the vertices of H that preserves the "edge structure" in the sense that there is an edge from vertex u to vertex v in G if and only if there is an edge from $f(u)$ to $f(v)$ in H. See graph isomorphism.

In linear algebra, an isomorphism can also be defined as a linear map between two vector spaces that is one-to-one and onto.

See also

- Automorphism
- Homomorphism
- Epimorphism
- Isomorphism class
- Monomorphism
- Morphism

- Isometry
- Metamorphic computer virus

External links

- Weisstein, Eric W., *Isomorphism (http://mathworld.wolfram.com/Isomorphism.html)* at MathWorld.

Retrieved from "http://en.wikipedia.org/wiki/Isomorphism"

Categories: Abstract algebra | Algebra | Category theory

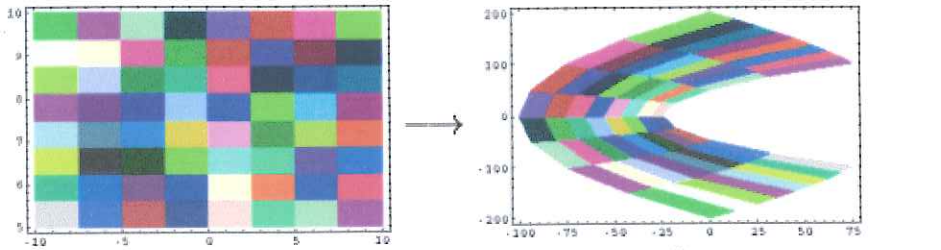
06-240/Linear Algebra - Why We Care

Contents [show]

06-240/Navigation Panel [Show]

Linear Algebra is the Small Scale Theory of *Everything* [edit]

- **To study the large, start with the small.**
- **In small scales, every space is a vector space.**
  - Indeed if you walk a mile east, a mile north, a mile west and a mile south, you're back were you started, but if you fly a 1,000 miles east, a 1,000 miles north, a 1,000 miles west and a 1,000 miles south, you're not back were you started (where will you be?).
- **In small scales, every function is a linear function.**



(code)

- **The world doesn't come with coordinates.**
  - Hence whenever we can we work without a basis, and when we do study bases, we study all of them.

Some Technical Remarks [edit]

The Term Test [edit]

Our one and only Term Test is coming up. It will take place in class on Tuesday October 24 2006, starting promptly at 1:10PM and ending at 3:00PM sharp. It will consist of 4-5 questions (each may have several parts) on everything that we will have covered in class by October 18: the axiomatic definition of fields and vector spaces, $\mathbb{Z}/p$ and other examples, spans, linear combinations and linear equations, linear dependence and independence, bases, the replacement lemma and its consequences, a bit about linear transformations and a few smaller topics that we touched but don't deserve their own headers.

Note my comments from 06-240/Homework Assignment 5:

Will there be "proof questions"?

Sure. What else have we done so far?

Do we need to know the proofs from class?

Sure. There's a reason why these proofs are in class to start with; if they weren't valuable, we wouldn't have covered them.

Note also that there may be some computations, but nothing that will require a calculator. Note also that I may include some questions from the homework assignments verbatim or nearly verbatim.

No electronic devices capable of displaying text or sounding speech will be allowed.

Extra Class Time [edit]

On Thursday October we will have a 2-hour class (1-3PM) to make up for the class time lost on the Term Test, and just one hour of orials (3-4PM) as no HW assignment is due on that week anyway.

Linear Algebra is the Small Scale Theory of *Everything*

[edit]

- **To study the large, start with the small.**
- **In small scales, every space is a vector space.**
  - Indeed if you walk a mile east, a mile north, a mile west and a mile south, you're back were you started, but if you fly a 1,000 miles east, a 1,000 miles north, a 1,000 miles west and a 1,000 miles south, you're not back were you started (where will you be?).
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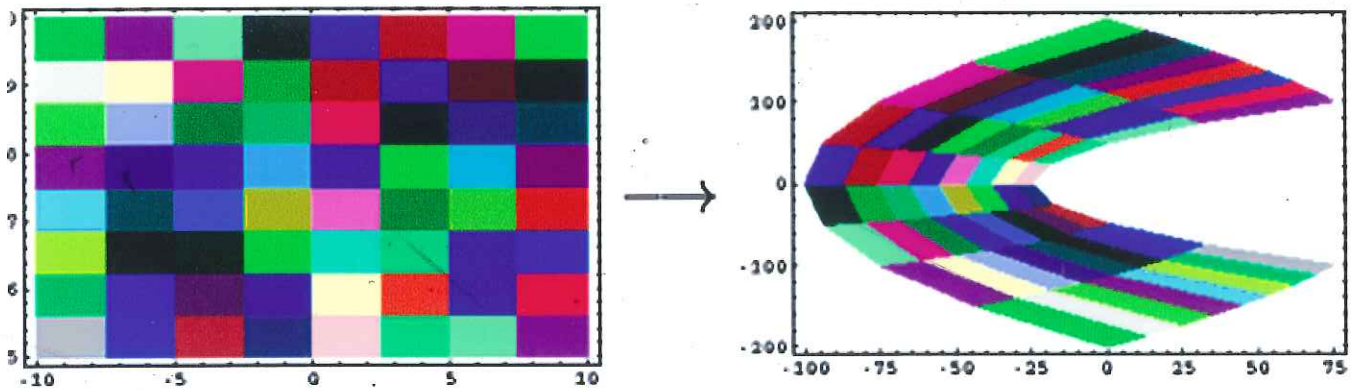


$z \mapsto z^2$ (code)

- **The world doesn't come with coordinates.**
  - Hence whenever we can we work without a basis, and when we do study bases, we study all of them.

ask to be reminded to go over handout 15 minutes before end.

Math 240 Algebra I, Tue Oct. 17 2006, hours 16-17

Linear Transformation... but first

Fishy thm: Every v.s. has a basis.

* $\mathbb{R}/\mathbb{Q}$: $\{1, \sqrt{2}, \pi, e, \ldots\}$ or $\{a_1, a_2, a_3, \pi, \ldots\}$

* $\{1, \pi, e\}$
* Lagrange
* Lin trans, examples, dim & iso.
* null space, range.
* nullity + rank = dim
* Propaganda
* TT

* Let x_i $i = 1 \ldots n+1$ be distinct pts in $\mathbb{R}$
  Let y_i $i = 1 \ldots n+1$ be in $\mathbb{R}$

Q: Can you find a polynomial $P \in P_n(\mathbb{R})$ s.t. $P(x_i) = y_i$? is it unique?

** who cares? 1. computer drawing programs
2. scientists

** Do it.

Let V & W be v.s. over the same field F.

A function $T: V \to W$ is a lin trans iff 1. $T(0) = 0$ 2. $T(x+y) = T(x) + T(y)$ 3. $T(cx) = cT(x)$

Annoying Claim: T is linear iff $\forall c, x, y$ $T(cx+y) = cT(x) + T(y)$

Silly claims: $T(x-y) = T(x) - T(y)$,

$T(\sum a_i x_i) = \sum a_i T(x_i)$

Examples 1. $T\begin{pmatrix} a_1 \\ a_2 \end{pmatrix} = \begin{pmatrix} 3a_1 - a_2 \\ 2a_1 + a_2 \end{pmatrix}$ $T: \mathbb{R}^2 \to \mathbb{R}^2$

2. $T(P) = P'$ $T: P_n(\mathbb{R}) \to P_{n-1}(\mathbb{R})$

3. T(keep) = counter clockwise by θ.

Thu Oct 19 hour 18.

Theorem If $(\alpha_i)_{i=1}^n$ is a basis of V and $w_i \in W$, there is a unique linear $T: V \to W$ s.t. ...finish

Cont.

$T(\alpha_i) = w_i$

Pf

Def V & W are "isomorphic" if...

Thm Any two vector spaces of dim n are isomorphic
In particular, all are isomorphic to F^n.

---

null space / kernel

range / image

Thm These are subspaces

Def nullity
rank

Thm nullity + rank = dim

159 348
168 357
249 456
258
267

2	9	4
7	5	3
6	1	8

*Return TT at end.

Math 240 Algebra I, Thu oct 26 20xx, hours 19-20

Plan: 1. All you can say about lin. trans. without fixing a basis,
2. Then choose a basis

Recall $T: V \to W$ is lin. trans if

$T(0)=0$
$T(x+y)=T(x)+T(y)$
$T(cx)=cT(x)$
claim
$T(cx+y)=cT(x)+T(y)$

Example
1. $0: V \to W$
2. $I_V: V \to V$

Def $N(T)=\ker T \quad R(T)=\mathrm{Im}(T)$

Prop $N(T) \subset V$ is a subspace; $\dim N(T) =: \mathrm{nullity}(T)$
$R(T) \subset W$ is a subspace. $\dim R(T) =: \mathrm{rank}(T)$

Examples 0, I_V, $D: P_n(\mathbb{R}) \to P_n(\mathbb{R})$

1-1, onto, invertible

Thm 1 $\mathrm{nullity}(T)+\mathrm{rank}(T)=\dim(V)$ (the dim theorem)

Thm 2 Given $T: V \to W$ and $T': V' \to W'$
s.t. $(\dim V, \dim W, \mathrm{rank}\, T) = (\dim V', \dim W', \mathrm{rank}\, T') = (n, m, r)$,
there is a "commutative square of isomorphisms"

$$\begin{array}{ccc} V & \xrightarrow{T} & W \\ \phi \downarrow & & \downarrow \psi \\ V' & \xrightarrow{T'} & W \end{array}$$

skippable

Cor of Thm 1 If $\dim V = \dim W$
TFAE
1. T is 1-1
2. T is onto
3. $\mathrm{rank}\, T = \dim(V)$
4. T is invertible.

... /cont.

Cont.

Let $\beta = (u_1 \dots u_n)$ be an <u>ordered</u> basis of a f.d. v.s. V. If $x = \sum a_i u_i$, write

$$[x]_\beta = \begin{pmatrix} a_1 \\ \vdots \\ a_n \end{pmatrix} \quad \left(= Tx, \text{ if } T \text{ is the iso. } V \to F^n \text{ given by } u_i \to e_i \right)$$

The coords of x rel. to β.

<u>Example</u> in $P_2(\mathbb{R})$

$$[x^2 - 2x + 3]_{(1, x, x^2)} = \begin{pmatrix} 3 \\ -2 \\ 1 \end{pmatrix}$$

<u>Def</u> Given $T: V \to W$ a lin trans, and ordered bases $\beta = (v_1 \dots v_n)$ of V & $\gamma = (w_1 \dots w_m)$ of W,

Let

$$A = [T]_\beta^\gamma = \left([Tv_1]_\gamma \middle| [Tv_2]_\gamma \middle| \dots \middle| [Tv_n]_\gamma \right) \in M_{m \times n}(F)$$

<u>Note</u> 1. T can be reconstructed from $[T]_\beta^\gamma$

2. Every matrix arises in this way.

<u>Examples</u> 2. $D: P_3(\mathbb{R}) \to P_2(\mathbb{R})$ by differentiation.

0:0 1:1

3 T_θ

<u>Def</u> $\mathcal{L}(V, W)$

<u>Claim</u> 1. $\mathcal{L}(V, W)$ is a v.s.

2. $T \longmapsto [T]_\beta^\gamma$ is an isomorphism of v.s.,

$$\mathcal{L}(V, W) \longrightarrow M_{m \times n}(F)$$

Math 240 Algebra I, Tue Oct 31 2006, hours 21-22

bases & matrices as planned for last time,

Composition & matrix multiplication.

Math 240 Algebra I, Thu Nov 2 2006 hour 23.

on board.

Abstract, general, coord. free. — merely numbers, choice dep., workable

$\mathcal{L}(V, W) \longrightarrow M_{m\times n}(F)$; dim = n, m ; basis β, γ

$T \longmapsto [T]_\beta^\gamma = A$

$T \to A$, $S \to B$, $T+S \to A+B$, ...

$A = \begin{pmatrix} a_{11} & & & a_{1n} \\ [Tv_1]_\gamma & [Tv_2]_\gamma & \cdots & [Tv_n]_\gamma \\ a_{m1} & & & a_{mn} \end{pmatrix} \Leftrightarrow \forall j \; Tv_j = \sum_{k=1}^{m} a_{kj} w_k$

---

derive

$U \xrightarrow[B]{S} V \xrightarrow[A]{T} W$, $T \circ S$

$C = A \cdot B$

$C_{ki} = A_{kj} B_{ji}$

example

$\begin{pmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ -1 & 1 \end{pmatrix} = \begin{pmatrix} & \end{pmatrix}$

Do $n\times n$ matrices form a field?

$w = Tv \qquad [v]_\beta = \bar{v} \qquad [w]_\gamma = \bar{w} \qquad [T]_\beta^\gamma = A$

claim

$\bar{w} = A \bar{v}$

($\bar{w}$: $m\times 1$, A: $m\times n$, $\bar{v}$: $n\times 1$)

Math 240 Algebra I, Tue Nov 7 2006 hours 24-25

on board:

$U \xrightarrow{S} V \xrightarrow{T} W$

$\alpha=(u_i)_{i=1}^{l}$ $B \in M_{n\times l}$ $\beta=(v_j)_{j=1}^{n}$ $A \in M_{m\times n}$ $\gamma=(w_k)_{k=1}^{m}$

$A=[T]_\beta^\gamma=(a_{kj})$ $Tv_j=\sum a_{kj}w_k$

$C=[T\circ S]_\alpha^\gamma=A\cdot B=(c_{ki}) \in M_{m\times l}$

$c_{ki}=\sum_{j=1}^{n} a_{kj}b_{ji}$ (*)

(A)(x)

The bad news about matrix algebra

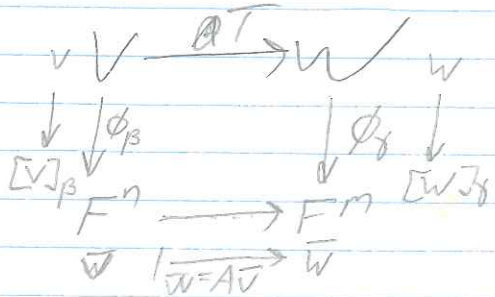
1. $A \cdot B$ defined only when dims match.
2. A^{-1} may not exist even if $A \neq 0$.
3. Generally $AB \neq BA$.

Today Goals:

1. The good news about matrix algebra
2. A computational interlude.

$w=Tv$ $[v]_\beta=\bar{v}$ $[w]_\gamma=\bar{w}$ $[T]_\beta^\gamma=A$

claim $\bar{w}=A\bar{v}$ (mx1 = mxn nx1)



pf of claim

The good news about matrix algebra:

1. $A+B=B+A$ $(A+B)+C=A+(B+C)$, ... (addition works just fine)
2. $A\cdot(B\cdot C)=(AB)C$, $A\cdot I=A$, $IA=A$,
   if $A\cdot A^{-1}=I$ then $A^{-1}A=I$
3. $(A+B)C=AC+BC$ $A(B+C)=AB+AC$

Computation 1. compute rank T/A

2. compute A^{-1} (when possible)
3. Solve systems of lin. eqns

Thm If P, Q are invertible, $\operatorname{rank} PTQ = \operatorname{rank} T$

---

elementary matrices & row/col ops

1. interchanges
2. mult. by a scalar
3. add a mult. of one row to another

Example on other side.

Example compute the rank of

$$\begin{pmatrix} 0 & 2 & 4 & 2 & 2 \\ 4 & 4 & 4 & 8 & 0 \\ 8 & 2 & 0 & 10 & 2 \\ 6 & 3 & 2 & 9 & 1 \end{pmatrix}$$

Logins are strongly encouraged (e)

Talk:06-240/Classnotes For Thursday November 9

< Talk:06-240

Does anyone have an intuitive way of understanding matrix multiplication? Specifically, why we take a column of B and a row of A for AxB? There are a few helpful indications of how this can be interpreted (in terms of linear transformations), but I was wondering if anyone had found a stronger (i.e. more natural/intuitive) way of justifying it to themselves. --Wanmike 13:44, 8 November 2006 (EST)

06-240/Classnotes For Thursday November 9

From Drorbn

Review of Last Class

06-240/Navigation Panel [Show]

Problem. Find the rank (the dimension of the image) of a linear transformation T whose matrix representation is the matrix A shown on the right.

$$A = \begin{pmatrix} 0 & 2 & 4 & 2 & 2 \\ 4 & 4 & 4 & 8 & 0 \\ 8 & 2 & 0 & 10 & 2 \\ 6 & 3 & 2 & 9 & 1 \end{pmatrix}$$

Theorem 1. If $T : V \to W$ is a linear transformation and $P : V \to V$ and $Q : W \to W$ are *invertible* linear transformations, then the rank of T is the same as the rank of QTP.

Proof. Owed.

Theorem 2. The following row/column operations can be applied to a matrix A by multiplying it on the left/right (respectively) by certain *invertible* "elementary matrices":

1. Swap two rows/columns
2. Multiply a row/column by a scalar.
3. Add a multiple of one row/column to another row/column.

Proof. Semi-owed.

Solution of the problem. using these (invertible!) row/column operations we aim to bring A to look as close as possible to an identity matrix, hoping it will be easy to determine the rank of the matrix we get at the end:

Do	Get	Do	Get
1. Bring a 1 to the upper left corner by swapping the first two rows and multiplying the first row (after the swap) by $1/4$.	$\begin{pmatrix} 1 & 1 & 1 & 2 & 0 \\ 0 & 2 & 4 & 2 & 2 \\ 8 & 2 & 0 & 10 & 2 \\ 6 & 3 & 2 & 9 & 1 \end{pmatrix}$	2. Add (-8) times the first row to the third row, in order to cancel the 8 in position 3-1.	$\begin{pmatrix} 1 & 1 & 1 & 2 & 0 \\ 0 & 2 & 4 & 2 & 2 \\ 0 & -6 & -8 & -6 & 2 \\ 6 & 3 & 2 & 9 & 1 \end{pmatrix}$
3. Likewise add (-6) times the first row to the fourth row, in order to cancel the 6 in position 4-1.	$\begin{pmatrix} 1 & 1 & 1 & 2 & 0 \\ 0 & 2 & 4 & 2 & 2 \\ 0 & -6 & -8 & -6 & 2 \\ 0 & -3 & -4 & -3 & 1 \end{pmatrix}$	4. With similar column operations (you need three of those) cancel all the entries in the first row (except, of course, the first, which is used in the canceling).	$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 2 & 4 & 2 & 2 \\ 0 & -6 & -8 & -6 & 2 \\ 0 & -3 & -4 & -3 & 1 \end{pmatrix}$
5. Turn the 2-2 entry to a 1 by multiplying the second row by $1/2$.	$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 2 & 1 & 1 \\ 0 & -6 & -8 & -6 & 2 \\ 0 & -3 & -4 & -3 & 1 \end{pmatrix}$	6. Using two row operations "clean" the second column; that is, cancel all entries in it other than the "pivot" 1 at position 2-2.	$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 2 & 1 & 1 \\ 0 & 0 & 4 & 0 & 8 \\ 0 & 0 & 2 & 0 & 4 \end{pmatrix}$
7. Using three column operations clean the second row except the pivot.	$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 4 & 0 & 8 \\ 0 & 0 & 2 & 0 & 4 \end{pmatrix}$	8. Clean up the row and the column of the 4 in position 3-3 by first multiplying the third row by $1/4$ and then performing the appropriate row and column transformations. Notice that by pure luck, the 4 at position 4-5 of the matrix gets killed in action.	$\begin{pmatrix} 1 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$

But the matrix we now have represents a linear transformation S satisfying $S(v_1, v_2, v_3, v_4 v_5) = (w_1, w_2, w_3, 0, 0)$ for some bases $(v_i)_{i=1}^5$ of V and $(w_j)_{j=1}^4$ of W. Thus the image (range) of S is spanned by $\{w_1, w_2, w_3\}$, and as these are independent, they form a basis of the image. Thus the rank of S is 3. Going backward through the "matrix reduction" process above and repeatedly using theorems 1 and 2, we find that the rank of T must also be 3.

Math 240 Algebra I, Thu Nov 9 2006, hour 26

* Go over "interpretation" question by Wanmike & over "review of last class" handout.

Math 240 Algebra I: The last 4 weeks, Fall 2006

Week 10: More about computations: Inverting matrices, solving systems of lin. eqn's, row echelon form.

Week 11: A very quick intro. to determinants.

Week 12: A very quick into to diagonalization.

Week 13: review

menu: 1. done lemma
2. a difficult principle on loan
3. An easy theorem
4. Corollaries
5. How far can you go with row ops. alone
6. inverting matrices

Math 240 Algebra I, Tue Nov 14 2006, hours 27-28.

Lemma If P & Q are invertible,

rank T = rank PT = rank TQ = rank PTQ

---

principle (to be revisited later) Changing a basis "is" multiplication by an invertible matrix lin. trans.

(hence rank A is well defined on a-priori grounds)

Thm Every matrix A can be row and column reduced to a block matrix of the form

$$\begin{pmatrix} I & 0 \\ 0 & 0 \end{pmatrix}$$

(so ranks can always be computed)

Claim rank A = rank A^T

Claim rank A = dim(col-space A) = dim(row-space(A))

Q: How far can you go with row ops only?

Ans: To "reduced row echelon form"

Claim: If B is in reduced row echelon form, rank B = number of non-zero rows.

Suppose A is n×n invertible & B is a r.r.e.f matrix row-equivalent to A. Then B is I.

$\Rightarrow E_4E_3E_2E_1A = I$ $\Rightarrow$ an algorithm for matrix inversion. → other side

Example Invert

$$\begin{pmatrix} 1 & 2 \\ 3 & 4 \end{pmatrix}$$

The Lost Sequence

I was playing around with sequences, and I thought I would screw around with the Shaw-Basho Polynomial:

$$\frac{1}{120}\left(42x^5 - 305x^4 + 1100x^3 - 895x^2 + 1018x + 480\right)$$

I plugged 0 into the polyomial to get 4, and then I plugged in 1 to get 12, etc. I got the following infinite sequence of numbers:

4 12 35 89 213 511 1194 2622 5346 10150 18093 ... (goes on forever)

Not too interesting, eh? Then I wrote out the differences of succeeding numbers in the sequence. For example, 12 - 4 = 8, 35 - 12 = 23, 89 - 35 = 54, etc.

8 23 54 124 298 683 1428 2624 4804 7943 ... (goes on forever)

I kept doing this process. The third sequence began 23 - 8 = 15, 54 - 23 = 31, etc. When you keep going, something completely unexpected happens! Here - I've done the work for you:

SEQUENCE 1: 4 12 35 89 213 511 1194 2622 5346 10150 18093 ... (goes on forever)
SEQUENCE 2: 8 23 54 124 298 683 1428 2624 4804 7943 12458... (goes on forever)
SEQUENCE 3: 15 31 70 174 385 745 1296 2080 3139 4515 6250... (goes on forever)
SEQUENCE 4: 16 39 104 211 360 551 784 1059 1376 1735 ... (goes on forever)
SEQUENCE 5: 23 65 107 149 191 233 275 317 359 ... (goes on forever)
SEQUENCE 6: 42 42 42 42 42 42 42 42 42 42... (goes on forever)
SEQUENCE 7: 0 0 0 0 0 0 0 (goes on forever)
SEQUENCE 8: 0 0 0 0 0 0 0 (goes on forever)
SEQUENCE 9: 0 0 0 0 0 0 0 (goes on forever)
SEQUENCE 10: 0 0 0 0 0 0 0 (goes on forever)
And it stays at zero forever. The sequence destroys itself.

NOW: Look at the first element of each sequence, and you have the LOST numbers. Weird, eh?

Lost Lost

2,1 Agenda 0. Wongpak 1. Lin eqn's & matrices. 2. Go as far as possible w/o row reduction. 3. go further with row reduction

Math 240 Algebra I, Thu Nov 16 2006, hour 29.

$(x,y)=2,1$

$$2x-7y=-3$$
$$-3x+2y=-4$$

$$a_{11}x_1+a_{12}x_2+\dots a_{1n}x_n=b_1$$
$$\vdots$$
$$a_{m1}x_1+\dots+a_{mn}x_n=b_m$$

$\Rightarrow Ax=b$ "inhomogenuous lin. eqn."

$Ax=0$ "homog" lin eqn

---

0. If we are lucky & A is square & invertible, $b=A^{-1}x$. Often we are lucky, but often we are not

---

homog case ~~# solutions iff $b\in\operatorname{ran}L_A$~~ ~~1. In that~~

1. 0 is always a sol.
2. x a sol'n iff $x\in$ null-space A. (ker)

non-homog case

1. Sol'n iff $b\in\operatorname{ran}A=$ col-space A
2. If x_0 solves then x_1 solves iff $x_1=x_0+x$ where x solves the homog version.

Solving $Ax=b \Leftrightarrow EAx=Eb$
$A'x=b'$

If A' is r.r.e.f

pivotal col. non-pivotal column.

non zero rows { pivots

zero rows {

$$\begin{pmatrix} 1 & 0 & & 0 & \\ & 0 & & 0 & \\ & 1 & & 0 & \\ & & & 1 & \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}=\begin{pmatrix} \\ \\ 0 \\ 0 \\ 0 \end{pmatrix}$$

Talk:06-240/Classnotes For Tuesday November 14

< Talk:06-240

Reduced row echelon form - Is there a reason to make column with entry 1 to the form of e_n (1 at n^{th} row, 0 for all other entries)? According to some books, matrix $\begin{pmatrix} 1 & 3 & 2 & 4 & 2 \\ 0 & 1 & 2 & 3 & 4 \\ 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$ is good enough to show that the rank of the matrix is 3. This is because the first three rows are linearly independent, they can't form linear combination for preceeding rows. Anyone could please explain why we have to reduce it to $\begin{pmatrix} 1 & 0 & -4 & 0 & -2 \\ 0 & 1 & 2 & 0 & -2 \\ 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$? Thank you. Wongpak 23:54, 15 November 2006 (EST)

Answer. For the purpose of figuring out the rank of a matrix, indeed there is no reason to go to "reduced row echelon form", and the slightly easier "row echelon form" (as in your first matrix above) is sufficient. But

1. "Row echelon form" doesn't enjoy the nicer characterization as "the most you can do with row reduction".
2. It is possible to prove (though most likely we won't) that the reduced row echelon form of a matrix is unique, while the row echelon form certainly isn't.
3. We did use the fact that we could get via row operation to the reduced row echelon form in our algorithm for matrix inversion.

--Drorbn 09:47, 16 November 2006 (EST)

4. We will use "reduced" r.e.f again today!

Math 240 Algebra I, Tue Nov 21 2006, hours 30-31

1. Go over Wongpak Matrices handout.

2. $A \longmapsto \det A$ $\quad |A| \in F$

1. Usefulness
2. Formula
3. "axiomatic" properties

1. Thm A^{-1} exists $\Leftrightarrow \det A \neq 0$.

2. Formula: $|(a_{11})| = a_{11}$

$$|A| = \sum_{j=1}^{n} (-1)^{1+j} A_{1j} |\tilde{A}_{1j}|$$

Examples $\begin{vmatrix} a & b \\ c & d \end{vmatrix} = ad - bc$

$$\begin{vmatrix} 1 & 2 & 3 \\ 4 & 5 & 6 \\ 7 & 8 & 9 \end{vmatrix} =$$

3. "Axiomatic" Properties:

Thm 1. $\det I = 1$

2. det is "multilinear" in the rows"

3. If A has two equal adjacent rows, $\det A = 0$.

Thm All there is to know about determinants can be learned from these three properties: If det' has same props, det = det'.

Proposition: 1. $\det(E_{ij} A) = \det A$; $\det E_{ij} = -1$

2. $\det E^2_{i,c} A = c \det A$; $\det E^2_{i,c} = c$ (also for $c=0$)

3. $\det E^3_{i,j,c} A = \det A$; $\det E^3_{i,j,c} = 1$

...follow side.

Cont.

Example Compute

$$\begin{vmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{vmatrix}$$

Proof of proposition

Proof of the axiomatic props

06-240/Classnotes For Tuesday November 21

From Drorbn

More about the Wongpak Matrices

06-240/Navigation Panel [Show]

In Talk:06-240/Classnotes_For_Tuesday_November_14, User:Wongpak asked something about row echelon form and reduced row echelon form, and gave the following matrices as specific examples:

$$A_1 = \begin{pmatrix} 1 & 3 & 2 & 4 & 2 \\ 0 & 1 & 2 & 3 & 4 \\ 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \qquad A_2 = \begin{pmatrix} 1 & 0 & -4 & 0 & -6 \\ 0 & 1 & 2 & 0 & -2 \\ 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

So let us assume row reduction leads us to the systems $A_1 x = b$ or $A_2 x = b$. What does it tell us about the solutions? Let us start from the second system:

$$\begin{pmatrix} 1 & 0 & -4 & 0 & -6 \\ 0 & 1 & 2 & 0 & -2 \\ 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{pmatrix} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{pmatrix} \quad \text{or} \quad \begin{aligned} x_1 \quad -4x_3 \quad -6x_5 &= b_1 \\ x_2 \; +2x_3 \quad -2x_5 &= b_2 \\ x_4 \; +2x_5 &= b_3 \\ 0 &= b_4 \end{aligned}$$

Well, quite clearly if $b_4 \neq 0$ this system has no solutions, but if $b_4 = 0$ it has solutions no matter what b_1, b_2 and b_3 are. Finally, for any given values of b_1, b_2 and b_3 we can choose the values of x_3 and x_5 (the variables corresponding the columns containing no pivots) as we please, and then get solutions by setting the "pivotal variables" in terms of the non-pivotal ones as follows: $x_1 = b_1 + 4x_3 + 6x_5$, $x_2 = b_2 - 2x_3 + 2x_5$ and $x_4 = b_3 - 2x_5$.

What about the system corresponding to A_1? It is

$$\begin{pmatrix} 1 & 3 & 2 & 4 & 2 \\ 0 & 1 & 2 & 3 & 4 \\ 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{pmatrix} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \\ b_4 \end{pmatrix} \quad \text{or} \quad \begin{aligned} x_1 \; +3x_2 \; +2x_3 \; +4x_4 \; +2x_5 &= b_1 \\ x_2 \; +2x_3 \; +3x_4 \; +4x_5 &= b_2 \\ x_4 \; +2x_5 &= b_3 \\ 0 &= b_4 \end{aligned}$$

Here too we have solutions iff $b_4 = 0$, and if $b_4 = 0$, we have the freedom to choose the non-pivotal variables x_3 and x_5 as we please. But now the formulas for fixing the pivotal variables x_1, x_2 and x_4 in terms of the non-pivotal ones are a bit harder.

Math 240 Algebra I, Thu Nov 23 2006, hour 32.

"Axiomatic" properties of det

1. $\det I = 1$
2. det is "multilinear in the rows"
3. If A has two equal adjacent rows, $\det A = 0$

First: $|A| = \sum_{j=1}^{n} (-1)^{i+j} A_{ij} |\widetilde{A_{ij}}|$

Thm 2. A is invertible $\Leftrightarrow$ $|A|$ is invertible

Thm 1. det is fully computable using row operations.

Proof of the props

More properties:

$\det(AB)$; $\det(A^{-1})$; $\det A^T$; columns; volume.

Math 240 Algebra I, Tue Nov 28 2006, hours 33-34

on board

Today's agenda:

Dets 1. $\det(AB) = \det(A)\det(B)$

Dets 2. $\det(A^{-1}) = (\det A)^{-1}$

3. $\det A^T$ & columns

4. Volumes in a word.

Nxt: The reproductive biology of rabbits. & powers.

Already know:

If E is elementary, $\det(EA) = (\det E)\cdot(\det A)$

Proof of 1 Annoying case A or B not invertible

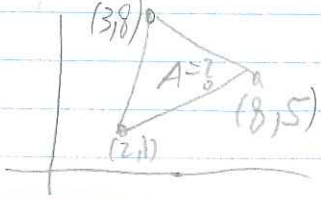
Interesting case $A = E_1^A E_2^A \cdots E_n^A \cdot I$

$B = E_1^B E_2^B \cdots E_m^B I$ - - -

Pf of 2

Pf of 3 $(AB)^T = B^T A^T$; $\det E^T = \det E$; Thus all that was true for rows is true for cols.

Pf of 4 Just the state & example:



Problem Given T, compute T^{365}

"knowing tomorrow in terms of today, compute a year ahead".

Example 1 Compute $\begin{pmatrix} 2 & 0 \\ 0 & -1 \end{pmatrix}^{365}$; $\begin{pmatrix} 8 & 9 \\ -6 & -7 \end{pmatrix}^{365}$

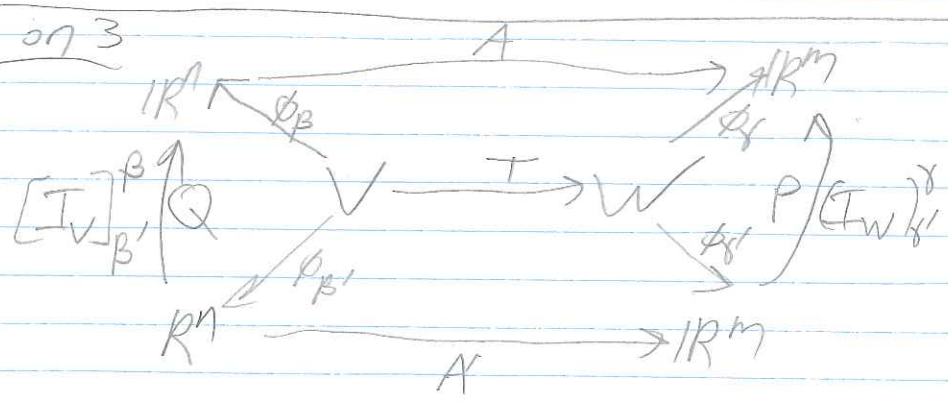
Example 2 The reproductive biology of rabbits

Actually do 1 & 2!

Math 240 Algebra I, Thu Nov 30 2006 hour 35

1. * A word about complexity & determinants (transparency)
2. * Finish an eigval/vec example
3. * Change of basis
4. * $\begin{pmatrix} 8 & 9 \\ -6 & -7 \end{pmatrix}^{365}$, $\begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix}^{n}$, Fibonacci. (next time)

More on 3



"A commutative diagram"

More on 2

$Tv = \lambda v \Leftrightarrow$

$Av = \lambda v \Leftrightarrow$

$(A - \lambda I)v = 0 \overset{\text{tough}}{\Leftrightarrow} \det(A - \lambda I) = 0$

$$\det\left(\begin{pmatrix} 8 & 9 \\ -6 & -7 \end{pmatrix} - \lambda I\right) = \det\begin{pmatrix} 8-\lambda & 9 \\ -6 & -7-\lambda \end{pmatrix} = \lambda^2 - \lambda - 2 \quad \lambda_{1,2} = 2, -1$$

$$\left(\begin{pmatrix} 8 & 9 \\ -6 & -7 \end{pmatrix} - 2I\right)v_1 = 0 \Rightarrow \begin{pmatrix} 6 & 9 \\ -6 & -9 \end{pmatrix} v_1 = 0 \Rightarrow v_1 \propto \begin{pmatrix} 3 \\ -2 \end{pmatrix}$$

06-240/Classnotes For Tuesday December 5

06-240/Navigation Panel [Show]

Our remaining goal for this semester is to study the following theorem:

Theorem. Let A be an $n \times n$ matrix (with entries in some field F) and let $\chi_A(\lambda) := \det(A - \lambda I)$ be the characteristic polynomial of A. Assume χ_A has n distinct roots $\lambda_1 \ldots \lambda_n$, that is, A has n distinct eigenvalues $\lambda_1 \ldots \lambda_n$, and let $v_1, \ldots, v_n$ be corresponding eigenvectors, so that $Av_i = \lambda_i v_i$ for all $1 \leq i \leq n$. Let D be the diagonal matrix that has λ_1 through λ_n on its main diagonal (in order) and let P be the matrix whose columns are these eigenvectors: $P := (v_1 | v_2 | \cdots | v_n)$. Then P is invertible and the following equalities hold:

1. $D = P^{-1}AP$ and $A = PDP^{-1}$.
2. For any positive integer k we have $A^k = PD^kP^{-1}$ and
$$D^k = \begin{pmatrix} \lambda_1^k & & 0 \\ & \ddots & \\ 0 & & \lambda_n^k \end{pmatrix}.$$
3. Likewise if $F = \mathbb{R}$ and $\exp(B) := \sum_{k=0}^{\infty} \frac{B^k}{k!}$ then
$$\exp(A) = P\exp(D)P^{-1} \text{ and } \exp(D) = \begin{pmatrix} e^{\lambda_1} & & 0 \\ & \ddots & \\ 0 & & e^{\lambda_n} \end{pmatrix}.$$

Order of the proceedings.

1. Assuming P is invertible, a proof of 1.
2. Proof of 2.
3. Example - the "reproduction of rabbits" matrix $A = \begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix}$ (see the mathematica session below).
4. Discussion of 3.
5. The relationship with linear transformations and changes of basis.
6. v_1 thorough v_n form a basis and P is invertible.

Reproduction of Rabbits.nb *

In[1]:=
$$\{A = \begin{pmatrix} 0 & 1 \\ 1 & 1 \end{pmatrix};\ \chi = \text{Det}\left[A - \lambda \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\right],\ \{\lambda_1, \lambda_2\} = \lambda\ /.\ \text{Solve}[\chi == 0,\ \lambda]\}$$

Out[1]=
$$\left\{-1 - \lambda + \lambda^2,\ \left\{\frac{1}{2}\left(1 - \sqrt{5}\right),\ \frac{1}{2}\left(1 + \sqrt{5}\right)\right\}\right\}$$

In[2]:=
$$\{\text{Simplify}\left[A - \lambda_1 \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\right]\ //\ \text{MatrixForm},\ v_1 = \text{First}\left[\text{NullSpace}\left[A - \lambda_1 \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\right]\right]\}$$

Out[2]=
$$\left\{\begin{pmatrix} \frac{1}{2}\left(-1 + \sqrt{5}\right) & 1 \\ 1 & \frac{1}{2}\left(1 + \sqrt{5}\right) \end{pmatrix},\ \left\{\frac{1}{2}\left(-1 - \sqrt{5}\right),\ 1\right\}\right\}$$

In[3]:=
$$\{\text{Simplify}\left[A - \lambda_2 \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\right]\ //\ \text{MatrixForm},\ v_2 = \text{First}\left[\text{NullSpace}\left[A - \lambda_2 \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\right]\right]\}$$

Out[3]=
$$\left\{\begin{pmatrix} \frac{1}{2}\left(-1 - \sqrt{5}\right) & 1 \\ 1 & \frac{1}{2}\left(1 - \sqrt{5}\right) \end{pmatrix},\ \left\{\frac{1}{2}\left(-1 + \sqrt{5}\right),\ 1\right\}\right\}$$

In[4]:=
$$\text{MatrixForm}\ /@\ \{P = \text{Transpose}[\{v_1,\ v_2\}],\ \text{Inverse}[P],\ \text{Simplify}[P.\text{Inverse}[P]]\}$$

Out[4]=
$$\left\{\begin{pmatrix} \frac{1}{2}\left(-1 - \sqrt{5}\right) & \frac{1}{2}\left(-1 + \sqrt{5}\right) \\ 1 & 1 \end{pmatrix},\ \begin{pmatrix} -\frac{1}{\sqrt{5}} & -\frac{1-\sqrt{5}}{2\sqrt{5}} \\ \frac{1}{\sqrt{5}} & -\frac{-1-\sqrt{5}}{2\sqrt{5}} \end{pmatrix},\ \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\right\}$$

In[5]:=
$$\text{MatrixForm}\ /@\ \{B = \text{Simplify}[\text{Inverse}[P].A.P],\ Bn = \text{Simplify}[B\,\hat{}\,n,\ n > 0]\}$$

Out[5]=
$$\left\{\begin{pmatrix} \frac{1}{2}\left(1 - \sqrt{5}\right) & 0 \\ 0 & \frac{1}{2}\left(1 + \sqrt{5}\right) \end{pmatrix},\ \begin{pmatrix} \left(\frac{1}{2}\left(1 - \sqrt{5}\right)\right)^n & 0 \\ 0 & \left(\frac{1}{2}\left(1 + \sqrt{5}\right)\right)^n \end{pmatrix}\right\}$$

In[6]:=
$$(An = \text{Simplify}[P.Bn.\text{Inverse}[P]])\ //\ \text{MatrixForm}$$

Out[6]//MatrixForm=
$$\begin{pmatrix} \frac{1}{5} 2^{-1-n}\left(-\left(-5 + \sqrt{5}\right)\left(1 + \sqrt{5}\right)^n + \left(1 - \sqrt{5}\right)^n\left(5 + \sqrt{5}\right)\right) & \frac{2^{-n}\left(-\left(1-\sqrt{5}\right)^n + \left(1+\sqrt{5}\right)^n\right)}{\sqrt{5}} \\ \frac{-\left(\frac{1}{2}\left(1-\sqrt{5}\right)\right)^n + \left(\frac{1}{2}\left(1+\sqrt{5}\right)\right)^n}{\sqrt{5}} & \frac{1}{5} 2^{-1-n}\left(-\left(1 - \sqrt{5}\right)^n\left(-5 + \sqrt{5}\right) + (1 \end{pmatrix}$$

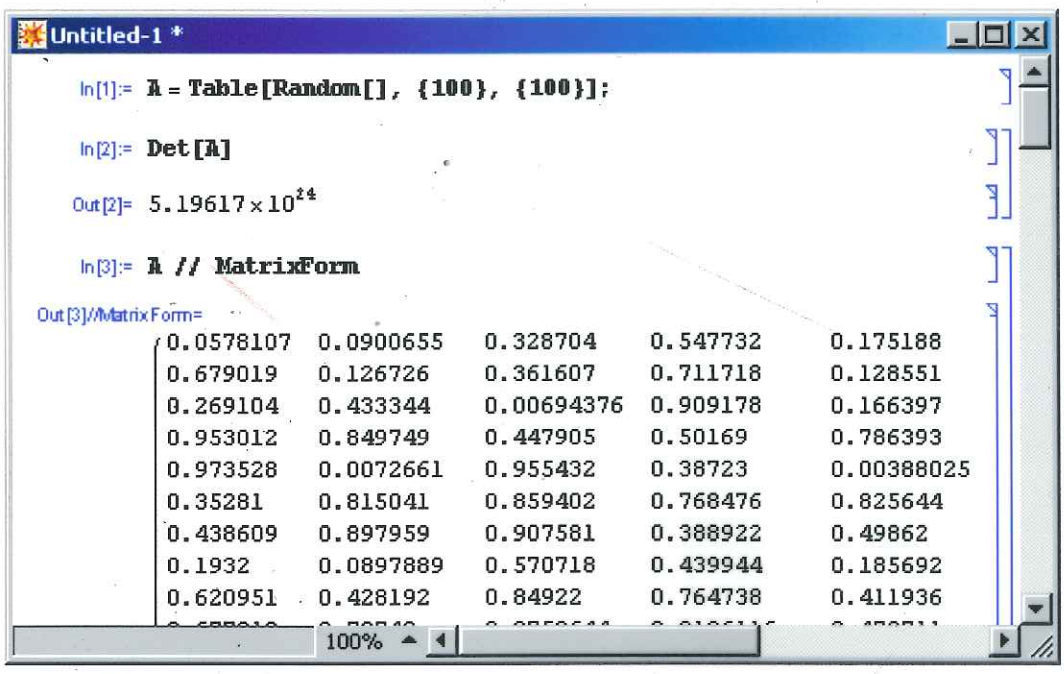
In[7]:=
$$\text{Simplify}[An\ /.\ n \to 7]\ //\ \text{MatrixForm}$$

Out[7]//MatrixForm=
$$\begin{pmatrix} 8 & 13 \\ 13 & 21 \end{pmatrix}$$

150%

Just for fun. A certain 100×100 matrix A of random numbers between 0 and 1 is fed into a computer called Golem, capable of about 10^9 arithmetic operations per second (between floating point numbers, at roughly 14 decimal digits of precision).

- Estimate how long it will take Golem to compute $\det A$ using the explicit recursive formula.
- As you may know, glass is really a liquid and it slowly flows with gravity. How many times will you need to replace your computer screen before the computation is done?
- Assuming you are ready to wait and shuffle screens, will you trust the results? (Remember that even if electrical power will be available to eternity and electronic components will never fail, every time a computer adds or multiplies two 14-digit numbers it makes a rounding error of size around 10^{-14}).
- Estimate how long it will take Golem to compute $\det A$ using row operations.
- Assuming you are ready to wait, will you trust the results (remembering the same comment as above)? How many screens will you go through this time?



Computed on Dror's laptop in a fraction of a second. The matrix is cropped, of course.

06-240/On The Final Exam

From Drorbn

Our final exam is coming up. It will take place at BN3 - room 3 of the Clara Benson Building, 320 Huron Street (south west of Harbord cross Huron, home of the Faculty of Physical Education and Health) on Wednesday December 13 from 2PM until 5PM. It will consist of 5-6 questions (each may have several parts) on everything that we will have covered in class this semester:

06-240/Navigation Panel [Show]

- Fields and vector spaces.
- Spans, independence, replacement and bases.
- Linear transformation, rank, nullity, matrices.
- Row and column reduction and elementary matrices, systems of linear equations.
- Determinants.
- Change of basis and diagonalization.
- Several other "smaller" topics.

As for the style -

- You can expect to be asked to reproduce some proofs that were given in class.
- You can expect some fresh things to prove, though generally not as hard as the previous type of proofs.
- You can expect questions (or parts of questions) that will be identical or nearly identical to questions that were assigned for homework.
- You can expect some calculations (but nothing that will require a calculator).

Basic calculators (not capable of displaying text or sounding speech) will be allowed but will not be necessary. You may wish to bring one nevertheless, as under pressure $5+7$ often comes out to be 13.

Remember. Neatness counts! Organization counts! Language counts! Proofs are best given as short and readable essays; without the English between the formulas one never knows how to interpret those formulas. When you write, say, "$v \in V$", does it mean "choose $v \in V$", or "we've just proven that $v \in V$", or "assume by contradiction that $v \in V$", or "for every $v \in V$" or "there exists $v \in V$"? If you don't say, your reader has no way of knowing. Also remember that long and roundabout solutions of simple problems, full of detours and irrelevant facts, are often an indication that their author didn't quite get the point, even if they are entirely correct. Avoid those!

Office hours. I (Dror) will hold two sessions of extended office hours at or near my office (Bahen 6178) before the final:

- On Tuesday December 12th (the day before), 1-4PM.
- On Wednesday December 13th (the day of), 10-12, just for last minute questions.

I would have loved to give more, but long before the final was scheduled I was asked to organize a session in a Canadian Mathematical Society conference on Saturday through Monday right before our exam. So unfortunately I will be completely unreachable on these three days.

- This page was last modified 11:41, 5 December 2006.

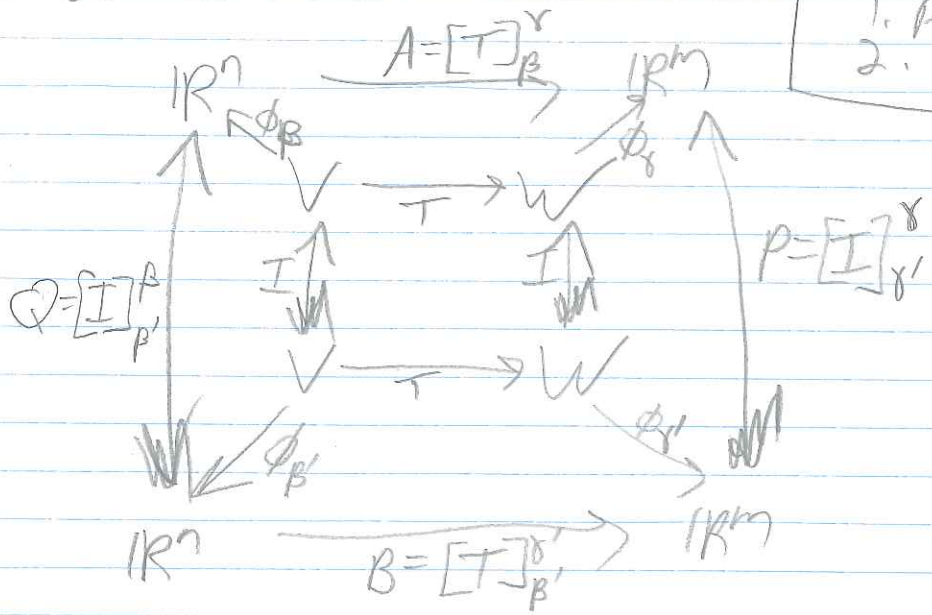
Math 240 Algebra I, Tue Dec 5 2006 hours 36-37

Remember: Follow handouts

Math 240 Algebra I, Thu Dec 7 2006, hour 38

If we had time....

Main goals:
1. Post Mortem
2. Evaluations



$$\Rightarrow A = PBQ^{-1} \qquad B = P^{-1}AQ$$

So what we saw last time was "change of basis"
(in the case $V=W$, $\beta=\beta'$, $\gamma=\gamma'$

Post Mortem: Classes / Tutorials / H-w / Tests / web / ~~Good books~~ / Text.
Material

UNIVERSITY OF TORONTO

Faculty of Arts and Sciences

DECEMBER EXAMINATIONS 2006

Math 240H1 Algebra I — Final Exam

Dror Bar-Natan

December 13, 2006

Solve all of the following 5 questions. The questions carry equal weight though different parts of the same question may be weighted differently.

Duration. You have 3 hours to write this exam.

Allowed Material. Basic calculators, not capable of displaying text or sounding speech.

Good Luck!

Problem 1. Prove the "replacement lemma": Let G be a set of g vectors in some vector space V and let L be some set of l linearly independent vectors in V (where g and l are both finite). Assume that $\operatorname{Span} L \subset \operatorname{Span} G$. Then $g \geq l$ and there is a subset R of G, consisting of $r := g - l$ vectors, so that $\operatorname{Span}(R \cup L) = \operatorname{Span} G$.

Problem 2.

1. Let $L : P_3(\mathbb{R}) \to \mathbb{R}^3$ be the linear transformation given by $L(p) = \begin{pmatrix} p(-2) \\ p(0) \\ p(2) \end{pmatrix}$. Find the matrix A representing L relative to the basis $\{1, x, x^2, x^3\}$ of $P_3(\mathbb{R})$ and the standard basis of $\mathbb{R}^3$.

2. Let $w = a + bi$ be a complex number and let $T : \mathbb{C} \to \mathbb{C}$ be defined by $T(z) = w \cdot z$. Considering $\mathbb{C}$ as a vector space over $\mathbb{R}$, find the matrix B representing T relative to the basis $\{1, i\}$ of $\mathbb{C}$.

Problem 3. Find all the solutions (if any exist) of the following two systems of linear equations:

$$\begin{pmatrix} -1 & 2 & -2 & -7 \\ -1 & 2 & 2 & 1 \\ -2 & 4 & 2 & -2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} -2 \\ 1 \\ -1 \end{pmatrix} \quad \text{and} \quad \begin{pmatrix} -1 & 2 & -2 & -7 \\ -1 & 2 & 2 & 1 \\ -2 & 4 & 2 & -2 \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \end{pmatrix} = \begin{pmatrix} 0 \\ -4 \\ -6 \end{pmatrix}.$$

Problem 4. Let $A \in M_{n \times n}(F)$ have the form

$$A = \begin{pmatrix} 0 & 0 & 0 & \cdots & 0 & a_0 \\ -1 & 0 & 0 & \cdots & 0 & a_1 \\ 0 & -1 & 0 & \cdots & 0 & a_2 \\ \vdots & \vdots & \vdots & & \vdots & \vdots \\ 0 & 0 & 0 & \cdots & -1 & a_{n-1} \end{pmatrix}.$$

1. Compute $\det(A + tI)$, where I is the $n \times n$ identity matrix.

2. (3 point bonus). What does your result tell you about characteristic polynomials?

Problem 5. Let A be the matrix $A = \begin{pmatrix} 1 & 0 \\ 3 & -2 \end{pmatrix}$.

1. Find a matrix P for which $P^{-1}AP$ is diagonal.

2. Compute A^7.

Good Luck!

STUDENT SURVEY FORM
UNIVERSITY OF TORONTO

FACULTY OF ARTS & SCIENCE

Note that survey results will be available to the instructor(s) only after final course marks have been submitted.

PART I: INSTRUCTIONS. PLEASE READ FIRST.

Using an HB pencil or a blue or black ball-point pen (but not a felt marking pen), fill completely the numbered oval corresponding to your response for each statement. If using a pen, do not alter original response by making another selection.

Part II *(on the reverse side)* requires a written answer.

Course Identification: Please print course and section you are evaluating

COURSE MAT240F SECTION

INSTRUCTOR(S):

A:	BAR-NATAN
B:	[redacted]

1. If evaluating only one instructor, write the name in the upper (A) box. If evaluating two instructors, write their names, one in box A and the other in box B.

DO NOT EVALUATE TEACHING ASSISTANTS ON THIS FORM

Statements about the instructor(s):

Respond to the statements below for instructor A (and instructor B) bearing in mind that there are wide variations in class size and subject matter in Arts and Science.

		extremely poor	very poor	poor	adequate	good	very good	outstanding
2. Communicates goals and requirements of the course clearly and explicitly.	A:	1	2	3	4	5	●	7
	B:	1	2	3	4	5	6	7
3. Uses methods of evaluation (e.g. papers, assignments, tests) that appropriately reflect the subject matter and provide a fair evaluation of student learning.	A:	1	2	3	4	5	●	7
	B:	1	2	3	4	5	6	7
4. Presents material in an organized, well-planned manner.	A:	1	2	3	4	5	●	7
	B:	1	2	3	4	5	6	7
5. Explains concepts clearly with appropriate use of examples.	A:	1	2	3	4	5	●	7
	B:	1	2	3	4	5	6	7
6. Communicates enthusiasm and interest in the course material.	A:	1	2	3	4	5	6	●
	B:	1	2	3	4	5	6	7
7. Attends to students' questions and answers them clearly and effectively.	A:	1	2	3	4	5	●	7
	B:	1	2	3	4	5	6	7
8. Is available for individual consultation, by appointment or stated office hours, to students with problems relating to the course.	A:	1	2	3	●	5	6	7
	B:	1	2	3	4	5	6	7
9. Ensures that student work is graded fairly, with helpful comments and feedback where appropriate.	A:	1	2	3	4	●	6	7
	B:	1	2	3	4	5	6	7
10. Ensures that student work is graded within a reasonable time.	A:	1	2	3	4	●	6	7
	B:	1	2	3	4	5	6	7
11. All things considered, performs effectively as a university teacher.	A:	1	2	3	4	5	●	7
	B:	1	2	3	4	5	6	7

PART I CONTINUES ON THE REVERSE SIDE

Statements about the course: Respond to the statements below, using the following 7-point scale. ***SIDE 2***

	very low	low	below average	average	above average	high	very high
12. Compared to other courses at the same level (100,200,300,400), the work load is	1	2	3	4	●	6	7
13. Compared to other courses at the same level, the level of difficulty of the material is	1	2	3	4	5	●	7
14. The value of the required reading is	1	2	3	4	5	●	7
15. (If applicable) The value of the tutorials is	1	2	3	4	5	●	7
16. (If applicable) The value of the laboratories is	1	2	3	4	5	6	7
17. (If applicable) The value of the seminars is	1	2	3	4	5	6	7
18. (If applicable) The value of the language conversation classes is	1	2	3	4	5	6	7
19. The value of the overall learning experience is	1	2	3	4	●	6	7

20. Considering your experience with this course, and disregarding your need for it to meet program or degree requirements, would you still have taken this course? ● Yes ○ No

Statements about yourself:

21. Number of full course credits already earned (prior to this session):
① 0-4½ ② 5-9½ ③ 10-14½ ④ 15-19½ ● ≥20

22. Status of the course for you:
① Program Requirement ● Selected from a required list in a program ③ Breadth Requirement ④ Optional

23. Your level of enthusiasm to take this course at the time of initial registration:
① low ② medium ● high

24. Your expected grade in this course:
① <50 ② 50-59 ③ 60-69 ④ 70-79 ● ≥80

Additional statements or questions which may be supplied in class:

25. ①②③④⑤⑥⑦	28. ①②③④⑤⑥⑦	31. ①②③④⑤⑥⑦	34. ①②③④⑤⑥⑦
26. ①②③④⑤⑥⑦	29. ①②③④⑤⑥⑦	32. ①②③④⑤⑥⑦	35. ①②③④⑤⑥⑦
27. ①②③④⑤⑥⑦	30. ①②③④⑤⑥⑦	33. ①②③④⑤⑥⑦	36. ①②③④⑤⑥⑦

PART II: PLEASE ANSWER ONLY AFTER COMPLETING PART I. Please use the space below to provide supplementary comments on the instructor(s) or course. For example, you may wish to give the reasons for your numerical evaluations or provide specific suggestions for improving the instruction in the course.

good

instructor was clear and enthusiastic

book was great

bad

too many "hand waving" proofs

too long on fields

not enough vector spaces / subspaces.

~~not there~~

END OF SURVEY. THANK YOU FOR YOUR COOPERATION AND PARTICIPATION.

STUDENT SURVEY FORM
UNIVERSITY OF TORONTO

FACULTY OF ARTS & SCIENCE

Note that survey results will be available to the instructor(s) only after final course marks have been submitted.

PART I: INSTRUCTIONS. PLEASE READ FIRST.

Using an HB pencil or a blue or black ball-point pen (but not a felt marking pen), fill completely the numbered oval corresponding to your response for each statement. If using a pen, do not alter original response by making another selection.

Part II *(on the reverse side)* requires a written answer.

Course Identification: Please print course and section you are evaluating

COURSE MAT240H1F SECTION

INSTRUCTOR(S):

A: D Bar-Natan

B:

1. **If evaluating only one instructor, write the name in the upper (A) box. If evaluating two instructors, write their names, one in box A and the other in box B.**

DO NOT EVALUATE TEACHING ASSISTANTS ON THIS FORM

Statements about the instructor(s):

Respond to the statements below for instructor A (and instructor B) bearing in mind that there are wide variations in class size and subject matter in Arts and Science.

		extremely poor	very poor	poor	adequate	good	very good	outstanding
2. Communicates goals and requirements of the course clearly and explicitly.	A:	1	2	3	4	5	●	7
	B:	1	2	3	4	5	6	7
3. Uses methods of evaluation (e.g. papers, assignments, tests) that appropriately reflect the subject matter and provide a fair evaluation of student learning.	A:	1	2	3	4	5	●	7
	B:	1	2	3	4	5	6	7
4. Presents material in an organized, well-planned manner.	A:	1	2	3	4	5	6	●
	B:	1	2	3	4	5	6	7
5. Explains concepts clearly with appropriate use of examples.	A:	1	2	3	4	5	●	7
	B:	1	2	3	4	5	6	7
6. Communicates enthusiasm and interest in the course material.	A:	1	2	3	4	5	6	●
	B:	1	2	3	4	5	6	7
7. Attends to students' questions and answers them clearly and effectively.	A:	1	2	3	4	●	6	7
	B:	1	2	3	4	5	6	7
8. Is available for individual consultation, by appointment or stated office hours, to students with problems relating to the course.	A:	1	2	3	4	●	6	7
	B:	1	2	3	4	5	6	7
9. Ensures that student work is graded fairly, with helpful comments and feedback where appropriate.	A:	1	2	3	4	●	6	7
	B:	1	2	3	4	5	6	7
10. Ensures that student work is graded within a reasonable time.	A:	1	2	3	4	●	6	7
	B:	1	2	3	4	5	6	7
11. All things considered, performs effectively as a university teacher.	A:	1	2	3	4	5	6	●
	B:	1	2	3	4	5	6	7

PART I CONTINUES ON THE REVERSE SIDE

Statements about the course: Respond to the statements below, using the following 7-point scale.

SIDE 2

	very low	low	below average	average	above average	high	very high
12. Compared to other courses at the same level (100,200,300,400), the work load is	1	2	3	4	●	6	7
13. Compared to other courses at the same level, the level of difficulty of the material is	1	2	3	●	5	6	7
14. The value of the required reading is	1	2	3	4	5	●	7
15. (If applicable) The value of the tutorials is	1	●	3	4	5	6	7
16. (If applicable) The value of the laboratories is	1	2	3	4	5	6	7
17. (If applicable) The value of the seminars is	1	2	3	4	5	6	7
18. (If applicable) The value of the language conversation classes is	1	2	3	4	5	6	7
19. The value of the overall learning experience is	1	2	3	4	5	6	●

20. Considering your experience with this course, and disregarding your need for it to meet program or degree requirements, would you still have taken this course? ● Yes ○ No

Statements about yourself:

21. Number of full course credits already earned (prior to this session):
① 0-4½ ② 5-9½ ● 10-14½ ④ 15-19½ ⑤ ≥20

22. Status of the course for you:
● Program Requirement ② Selected from a required list in a program ③ Breadth Requirement ④ Optional

23. Your level of enthusiasm to take this course at the time of initial registration:
① low ② medium ● high

24. Your expected grade in this course:
① <50 ② 50-59 ③ 60-69 ④ 70-79 ● ≥80

Additional statements or questions which may be supplied in class:

25. ① ② ③ ④ ⑤ ⑥ ⑦	28. ① ② ③ ④ ⑤ ⑥ ⑦	31. ① ② ③ ④ ⑤ ⑥ ⑦	34. ① ② ③ ④ ⑤ ⑥ ⑦
26. ① ② ③ ④ ⑤ ⑥ ⑦	29. ① ② ③ ④ ⑤ ⑥ ⑦	32. ① ② ③ ④ ⑤ ⑥ ⑦	35. ① ② ③ ④ ⑤ ⑥ ⑦
27. ① ② ③ ④ ⑤ ⑥ ⑦	30. ① ② ③ ④ ⑤ ⑥ ⑦	33. ① ② ③ ④ ⑤ ⑥ ⑦	36. ① ② ③ ④ ⑤ ⑥ ⑦

PART II: PLEASE ANSWER ONLY AFTER COMPLETING PART I. Please use the space below to provide supplementary comments on the instructor(s) or course. For example, you may wish to give the reasons for your numerical evaluations or provide specific suggestions for improving the instruction in the course.

- excellent book, excellent lectures
- Dror was the best Math Prof. I have had at U of T (I've had quite a few)

END OF SURVEY. THANK YOU FOR YOUR COOPERATION AND PARTICIPATION.

STUDENT SURVEY FORM
UNIVERSITY OF TORONTO

FACULTY OF ARTS & SCIENCE

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Part II *(on the reverse side)* requires a written answer.

Course Identification: Please print course and section you are evaluating

COURSE | M | A | T | 2 | 4 | 0 | H | 1 | F | SECTION | | | | | | |

INSTRUCTOR(S):

A:	Dror Bar Natan
B:	

1. If evaluating only one instructor, write the name in the upper (A) box. If evaluating two instructors, write their names, one in box A and the other in box B.

DO NOT EVALUATE TEACHING ASSISTANTS ON THIS FORM

Statements about the instructor(s):

Respond to the statements below for instructor A (and instructor B) bearing in mind that there are wide variations in class size and subject matter in Arts and Science.

		extremely poor	very poor	poor	adequate	good	very good	outstanding
2. Communicates goals and requirements of the course clearly and explicitly.	A:	1	2	3	4	5	6	●
	B:	1	2	3	4	5	6	7
3. Uses methods of evaluation (e.g. papers, assignments, tests) that appropriately reflect the subject matter and provide a fair evaluation of student learning.	A:	1	2	3	4	5	6	●
	B:	1	2	3	4	5	6	7
4. Presents material in an organized, well-planned manner.	A:	1	2	3	4	5	6	●
	B:	1	2	3	4	5	6	7
5. Explains concepts clearly with appropriate use of examples.	A:	1	2	3	4	5	6	●
	B:	1	2	3	4	5	6	7
6. Communicates enthusiasm and interest in the course material.	A:	1	2	3	4	5	6	●
	B:	1	2	3	4	5	6	7
7. Attends to students' questions and answers them clearly and effectively.	A:	1	2	3	4	5	6	●
	B:	1	2	3	4	5	6	7
8. Is available for individual consultation, by appointment or stated office hours, to students with problems relating to the course.	A:	1	2	3	4	5	6	●
	B:	1	2	3	4	5	6	7
9. Ensures that student work is graded fairly, with helpful comments and feedback where appropriate.	A:	1	2	3	4	5	6	●
	B:	1	2	3	4	5	6	7
10. Ensures that student work is graded within a reasonable time.	A:	1	2	3	4	5	6	●
	B:	1	2	3	4	5	6	7
11. All things considered, performs effectively as a university teacher.	A:	1	2	3	4	5	6	●
	B:	1	2	3	4	5	6	7

PART I CONTINUES ON THE REVERSE SIDE

Statements about the course: Respond to the statements below, using the following 7-point scale.

SIDE 2

	very low	low	below average	average	above average	high	very high
12. Compared to other courses at the same level (100,200,300,400), the work load is	1	2	3	●	5	6	7
13. Compared to other courses at the same level, the level of difficulty of the material is	1	2	3	4	5	●	7
14. The value of the required reading is	1	2	3	4	5	6	●
15. (If applicable) The value of the tutorials is	1	●	3	4	5	6	7
16. (If applicable) The value of the laboratories is	1	2	3	4	5	6	7
17. (If applicable) The value of the seminars is	1	2	3	4	5	6	7
18. (If applicable) The value of the language conversation classes is	1	2	3	4	5	6	7
19. The value of the overall learning experience is	1	2	3	4	5	●	7

20. Considering your experience with this course, and disregarding your need for it to meet program or degree requirements, would you still have taken this course? ● Yes ○ No

Statements about yourself:

21. Number of full course credits already earned (prior to this session):
① 0-4½ ● 5-9½ ③ 10-14½ ④ 15-19½ ⑤ ≥20

22. Status of the course for you:
● Program Requirement ② Selected from a required list in a program ③ Breadth Requirement ④ Optional

23. Your level of enthusiasm to take this course at the time of initial registration:
① low ● medium ③ high

24. Your expected grade in this course:
① <50 ② 50-59 ③ 60-69 ④ 70-79 ● ≥80

Additional statements or questions which may be supplied in class:

25. ① ② ③ ④ ⑤ ⑥ ⑦	28. ① ② ③ ④ ⑤ ⑥ ⑦	31. ① ② ③ ④ ⑤ ⑥ ⑦	34. ① ② ③ ④ ⑤ ⑥ ⑦
26. ① ② ③ ④ ⑤ ⑥ ⑦	29. ① ② ③ ④ ⑤ ⑥ ⑦	32. ① ② ③ ④ ⑤ ⑥ ⑦	35. ① ② ③ ④ ⑤ ⑥ ⑦
27. ① ② ③ ④ ⑤ ⑥ ⑦	30. ① ② ③ ④ ⑤ ⑥ ⑦	33. ① ② ③ ④ ⑤ ⑥ ⑦	36. ① ② ③ ④ ⑤ ⑥ ⑦

PART II: PLEASE ANSWER ONLY AFTER COMPLETING PART I. Please use the space below to provide supplementary comments on the instructor(s) or course. For example, you may wish to give the reasons for your numerical evaluations or provide specific suggestions for improving the instruction in the course.

Dror has a well structured lecture style

END OF SURVEY. THANK YOU FOR YOUR COOPERATION AND PARTICIPATION.

STUDENT SURVEY FORM
UNIVERSITY OF TORONTO

FACULTY OF ARTS & SCIENCE

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Part II *(on the reverse side)* requires a written answer.

Course Identification: Please print course and section you are evaluating

COURSE MAT240H1F SECTION L0101

INSTRUCTOR(S):

A:	Bar-Natan.
B:	

1. **If evaluating only one instructor, write the name in the upper (A) box. If evaluating two instructors, write their names, one in box A and the other in box B.**

DO NOT EVALUATE TEACHING ASSISTANTS ON THIS FORM

Statements about the instructor(s):

Respond to the statements below for instructor A (and instructor B) bearing in mind that there are wide variations in class size and subject matter in Arts and Science.

		extremely poor	very poor	poor	adequate	good	very good	outstanding
2. Communicates goals and requirements of the course clearly and explicitly.	A:	1	2	3	4	●	6	7
	B:	1	2	3	4	5	6	7
3. Uses methods of evaluation (e.g. papers, assignments, tests) that appropriately reflect the subject matter and provide a fair evaluation of student learning.	A:	1	2	3	●	5	6	7
	B:	1	2	3	4	5	6	7
4. Presents material in an organized, well-planned manner.	A:	1	2	3	4	5	6	●
	B:	1	2	3	4	5	6	7
5. Explains concepts clearly with appropriate use of examples.	A:	1	2	3	4	5	6	●
	B:	1	2	3	4	5	6	7
6. Communicates enthusiasm and interest in the course material.	A:	1	2	3	4	5	6	●
	B:	1	2	3	4	5	6	7
7. Attends to students' questions and answers them clearly and effectively.	A:	1	2	3	4	5	6	●
	B:	1	2	3	4	5	6	7
8. Is available for individual consultation, by appointment or stated office hours, to students with problems relating to the course.	A:	1	2	3	4	5	●	7
	B:	1	2	3	4	5	6	7
9. Ensures that student work is graded fairly, with helpful comments and feedback where appropriate.	A:	1	2	3	4	5	●	7
	B:	1	2	3	4	5	6	7
10. Ensures that student work is graded within a reasonable time.	A:	1	2	3	4	5	6	●
	B:	1	2	3	4	5	6	7
11. All things considered, performs effectively as a university teacher.	A:	1	2	3	4	5	6	●
	B:	1	2	3	4	5	6	7

PART I CONTINUES ON THE REVERSE SIDE

Statements about the course: Respond to the statements below, using the following 7-point scale.

SIDE 2

	very low	low	below average	average	above average	high	very high
12. Compared to other courses at the same level (100,200,300,400), the work load is	1	2	3	4	●	6	7
13. Compared to other courses at the same level, the level of difficulty of the material is	1	2	3	4	●	6	7
14. The value of the required reading is	1	2	3	4	5	●	7
15. (If applicable) The value of the tutorials is	1	2	3	●	5	6	7
16. (If applicable) The value of the laboratories is	1	2	3	4	5	6	7
17. (If applicable) The value of the seminars is	1	2	3	4	5	6	7
18. (If applicable) The value of the language conversation classes is	1	2	3	4	5	6	7
19. The value of the overall learning experience is	1	2	3	4	●	6	7

20. Considering your experience with this course, and disregarding your need for it to meet program or degree requirements, would you still have taken this course? ● Yes ○ No

Statements about yourself:

21. Number of full course credits already earned (prior to this session):
① 0-4½ ② 5-9½ ● 10-14½ ④ 15-19½ ⑤ ≥20

22. Status of the course for you:
① Program Requirement ● Selected from a required list in a program ③ Breadth Requirement ④ Optional

23. Your level of enthusiasm to take this course at the time of initial registration:
① low ● medium ③ high

24. Your expected grade in this course:
① <50 ② 50-59 ③ 60-69 ④ 70-79 ● ≥80

Additional statements or questions which may be supplied in class:

25. ① ② ③ ④ ⑤ ⑥ ⑦	28. ① ② ③ ④ ⑤ ⑥ ⑦	31. ① ② ③ ④ ⑤ ⑥ ⑦	34. ① ② ③ ④ ⑤ ⑥ ⑦
26. ① ② ③ ④ ⑤ ⑥ ⑦	29. ① ② ③ ④ ⑤ ⑥ ⑦	32. ① ② ③ ④ ⑤ ⑥ ⑦	35. ① ② ③ ④ ⑤ ⑥ ⑦
27. ① ② ③ ④ ⑤ ⑥ ⑦	30. ① ② ③ ④ ⑤ ⑥ ⑦	33. ① ② ③ ④ ⑤ ⑥ ⑦	36. ① ② ③ ④ ⑤ ⑥ ⑦

PART II: PLEASE ANSWER ONLY AFTER COMPLETING PART I. Please use the space below to provide supplementary comments on the instructor(s) or course. For example, you may wish to give the reasons for your numerical evaluations or provide specific suggestions for improving the instruction in the course.

Tutorials could have been more organized.

END OF SURVEY. THANK YOU FOR YOUR COOPERATION AND PARTICIPATION.

STUDENT SURVEY FORM
UNIVERSITY OF TORONTO

FACULTY OF ARTS & SCIENCE

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Part II *(on the reverse side)* requires a written answer.

Course Identification: Please print course and section you are evaluating

COURSE M A 1 1 4 0 SECTION

INSTRUCTOR(S):

A: Dror Ben-Naim

B:

1. If evaluating only one instructor, write the name in the upper (A) box. If evaluating two instructors, write their names, one in box A and the other in box B.

DO NOT EVALUATE TEACHING ASSISTANTS ON THIS FORM

Statements about the instructor(s):

Respond to the statements below for instructor A (and instructor B) bearing in mind that there are wide variations in class size and subject matter in Arts and Science.

		extremely poor	very poor	poor	adequate	good	very good	outstanding
2. Communicates goals and requirements of the course clearly and explicitly.	A:	1	2	3	4	5	6	● 7
	B:	1	2	3	4	5	6	7
3. Uses methods of evaluation (e.g. papers, assignments, tests) that appropriately reflect the subject matter and provide a fair evaluation of student learning.	A:	1	2	3	4	5	6	● 7
	B:	1	2	3	4	5	6	7
4. Presents material in an organized, well-planned manner.	A:	1	2	3	4	5	6	● 7
	B:	1	2	3	4	5	6	7
5. Explains concepts clearly with appropriate use of examples.	A:	1	2	3	4	5	6	● 7
	B:	1	2	3	4	5	6	7
6. Communicates enthusiasm and interest in the course material.	A:	1	2	3	4	5	6	● 7
	B:	1	2	3	4	5	6	7
7. Attends to students' questions and answers them clearly and effectively.	A:	1	2	3	4	5	6	● 7
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10. Ensures that student work is graded within a reasonable time.	A:	1	2	3	4	●	6	7
	B:	1	2	3	4	5	6	7
11. All things considered, performs effectively as a university teacher.	A:	1	2	3	4	5	6	● 7
	B:	1	2	3	4	5	6	7

PART I CONTINUES ON THE REVERSE SIDE

Statements about the course: Respond to the statements below, using the following 7-point scale.

SIDE 2

	very low	low	below average	average	above average	high	very high
12. Compared to other courses at the same level (100,200,300,400), the work load is	1	2	3	4	●	6	7
13. Compared to other courses at the same level, the level of difficulty of the material is	1	2	3	4	●	6	7
14. The value of the required reading is	1	2	3	4	5	6	7
15. (If applicable) The value of the tutorials is	1	2	3	4	●	6	7
16. (If applicable) The value of the laboratories is	1	2	3	4	5	6	7
17. (If applicable) The value of the seminars is	1	2	3	4	5	6	7
18. (If applicable) The value of the language conversation classes is	1	2	3	4	5	6	7
19. The value of the overall learning experience is	1	2	3	4	5	6	●

20. Considering your experience with this course, and disregarding your need for it to meet program or degree requirements, would you still have taken this course? ● Yes ○ No

Statements about yourself:

21. Number of full course credits already earned (prior to this session):
① 0-4½ ② 5-9½ ● 10-14½ ④ 15-19½ ⑤ ≥20

22. Status of the course for you:
● Program Requirement ② Selected from a required list in a program ③ Breadth Requirement ④ Optional

23. Your level of enthusiasm to take this course at the time of initial registration:
① low ② medium ● high

24. Your expected grade in this course:
① <50 ② 50-59 ③ 60-69 ④ 70-79 ● ≥80

Additional statements or questions which may be supplied in class:

25. ① ② ③ ④ ⑤ ⑥ ⑦	28. ① ② ③ ④ ⑤ ⑥ ⑦	31. ① ② ③ ④ ⑤ ⑥ ⑦	34. ① ② ③ ④ ⑤ ⑥ ⑦
26. ① ② ③ ④ ⑤ ⑥ ⑦	29. ① ② ③ ④ ⑤ ⑥ ⑦	32. ① ② ③ ④ ⑤ ⑥ ⑦	35. ① ② ③ ④ ⑤ ⑥ ⑦
27. ① ② ③ ④ ⑤ ⑥ ⑦	30. ① ② ③ ④ ⑤ ⑥ ⑦	33. ① ② ③ ④ ⑤ ⑥ ⑦	36. ① ② ③ ④ ⑤ ⑥ ⑦

PART II: PLEASE ANSWER ONLY AFTER COMPLETING PART I. Please use the space below to provide supplementary comments on the instructor(s) or course. For example, you may wish to give the reasons for your numerical evaluations or provide specific suggestions for improving the instruction in the course.

Best Math Teacher I've ever had.

END OF SURVEY. THANK YOU FOR YOUR COOPERATION AND PARTICIPATION.

STUDENT SURVEY FORM
UNIVERSITY OF TORONTO

FACULTY OF ARTS & SCIENCE

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Part II *(on the reverse side)* requires a written answer.

Course Identification: Please print course and section you are evaluating

COURSE MAT 241 UH1 F SECTION L0101

INSTRUCTOR(S):

A:	PROF. DROR BAR-NATAN
B:	

1. If evaluating only one instructor, write the name in the upper (A) box. If evaluating two instructors, write their names, one in box A and the other in box B.

DO NOT EVALUATE TEACHING ASSISTANTS ON THIS FORM

Statements about the instructor(s):

Respond to the statements below for instructor A (and instructor B) bearing in mind that there are wide variations in class size and subject matter in Arts and Science.

		extremely poor	very poor	poor	adequate	good	very good	outstanding
2. Communicates goals and requirements of the course clearly and explicitly.	A:	1	2	3	4	5	6	● 7
	B:	1	2	3	4	5	6	7
3. Uses methods of evaluation (e.g. papers, assignments, tests) that appropriately reflect the subject matter and provide a fair evaluation of student learning.	A:	1	2	3	4	5	● 6	7
	B:	1	2	3	4	5	6	7
4. Presents material in an organized, well-planned manner.	A:	1	2	3	4	5	6	● 7
	B:	1	2	3	4	5	6	7
5. Explains concepts clearly with appropriate use of examples.	A:	1	2	3	4	5	6	● 7
	B:	1	2	3	4	5	6	7
6. Communicates enthusiasm and interest in the course material.	A:	1	2	3	4	5	6	● 7
	B:	1	2	3	4	5	6	7
7. Attends to students' questions and answers them clearly and effectively.	A:	1	2	3	4	5	6	● 7
	B:	1	2	3	4	5	6	7
8. Is available for individual consultation, by appointment or stated office hours, to students with problems relating to the course.	A:	1	2	3	4	5	6	● 7
	B:	1	2	3	4	5	6	7
9. Ensures that student work is graded fairly, with helpful comments and feedback where appropriate.	A:	1	2	3	4	5	● 6	7
	B:	1	2	3	4	5	6	7
10. Ensures that student work is graded within a reasonable time.	A:	1	2	3	4	5	6	● 7
	B:	1	2	3	4	5	6	7
11. All things considered, performs effectively as a university teacher.	A:	1	2	3	4	5	6	● 7
	B:	1	2	3	4	5	6	7

PART I CONTINUES ON THE REVERSE SIDE

Statements about the course: Respond to the statements below, using the following 7-point scale.

SIDE 2

	very low	low	below average	average	above average	high	very high
12. Compared to other courses at the same level (100,200,300,400), the work load is	1	2	3	4	●	6	7
13. Compared to other courses at the same level, the level of difficulty of the material is	1	2	3	4	●	6	7
14. The value of the required reading is	1	2	3	4	●	6	7
15. (If applicable) The value of the tutorials is	1	2	●	4	5	6	7
16. (If applicable) The value of the laboratories is	1	2	3	4	5	6	7
17. (If applicable) The value of the seminars is	1	2	3	4	5	6	7
18. (If applicable) The value of the language conversation classes is	1	2	3	4	5	6	7
19. The value of the overall learning experience is	1	2	3	4	5	●	7

20. Considering your experience with this course, and disregarding your need for it to meet program or degree requirements, would you still have taken this course? ● Yes ○ No

Statements about yourself:

21. Number of full course credits already earned (prior to this session):
● 0-4½ ② 5-9½ ③ 10-14½ ④ 15-19½ ⑤ ≥20

22. Status of the course for you:
● Program Requirement ② Selected from a required list in a program ③ Breadth Requirement ④ Optional

23. Your level of enthusiasm to take this course at the time of initial registration:
① low ② medium ● high

24. Your expected grade in this course:
① <50 ② 50-59 ③ 60-69 ④ 70-79 ● ≥80

Additional statements or questions which may be supplied in class:

25. ① ② ③ ④ ⑤ ⑥ ⑦	28. ① ② ③ ④ ⑤ ⑥ ⑦	31. ① ② ③ ④ ⑤ ⑥ ⑦	34. ① ② ③ ④ ⑤ ⑥ ⑦
26. ① ② ③ ④ ⑤ ⑥ ⑦	29. ① ② ③ ④ ⑤ ⑥ ⑦	32. ① ② ③ ④ ⑤ ⑥ ⑦	35. ① ② ③ ④ ⑤ ⑥ ⑦
27. ① ② ③ ④ ⑤ ⑥ ⑦	30. ① ② ③ ④ ⑤ ⑥ ⑦	33. ① ② ③ ④ ⑤ ⑥ ⑦	36. ① ② ③ ④ ⑤ ⑥ ⑦

PART II: PLEASE ANSWER ONLY AFTER COMPLETING PART I. Please use the space below to provide supplementary comments on the instructor(s) or course. For example, you may wish to give the reasons for your numerical evaluations or provide specific suggestions for improving the instruction in the course.

It was an honour to be taught by Dror. Apart from the course material, his classes led to an increased appreciation of Mathematics in me.

END OF SURVEY. THANK YOU FOR YOUR COOPERATION AND PARTICIPATION.

STUDENT SURVEY FORM
UNIVERSITY OF TORONTO

FACULTY OF ARTS & SCIENCE

Note that survey results will be available to the instructor(s) only after final course marks have been submitted.

PART I: INSTRUCTIONS. PLEASE READ FIRST.

Using an HB pencil or a blue or black ball-point pen (but not a felt marking pen), fill completely the numbered oval corresponding to your response for each statement. If using a pen, do not alter original response by making another selection.

Part II *(on the reverse side)* requires a written answer.

Course Identification: Please print course and section you are evaluating

COURSE MAT240M1F SECTION L0101

INSTRUCTOR(S):

A:	P. Dror BarNatan
B:	

1. **If evaluating only one instructor, write the name in the upper (A) box. If evaluating two instructors, write their names, one in box A and the other in box B.**

DO NOT EVALUATE TEACHING ASSISTANTS ON THIS FORM

Statements about the instructor(s):

Respond to the statements below for instructor A (and instructor B) bearing in mind that there are wide variations in class size and subject matter in Arts and Science.

		extremely poor	very poor	poor	adequate	good	very good	outstanding
2. Communicates goals and requirements of the course clearly and explicitly.	A:	1	2	3	4	5	6	●
	B:	1	2	3	4	5	6	7
3. Uses methods of evaluation (e.g. papers, assignments, tests) that appropriately reflect the subject matter and provide a fair evaluation of student learning.	A:	1	2	3	4	5	6	●
	B:	1	2	3	4	5	6	7
4. Presents material in an organized, well-planned manner.	A:	1	2	3	4	5	6	●
	B:	1	2	3	4	5	6	7
5. Explains concepts clearly with appropriate use of examples.	A:	1	2	3	4	5	6	●
	B:	1	2	3	4	5	6	7
6. Communicates enthusiasm and interest in the course material.	A:	1	2	3	4	5	6	●
	B:	1	2	3	4	5	6	7
7. Attends to students' questions and answers them clearly and effectively.	A:	1	2	3	4	5	6	●
	B:	1	2	3	4	5	6	7
8. Is available for individual consultation, by appointment or stated office hours, to students with problems relating to the course.	A:	1	2	3	4	5	6	●
	B:	1	2	3	4	5	6	7
9. Ensures that student work is graded fairly, with helpful comments and feedback where appropriate.	A:	1	2	3	4	5	6	●
	B:	1	2	3	4	5	6	7
10. Ensures that student work is graded within a reasonable time.	A:	1	2	3	4	5	6	●
	B:	1	2	3	4	5	6	7
11. All things considered, performs effectively as a university teacher.	A:	1	2	3	4	5	6	●
	B:	1	2	3	4	5	6	7

PART I CONTINUES ON THE REVERSE SIDE

Statements about the course: Respond to the statements below, using the following 7-point scale.

SIDE 2

	very low	low	below average	average	above average	high	very high
12. Compared to other courses at the same level (100,200,300,400), the work load is	1	2	3	4	●	6	7
13. Compared to other courses at the same level, the level of difficulty of the material is	1	2	3	4	●	6	7
14. The value of the required reading is	1	2	3	4	5	●	7
15. (If applicable) The value of the tutorials is	1	2	3	4	5	6	7
16. (If applicable) The value of the laboratories is	1	2	3	4	5	6	7
17. (If applicable) The value of the seminars is	1	2	3	4	5	6	7
18. (If applicable) The value of the language conversation classes is	1	2	3	4	5	6	7
19. The value of the overall learning experience is	1	2	3	4	5	6	●

20. Considering your experience with this course, and disregarding your need for it to meet program or degree requirements, would you still have taken this course? ● Yes ○ No

Statements about yourself:

21. Number of full course credits already earned (prior to this session):
● 0-4½ 2 5-9½ 3 10-14½ 4 15-19½ 5 ≥20

22. Status of the course for you:
● Program Requirement 2 Selected from a required list in a program 3 Breadth Requirement 4 Optional

23. Your level of enthusiasm to take this course at the time of initial registration:
1 low 2 medium ● high

24. Your expected grade in this course:
1 <50 2 50-59 3 60-69 ● 70-79 5 ≥80

Additional statements or questions which may be supplied in class:

25. 1 2 3 4 5 6 7	28. 1 2 3 4 5 6 7	31. 1 2 3 4 5 6 7	34. 1 2 3 4 5 6 7
26. 1 2 3 4 5 6 7	29. 1 2 3 4 5 6 7	32. 1 2 3 4 5 6 7	35. 1 2 3 4 5 6 7
27. 1 2 3 4 5 6 7	30. 1 2 3 4 5 6 7	33. 1 2 3 4 5 6 7	36. 1 2 3 4 5 6 7

PART II: PLEASE ANSWER ONLY AFTER COMPLETING PART I. Please use the space below to provide supplementary comments on the instructor(s) or course. For example, you may wish to give the reasons for your numerical evaluations or provide specific suggestions for improving the instruction in the course.

Give Dror a raise!

END OF SURVEY. THANK YOU FOR YOUR COOPERATION AND PARTICIPATION.

STUDENT SURVEY FORM
UNIVERSITY OF TORONTO

FACULTY OF ARTS & SCIENCE

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PART I: INSTRUCTIONS. PLEASE READ FIRST.

Using an HB pencil or a blue or black ball-point pen (but not a felt marking pen), fill completely the numbered oval corresponding to your response for each statement. If using a pen, do not alter original response by making another selection.

Part II *(on the reverse side)* requires a written answer.

Course Identification: Please print course and section you are evaluating

COURSE MAT240H1F SECTION L0101

INSTRUCTOR(S):

A:	DROR BAR-NATAN
B:	

1. **If evaluating only one instructor, write the name in the upper (A) box. If evaluating two instructors, write their names, one in box A and the other in box B.**

DO NOT EVALUATE TEACHING ASSISTANTS ON THIS FORM

Statements about the instructor(s):

Respond to the statements below for instructor A (and instructor B) bearing in mind that there are wide variations in class size and subject matter in Arts and Science.

		extremely poor	very poor	poor	adequate	good	very good	outstanding
2. Communicates goals and requirements of the course clearly and explicitly.	A:	1	2	3	4	5	6	●
	B:	1	2	3	4	5	6	7
3. Uses methods of evaluation (e.g. papers, assignments, tests) that appropriately reflect the subject matter and provide a fair evaluation of student learning.	A:	1	2	3	4	5	6	●
	B:	1	2	3	4	5	6	7
4. Presents material in an organized, well-planned manner.	A:	1	2	3	4	5	6	●
	B:	1	2	3	4	5	6	7
5. Explains concepts clearly with appropriate use of examples.	A:	1	2	3	4	5	6	●
	B:	1	2	3	4	5	6	7
6. Communicates enthusiasm and interest in the course material.	A:	1	2	3	4	5	6	●
	B:	1	2	3	4	5	6	7
7. Attends to students' questions and answers them clearly and effectively.	A:	1	2	3	4	5	6	●
	B:	1	2	3	4	5	6	7
8. Is available for individual consultation, by appointment or stated office hours, to students with problems relating to the course.	A:	1	2	3	4	5	6	●
	B:	1	2	3	4	5	6	7
9. Ensures that student work is graded fairly, with helpful comments and feedback where appropriate.	A:	1	2	3	●	5	6	7
	B:	1	2	3	4	5	6	7
10. Ensures that student work is graded within a reasonable time.	A:	1	2	3	4	5	6	●
	B:	1	2	3	4	5	6	7
11. All things considered, performs effectively as a university teacher.	A:	1	2	3	4	5	6	●
	B:	1	2	3	4	5	6	7

PART I CONTINUES ON THE REVERSE SIDE

Statements about the course: Respond to the statements below, using the following 7-point scale.

SIDE 2

	very low	low	below average	average	above average	high	very high
12. Compared to other courses at the same level (100,200,300,400), the work load is	1	2	3	●	5	6	7
13. Compared to other courses at the same level, the level of difficulty of the material is	1	2	3	●	5	6	7
14. The value of the required reading is	1	2	3	4	5	6	●
15. (If applicable) The value of the tutorials is	●	2	3	4	5	6	7
16. (If applicable) The value of the laboratories is	1	2	3	4	5	6	7
17. (If applicable) The value of the seminars is	1	2	3	4	5	6	7
18. (If applicable) The value of the language conversation classes is	1	2	3	4	5	6	7
19. The value of the overall learning experience is	1	2	3	4	5	6	●

20. Considering your experience with this course, and disregarding your need for it to meet program or degree requirements, would you still have taken this course? ○ Yes ○ No

Statements about yourself:

21. Number of full course credits already earned (prior to this session):
● 0-4½ 2 5-9½ 3 10-14½ 4 15-19½ 5 ≥20

22. Status of the course for you:
● Program Requirement 2 Selected from a required list in a program 3 Breadth Requirement 4 Optional

23. Your level of enthusiasm to take this course at the time of initial registration:
1 low ● medium 3 high

24. Your expected grade in this course:
1 <50 2 50-59 3 60-69 ● 70-79 5 ≥80

Additional statements or questions which may be supplied in class:

25. 1 2 3 4 5 6 7	28. 1 2 3 4 5 6 7	31. 1 2 3 4 5 6 7	34. 1 2 3 4 5 6 7
26. 1 2 3 4 5 6 7	29. 1 2 3 4 5 6 7	32. 1 2 3 4 5 6 7	35. 1 2 3 4 5 6 7
27. 1 2 3 4 5 6 7	30. 1 2 3 4 5 6 7	33. 1 2 3 4 5 6 7	36. 1 2 3 4 5 6 7

PART II: PLEASE ANSWER ONLY AFTER COMPLETING PART I. Please use the space below to provide supplementary comments on the instructor(s) or course. For example, you may wish to give the reasons for your numerical evaluations or provide specific suggestions for improving the instruction in the course.

Excellent teaching style - enthusiasm about topic was contageous

END OF SURVEY. THANK YOU FOR YOUR COOPERATION AND PARTICIPATION.

STUDENT SURVEY FORM
UNIVERSITY OF TORONTO

FACULTY OF ARTS & SCIENCE

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Part II *(on the reverse side)* requires a written answer.

Course Identification: Please print course and section you are evaluating

COURSE MAT240H1F SECTION 0101

INSTRUCTOR(S):

A: Dror Bar-Natan

B:

1. If evaluating only one instructor, write the name in the upper (A) box. If evaluating two instructors, write their names, one in box A and the other in box B.

DO NOT EVALUATE TEACHING ASSISTANTS ON THIS FORM

Statements about the instructor(s):

Respond to the statements below for instructor A (and instructor B) bearing in mind that there are wide variations in class size and subject matter in Arts and Science.

		extremely poor	very poor	poor	adequate	good	very good	outstanding
2. Communicates goals and requirements of the course clearly and explicitly.	A:	1	2	3	4	5	6	7
	B:	1	2	3	4	5	6	7
3. Uses methods of evaluation (e.g. papers, assignments, tests) that appropriately reflect the subject matter and provide a fair evaluation of student learning.	A:	1	2	3	4	5	6	7
	B:	1	2	3	4	5	6	7
4. Presents material in an organized, well-planned manner.	A:	1	2	3	4	5	6	7
	B:	1	2	3	4	5	6	7
5. Explains concepts clearly with appropriate use of examples.	A:	1	2	3	4	5	6	7
	B:	1	2	3	4	5	6	7
6. Communicates enthusiasm and interest in the course material.	A:	1	2	3	4	5	6	7
	B:	1	2	3	4	5	6	7
7. Attends to students' questions and answers them clearly and effectively.	A:	1	2	3	4	5	6	7
	B:	1	2	3	4	5	6	7
8. Is available for individual consultation, by appointment or stated office hours, to students with problems relating to the course.	A:	1	2	3	4	5	6	7
	B:	1	2	3	4	5	6	7
9. Ensures that student work is graded fairly, with helpful comments and feedback where appropriate.	A:	1	2	3	4	5	6	7
	B:	1	2	3	4	5	6	7
10. Ensures that student work is graded within a reasonable time.	A:	1	2	3	4	5	6	7
	B:	1	2	3	4	5	6	7
11. All things considered, performs effectively as a university teacher.	A:	1	2	3	4	5	6	7
	B:	1	2	3	4	5	6	7

PART I CONTINUES ON THE REVERSE SIDE

Statements about the course: Respond to the statements below, using the following 7-point scale.

SIDE 2

	very low	low	below average	average	above average	high	very high
12. Compared to other courses at the same level (100,200,300,400), the work load is	1	2	3	4	5	6	7
13. Compared to other courses at the same level, the level of difficulty of the material is	1	2	3	4	5	6	7
14. The value of the required reading is	1	2	3	4	5	6	7
15. (If applicable) The value of the tutorials is	1	2	3	4	5	6	7
16. (If applicable) The value of the laboratories is	1	2	3	4	5	6	7
17. (If applicable) The value of the seminars is	1	2	3	4	5	6	7
18. (If applicable) The value of the language conversation classes is	1	2	3	4	5	6	7
19. The value of the overall learning experience is	1	2	3	4	5	6	7

20. Considering your experience with this course, and disregarding your need for it to meet program or degree requirements, would you still have taken this course? (3) Yes (4) No

Statements about yourself:

21. Number of full course credits already earned (prior to this session):
(1) 0-4½ (2) 5-9½ (3) 10-14½ (4) 15-19½ (5) ≥20

22. Status of the course for you:
(1) Program Requirement (2) Selected from a required list in a program (3) Breadth Requirement (4) Optional

23. Your level of enthusiasm to take this course at the time of initial registration:
(1) low (2) medium (3) high

24. Your expected grade in this course:
(1) <50 (2) 50-59 (3) 60-69 (4) 70-79 (5) ≥80

Additional statements or questions which may be supplied in class:

25. 1 2 3 4 5 6 7	28. 1 2 3 4 5 6 7	31. 1 2 3 4 5 6 7	34. 1 2 3 4 5 6 7
26. 1 2 3 4 5 6 7	29. 1 2 3 4 5 6 7	32. 1 2 3 4 5 6 7	35. 1 2 3 4 5 6 7
27. 1 2 3 4 5 6 7	30. 1 2 3 4 5 6 7	33. 1 2 3 4 5 6 7	36. 1 2 3 4 5 6 7

PART II: PLEASE ANSWER ONLY AFTER COMPLETING PART I. Please use the space below to provide supplementary comments on the instructor(s) or course. For example, you may wish to give the reasons for your numerical evaluations or provide specific suggestions for improving the instruction in the course.

BAR-NATAN IS NO BEST PROFESSOR.

END OF SURVEY. THANK YOU FOR YOUR COOPERATION AND PARTICIPATION.

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UNIVERSITY OF TORONTO

FACULTY OF ARTS & SCIENCE

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Part II *(on the reverse side)* requires a written answer.

Course Identification: Please print course and section you are evaluating

COURSE MAT240H1F SECTION L0101

INSTRUCTOR(S):

A:	DROR BAR NATAN
B:	

1. **If evaluating only one instructor, write the name in the upper (A) box. If evaluating two instructors, write their names, one in box A and the other in box B.**

DO NOT EVALUATE TEACHING ASSISTANTS ON THIS FORM

Statements about the instructor(s):

Respond to the statements below for instructor A (and instructor B) bearing in mind that there are wide variations in class size and subject matter in Arts and Science.

		extremely poor	very poor	poor	adequate	good	very good	outstanding
2. Communicates goals and requirements of the course clearly and explicitly.	A:	1	2	3	4	5	6	● 7
	B:	1	2	3	4	5	6	7
3. Uses methods of evaluation (e.g. papers, assignments, tests) that appropriately reflect the subject matter and provide a fair evaluation of student learning.	A:	1	2	3	● 4	5	6	7
	B:	1	2	3	4	5	6	7
4. Presents material in an organized, well-planned manner.	A:	1	2	3	4	5	6	● 7
	B:	1	2	3	4	5	6	7
5. Explains concepts clearly with appropriate use of examples.	A:	1	2	3	4	5	6	● 7
	B:	1	2	3	4	5	6	7
6. Communicates enthusiasm and interest in the course material.	A:	1	2	3	4	5	6	● 7
	B:	1	2	3	4	5	6	7
7. Attends to students' questions and answers them clearly and effectively.	A:	1	2	3	● 4	5	6	7
	B:	1	2	3	4	5	6	7
8. Is available for individual consultation, by appointment or stated office hours, to students with problems relating to the course.	A:	1	2	3	● 4	5	6	7
	B:	1	2	3	4	5	6	7
9. Ensures that student work is graded fairly, with helpful comments and feedback where appropriate.	A:	1	2	3	4	● 5	6	7
	B:	1	2	3	4	5	6	7
10. Ensures that student work is graded within a reasonable time.	A:	1	2	3	4	5	6	● 7
	B:	1	2	3	4	5	6	7
11. All things considered, performs effectively as a university teacher.	A:	1	2	3	4	5	● 6	7
	B:	1	2	3	4	5	6	7

PART I CONTINUES ON THE REVERSE SIDE

Statements about the course: Respond to the statements below, using the following 7-point scale.

SIDE 2

	very low	low	below average	average	above average	high	very high
12. Compared to other courses at the same level (100,200,300,400), the work load is	1	2	3	4	5	6	●
13. Compared to other courses at the same level, the level of difficulty of the material is	1	2	3	4	5	●	7
14. The value of the required reading is	1	2	3	4	5	●	7
15. (If applicable) The value of the tutorials is	●	2	3	4	5	6	7
16. (If applicable) The value of the laboratories is	1	2	3	4	5	6	7
17. (If applicable) The value of the seminars is	1	2	3	4	5	6	7
18. (If applicable) The value of the language conversation classes is	1	2	3	4	5	6	7
19. The value of the overall learning experience is	1	2	3	4	5	6	●

20. Considering your experience with this course, and disregarding your need for it to meet program or degree requirements, would you still have taken this course? ● Yes ○ No

Statements about yourself:

21. Number of full course credits already earned (prior to this session):
● 0-4½ ② 5-9½ ③ 10-14½ ④ 15-19½ ⑤ ≥20

22. Status of the course for you:
● Program Requirement ② Selected from a required list in a program ③ Breadth Requirement ④ Optional

23. Your level of enthusiasm to take this course at the time of initial registration:
① low ② medium ● high

24. Your expected grade in this course:
① <50 ② 50-59 ③ 60-69 ④ 70-79 ● ≥80

Additional statements or questions which may be supplied in class:

25. ① ② ③ ④ ⑤ ⑥ ⑦	28. ① ② ③ ④ ⑤ ⑥ ⑦	31. ① ② ③ ④ ⑤ ⑥ ⑦	34. ① ② ③ ④ ⑤ ⑥ ⑦
26. ① ② ③ ④ ⑤ ⑥ ⑦	29. ① ② ③ ④ ⑤ ⑥ ⑦	32. ① ② ③ ④ ⑤ ⑥ ⑦	35. ① ② ③ ④ ⑤ ⑥ ⑦
27. ① ② ③ ④ ⑤ ⑥ ⑦	30. ① ② ③ ④ ⑤ ⑥ ⑦	33. ① ② ③ ④ ⑤ ⑥ ⑦	36. ① ② ③ ④ ⑤ ⑥ ⑦

PART II: PLEASE ANSWER ONLY AFTER COMPLETING PART I. Please use the space below to provide supplementary comments on the instructor(s) or course. For example, you may wish to give the reasons for your numerical evaluations or provide specific suggestions for improving the instruction in the course.

BAR NAFTAN IS AN EXCEPTIONAL PROFESSOR.

PART I CONTINUES ON THE REVERSE SIDE

9. Ensures that student work is graded fairly, with helpful comments and feedback where appropriate. ..... A: ① ② ③ ④ ⑤ ⑥ ⑦ B: ① ② ③ ④ ⑤ ⑥ ⑦

10. Ensures that student work is graded within a reasonable time. ..... A: ① ② ③ ④ ⑤ ⑥ ⑦ B: ① ② ③ ④ ⑤ ⑥ ⑦

11. All things considered, performs effectively as a university teacher. ..... A: ① ② ③ ④ ⑤ ⑥ ⑦ B: ① ② ③ ④ ⑤ ⑥ ⑦

STUDENT SURVEY FORM
UNIVERSITY OF TORONTO

FACULTY OF ARTS & SCIENCE

Note that survey results will be available to the instructor(s) only after final course marks have been submitted.

PART I: INSTRUCTIONS. PLEASE READ FIRST.

Using an HB pencil or a blue or black ball-point pen (but not a felt marking pen), fill completely the numbered oval corresponding to your response for each statement. If using a pen, do not alter original response by making another selection.

Part II *(on the reverse side)* requires a written answer.

Course Identification: Please print course and section you are evaluating

COURSE | M | A | T | 2 | 4 | 0 | H | 1 | F | SECTION | | | | | | |

INSTRUCTOR(S):

A:	TA = Dmitry Donin
B:	

1. **If evaluating only one instructor, write the name in the upper (A) box. If evaluating two instructors, write their names, one in box A and the other in box B.**

DO NOT EVALUATE TEACHING ASSISTANTS ON THIS FORM

Statements about the instructor(s):

Respond to the statements below for instructor A (and instructor B) bearing in mind that there are wide variations in class size and subject matter in Arts and Science.

		extremely poor	very poor	poor	adequate	good	very good	outstanding
2. Communicates goals and requirements of the course clearly and explicitly.	A:	1	2	3	4	5	●	7
	B:	1	2	3	4	5	6	7
3. Uses methods of evaluation (e.g. papers, assignments, tests) that appropriately reflect the subject matter and provide a fair evaluation of student learning.	A:	1	2	3	4	●	6	7
	B:	1	2	3	4	5	6	7
4. Presents material in an organized, well-planned manner.	A:	1	2	3	4	5	●	7
	B:	1	2	3	4	5	6	7
5. Explains concepts clearly with appropriate use of examples.	A:	1	2	3	4	●	6	7
	B:	1	2	3	4	5	6	7
6. Communicates enthusiasm and interest in the course material.	A:	1	2	3	4	5	6	●
	B:	1	2	3	4	5	6	7
7. Attends to students' questions and answers them clearly and effectively.	A:	1	2	3	4	5	●	7
	B:	1	2	3	4	5	6	7
8. Is available for individual consultation, by appointment or stated office hours, to students with problems relating to the course.	A:	1	2	3	4	5	6	●
	B:	1	2	3	4	5	6	7
9. Ensures that student work is graded fairly, with helpful comments and feedback where appropriate.	A:	1	2	3	4	5	6	●
	B:	1	2	3	4	5	6	7
10. Ensures that student work is graded within a reasonable time.	A:	1	2	3	4	5	6	●
	B:	1	2	3	4	5	6	7
11. All things considered, performs effectively as a university teacher.	A:	1	2	3	4	5	●	7
	B:	1	2	3	4	5	6	7

PART I CONTINUES ON THE REVERSE SIDE

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Statements about the course: Respond to the statements below, using the following 7-point scale.

SIDE 2

	very low	low	below average	average	above average	high	very high
12. Compared to other courses at the same level (100,200,300,400), the work load is	1	2	3	4	●	6	7
13. Compared to other courses at the same level, the level of difficulty of the material is	1	2	3	4	5	●	7
14. The value of the required reading is	1	2	3	4	5	●	7
15. (If applicable) The value of the tutorials is	1	2	3	4	5	●	7
16. (If applicable) The value of the laboratories is	1	2	3	4	5	6	7
17. (If applicable) The value of the seminars is	1	2	3	4	5	6	7
18. (If applicable) The value of the language conversation classes is	1	2	3	4	5	6	7
19. The value of the overall learning experience is	1	2	3	4	5	●	7

20. Considering your experience with this course, and disregarding your need for it to meet program or degree requirements, would you still have taken this course? ● Yes ○ No

Statements about yourself:

21. Number of full course credits already earned (prior to this session):
● 0-4½ 2 5-9½ 3 10-14½ 4 15-19½ 5 ≥20

22. Status of the course for you:
● Program Requirement 2 Selected from a required list in a program 3 Breadth Requirement 4 Optional

23. Your level of enthusiasm to take this course at the time of initial registration:
1 low 2 medium ● high

24. Your expected grade in this course:
1 <50 2 50-59 ● 60-69 4 70-79 5 ≥80

Additional statements or questions which may be supplied in class:

25. 1 2 3 4 5 6 7	28. 1 2 3 4 5 6 7	31. 1 2 3 4 5 6 7	34. 1 2 3 4 5 6 7
26. 1 2 3 4 5 6 7	29. 1 2 3 4 5 6 7	32. 1 2 3 4 5 6 7	35. 1 2 3 4 5 6 7
27. 1 2 3 4 5 6 7	30. 1 2 3 4 5 6 7	33. 1 2 3 4 5 6 7	36. 1 2 3 4 5 6 7

PART II: PLEASE ANSWER ONLY AFTER COMPLETING PART I. Please use the space below to provide supplementary comments on the instructor(s) or course. For example, you may wish to give the reasons for your numerical evaluations or provide specific suggestions for improving the instruction in the course.

- got better as the class went on
- good choices of examples

END OF SURVEY. THANK YOU FOR YOUR COOPERATION AND PARTICIPATION.

STUDENT SURVEY FORM
UNIVERSITY OF TORONTO

FACULTY OF ARTS & SCIENCE

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Part II *(on the reverse side)* requires a written answer.

Course Identification: Please print course and section you are evaluating

COURSE MAT1240H SECTION

INSTRUCTOR(S):

A: Oror Bar-Natan

B:

1. If evaluating only one instructor, write the name in the upper (A) box. If evaluating two instructors, write their names, one in box A and the other in box B.

DO NOT EVALUATE TEACHING ASSISTANTS ON THIS FORM

Statements about the instructor(s):

Respond to the statements below for instructor A (and instructor B) bearing in mind that there are wide variations in class size and subject matter in Arts and Science.

		extremely poor	very poor	poor	adequate	good	very good	outstanding
2. Communicates goals and requirements of the course clearly and explicitly.	A:	1	2	3	4	5	6	●
	B:	1	2	3	4	5	6	7
3. Uses methods of evaluation (e.g. papers, assignments, tests) that appropriately reflect the subject matter and provide a fair evaluation of student learning.	A:	1	2	3	4	5	6	●
	B:	1	2	3	4	5	6	7
4. Presents material in an organized, well-planned manner.	A:	1	2	3	4	5	6	●
	B:	1	2	3	4	5	6	7
5. Explains concepts clearly with appropriate use of examples.	A:	1	2	3	4	5	6	●
	B:	1	2	3	4	5	6	7
6. Communicates enthusiasm and interest in the course material.	A:	1	2	3	4	5	6	●
	B:	1	2	3	4	5	6	7
7. Attends to students' questions and answers them clearly and effectively.	A:	1	2	3	4	5	6	●
	B:	1	2	3	4	5	6	7
8. Is available for individual consultation, by appointment or stated office hours, to students with problems relating to the course.	A:	1	2	3	4	5	6	●
	B:	1	2	3	4	5	6	7
9. Ensures that student work is graded fairly, with helpful comments and feedback where appropriate.	A:	1	2	3	4	5	6	●
	B:	1	2	3	4	5	6	7
10. Ensures that student work is graded within a reasonable time.	A:	1	2	3	4	5	6	●
	B:	1	2	3	4	5	6	7
11. All things considered, performs effectively as a university teacher.	A:	1	2	3	4	5	6	●
	B:	1	2	3	4	5	6	7

PART I CONTINUES ON THE REVERSE SIDE

Statements about the course: Respond to the statements below, using the following 7-point scale.

SIDE 2

	very low	low	below average	average	above average	high	very high
12. Compared to other courses at the same level (100,200,300,400), the work load is	1	2	3	4	●	6	7
13. Compared to other courses at the same level, the level of difficulty of the material is	1	2	3	4	●	6	7
14. The value of the required reading is	1	2	3	4	5	●	7
15. (If applicable) The value of the tutorials is	1	2	●	4	5	6	7
16. (If applicable) The value of the laboratories is	1	2	3	4	5	6	7
17. (If applicable) The value of the seminars is	1	2	3	4	5	6	7
18. (If applicable) The value of the language conversation classes is	1	2	3	4	5	6	7
19. The value of the overall learning experience is	1	2	3	4	5	6	●

20. Considering your experience with this course, and disregarding your need for it to meet program or degree requirements, would you still have taken this course? ● Yes ○ No

Statements about yourself:

21. Number of full course credits already earned (prior to this session):
● 0-4½ ② 5-9½ ③ 10-14½ ④ 15-19½ ⑤ ≥20

22. Status of the course for you:
● Program Requirement ② Selected from a required list in a program ③ Breadth Requirement ④ Optional

23. Your level of enthusiasm to take this course at the time of initial registration:
① low ● medium ③ high

24. Your expected grade in this course:
① <50 ② 50-59 ③ 60-69 ④ 70-79 ● ≥80

Additional statements or questions which may be supplied in class:

25. ① ② ③ ④ ⑤ ⑥ ⑦	28. ① ② ③ ④ ⑤ ⑥ ⑦	31. ① ② ③ ④ ⑤ ⑥ ⑦	34. ① ② ③ ④ ⑤ ⑥ ⑦
26. ① ② ③ ④ ⑤ ⑥ ⑦	29. ① ② ③ ④ ⑤ ⑥ ⑦	32. ① ② ③ ④ ⑤ ⑥ ⑦	35. ① ② ③ ④ ⑤ ⑥ ⑦
27. ① ② ③ ④ ⑤ ⑥ ⑦	30. ① ② ③ ④ ⑤ ⑥ ⑦	33. ① ② ③ ④ ⑤ ⑥ ⑦	36. ① ② ③ ④ ⑤ ⑥ ⑦

PART II: PLEASE ANSWER ONLY AFTER COMPLETING PART I. Please use the space below to provide supplementary comments on the instructor(s) or course. For example, you may wish to give the reasons for your numerical evaluations or provide specific suggestions for improving the instruction in the course.

I loved this course! ☺

DROR'S RESPONSES TO QUESTIONS WERE EXTREMELY INSIGHTFUL AND HELPFUL IN UNDERSTANDING!!

END OF SURVEY. THANK YOU FOR YOUR COOPERATION AND PARTICIPATION.

STUDENT SURVEY FORM
UNIVERSITY OF TORONTO

FACULTY OF ARTS & SCIENCE

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Part II *(on the reverse side)* requires a written answer.

Course Identification: Please print course and section you are evaluating

COURSE | M | A | T | 2 | 4 | 0 | H | 1 | F | SECTION | | | | | | |

INSTRUCTOR(S):

A:	Prof. Dror Bar-Natan
B:	

1. If evaluating only one instructor, write the name in the upper (A) box. If evaluating two instructors, write their names, one in box A and the other in box B.

DO NOT EVALUATE TEACHING ASSISTANTS ON THIS FORM

Statements about the instructor(s):

Respond to the statements below for instructor A (and instructor B) bearing in mind that there are wide variations in class size and subject matter in Arts and Science.

		extremely poor	very poor	poor	adequate	good	very good	outstanding
2. Communicates goals and requirements of the course clearly and explicitly.	A:	1	2	3	4	5	●	7
	B:	1	2	3	4	5	6	7
3. Uses methods of evaluation (e.g. papers, assignments, tests) that appropriately reflect the subject matter and provide a fair evaluation of student learning.	A:	1	2	3	4	5	●	7
	B:	1	2	3	4	5	6	7
4. Presents material in an organized, well-planned manner.	A:	1	2	3	●	5	6	7
	B:	1	2	3	4	5	6	7
5. Explains concepts clearly with appropriate use of examples.	A:	1	2	3	4	●	6	7
	B:	1	2	3	4	5	6	7
6. Communicates enthusiasm and interest in the course material.	A:	1	2	3	4	5	6	●
	B:	1	2	3	4	5	6	7
7. Attends to students' questions and answers them clearly and effectively.	A:	1	2	3	4	5	●	7
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8. Is available for individual consultation, by appointment or stated office hours, to students with problems relating to the course.	A:	1	2	3	4	5	6	●
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9. Ensures that student work is graded fairly, with helpful comments and feedback where appropriate.	A:	1	2	3	4	5	6	●
	B:	1	2	3	4	5	6	7
10. Ensures that student work is graded within a reasonable time.	A:	1	2	3	4	5	6	●
	B:	1	2	3	4	5	6	7
11. All things considered, performs effectively as a university teacher.	A:	1	2	3	4	5	●	7
	B:	1	2	3	4	5	6	7

PART I CONTINUES ON THE REVERSE SIDE

Statements about the course: Respond to the statements below, using the following 7-point scale.

SIDE 2

	very low	low	below average	average	above average	high	very high
12. Compared to other courses at the same level (100,200,300,400), the work load is	1	2	3	4	●	6	7
13. Compared to other courses at the same level, the level of difficulty of the material is	1	2	3	4	5	●	7
14. The value of the required reading is	1	2	3	4	5	●	7
15. (If applicable) The value of the tutorials is	1	2	3	4	5	6	●
16. (If applicable) The value of the laboratories is	1	2	3	4	5	6	7
17. (If applicable) The value of the seminars is	1	2	3	4	5	6	7
18. (If applicable) The value of the language conversation classes is	1	2	3	4	5	6	7
19. The value of the overall learning experience is	1	2	3	4	5	●	7

20. Considering your experience with this course, and disregarding your need for it to meet program or degree requirements, would you still have taken this course? ● Yes ○ No

Statements about yourself:

21. Number of full course credits already earned (prior to this session):
● 0-4½ ② 5-9½ ③ 10-14½ ④ 15-19½ ⑤ ≥20

22. Status of the course for you:
● Program Requirement ② Selected from a required list in a program ③ Breadth Requirement ④ Optional

23. Your level of enthusiasm to take this course at the time of initial registration:
① low ② medium ● high

24. Your expected grade in this course:
① <50 ② 50-59 ● 60-69 ④ 70-79 ⑤ ≥80

Additional statements or questions which may be supplied in class:

25. ① ② ③ ④ ⑤ ⑥ ⑦	28. ① ② ③ ④ ⑤ ⑥ ⑦	31. ① ② ③ ④ ⑤ ⑥ ⑦	34. ① ② ③ ④ ⑤ ⑥ ⑦
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PART II: PLEASE ANSWER ONLY AFTER COMPLETING PART I. Please use the space below to provide supplementary comments on the instructor(s) or course. For example, you may wish to give the reasons for your numerical evaluations or provide specific suggestions for improving the instruction in the course.

- more organization on the chalkboard would be nice, but speech was good.
- Tutorials were pretty good, but needed to be a bit more focused on class material
- ~~From~~ Can the tutorial not be right after the class?
- The wiki was AMAZING. Good deed points are not needed, but the discussion was cool

END OF SURVEY. THANK YOU FOR YOUR COOPERATION AND PARTICIPATION.

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UNIVERSITY OF TORONTO

FACULTY OF ARTS & SCIENCE

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Part II *(on the reverse side)* requires a written answer.

Course Identification: Please print course and section you are evaluating

COURSE MAT224OM1F SECTION L0101

INSTRUCTOR(S):

A:	Dror Bar-Natan
B:	

1. If evaluating only one instructor, write the name in the upper (A) box. If evaluating two instructors, write their names, one in box A and the other in box B.

DO NOT EVALUATE TEACHING ASSISTANTS ON THIS FORM

Statements about the instructor(s):

Respond to the statements below for instructor A (and instructor B) bearing in mind that there are wide variations in class size and subject matter in Arts and Science.

		extremely poor	very poor	poor	adequate	good	very good	outstanding
2. Communicates goals and requirements of the course clearly and explicitly.	A:	1	2	3	4	5	6	7
	B:	1	2	3	4	5	6	7
3. Uses methods of evaluation (e.g. papers, assignments, tests) that appropriately reflect the subject matter and provide a fair evaluation of student learning.	A:	1	2	3	4	5	6	7
	B:	1	2	3	4	5	6	7
4. Presents material in an organized, well-planned manner.	A:	1	2	3	4	5	6	7
	B:	1	2	3	4	5	6	7
5. Explains concepts clearly with appropriate use of examples.	A:	1	2	3	4	5	6	7
	B:	1	2	3	4	5	6	7
6. Communicates enthusiasm and interest in the course material.	A:	1	2	3	4	5	6	7
	B:	1	2	3	4	5	6	7
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8. Is available for individual consultation, by appointment or stated office hours, to students with problems relating to the course.	A:	1	2	3	4	5	6	7
	B:	1	2	3	4	5	6	7
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	B:	1	2	3	4	5	6	7
10. Ensures that student work is graded within a reasonable time.	A:	1	2	3	4	5	6	7
	B:	1	2	3	4	5	6	7
11. All things considered, performs effectively as a university teacher.	A:	1	2	3	4	5	6	7
	B:	1	2	3	4	5	6	7

PART I CONTINUES ON THE REVERSE SIDE

Statements about the course: Respond to the statements below, using the following 7-point scale.

SIDE 2

	very low	low	below average	average	above average	high	very high
12. Compared to other courses at the same level (100,200,300,400), the work load is	1	2	3	4	5	6	7
13. Compared to other courses at the same level, the level of difficulty of the material is	1	2	3	4	5	6	7
14. The value of the required reading is	1	2	3	4	5	6	7
15. (If applicable) The value of the tutorials is	1	2	3	4	5	6	7
16. (If applicable) The value of the laboratories is	1	2	3	4	5	6	7
17. (If applicable) The value of the seminars is	1	2	3	4	5	6	7
18. (If applicable) The value of the language conversation classes is	1	2	3	4	5	6	7
19. The value of the overall learning experience is	1	2	3	4	5	6	7

20. Considering your experience with this course, and disregarding your need for it to meet program or degree requirements, would you still have taken this course? (1) Yes (2) No

Statements about yourself:

21. Number of full course credits already earned (prior to this session):
(1) 0-4½ (2) 5-9½ (3) 10-14½ (4) 15-19½ (5) ≥20

22. Status of the course for you:
(1) Program Requirement (2) Selected from a required list in a program (3) Breadth Requirement (4) Optional

23. Your level of enthusiasm to take this course at the time of initial registration:
(1) low (2) medium (3) high

24. Your expected grade in this course:
(1) <50 (2) 50-59 (3) 60-69 (4) 70-79 (5) ≥80

Additional statements or questions which may be supplied in class:

25. 1 2 3 4 5 6 7	28. 1 2 3 4 5 6 7	31. 1 2 3 4 5 6 7	34. 1 2 3 4 5 6 7
26. 1 2 3 4 5 6 7	29. 1 2 3 4 5 6 7	32. 1 2 3 4 5 6 7	35. 1 2 3 4 5 6 7
27. 1 2 3 4 5 6 7	30. 1 2 3 4 5 6 7	33. 1 2 3 4 5 6 7	36. 1 2 3 4 5 6 7

PART II: PLEASE ANSWER ONLY AFTER COMPLETING PART I. Please use the space below to provide supplementary comments on the instructor(s) or course. For example, you may wish to give the reasons for your numerical evaluations or provide specific suggestions for improving the instruction in the course.

The homework assignments are very good, making one aware the material to do the problems effectively.

END OF SURVEY. THANK YOU FOR YOUR COOPERATION AND PARTICIPATION.

STUDENT SURVEY FORM
UNIVERSITY OF TORONTO

FACULTY OF ARTS & SCIENCE

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Part II *(on the reverse side)* requires a written answer.

Course Identification: Please print course and section you are evaluating

COURSE MAT240H1S SECTION L0101

INSTRUCTOR(S):

A:	Dror Bar-Natan
B:	

1. If evaluating only one instructor, write the name in the upper (A) box. If evaluating two instructors, write their names, one in box A and the other in box B.

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Statements about the instructor(s):

Respond to the statements below for instructor A (and instructor B) bearing in mind that there are wide variations in class size and subject matter in Arts and Science.

		extremely poor	very poor	poor	adequate	good	very good	outstanding
2. Communicates goals and requirements of the course clearly and explicitly.	A:	1	2	3	4	5	6	● 7
	B:	1	2	3	4	5	6	7
3. Uses methods of evaluation (e.g. papers, assignments, tests) that appropriately reflect the subject matter and provide a fair evaluation of student learning.	A:	1	2	3	4	5	● 6	7
	B:	1	2	3	4	5	6	7
4. Presents material in an organized, well-planned manner.	A:	1	2	3	4	5	6	● 7
	B:	1	2	3	4	5	6	7
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	B:	1	2	3	4	5	6	7
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	B:	1	2	3	4	5	6	7
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9. Ensures that student work is graded fairly, with helpful comments and feedback where appropriate.	A:	1	2	3	4	5	● 6	7
	B:	1	2	3	4	5	6	7
10. Ensures that student work is graded within a reasonable time.	A:	1	2	3	4	5	6	● 7
	B:	1	2	3	4	5	6	7
11. All things considered, performs effectively as a university teacher.	A:	1	2	3	4	5	6	● 7
	B:	1	2	3	4	5	6	7

PART I CONTINUES ON THE REVERSE SIDE

Statements about the course: Respond to the statements below, using the following 7-point scale.

SIDE 2

	very low	low	below average	average	above average	high	very high
12. Compared to other courses at the same level (100,200,300,400), the work load is	1	2	3	4	**5**	6	7
13. Compared to other courses at the same level, the level of difficulty of the material is	1	2	3	4	5	**6**	7
14. The value of the required reading is	1	2	3	4	5	6	**7**
15. (If applicable) The value of the tutorials is	**1**	2	3	4	5	6	7
16. (If applicable) The value of the laboratories is	1	2	3	4	5	6	7
17. (If applicable) The value of the seminars is	1	2	3	4	5	6	7
18. (If applicable) The value of the language conversation classes is	1	2	3	4	5	6	7
19. The value of the overall learning experience is	1	2	3	4	5	6	**7**

20. Considering your experience with this course, and disregarding your need for it to meet program or degree requirements, would you still have taken this course? **Yes** ○ No

Statements about yourself:

21. Number of full course credits already earned (prior to this session):
1 0-4½ 2 5-9½ 3 10-14½ 4 15-19½ 5 ≥20

22. Status of the course for you:
1 Program Requirement 2 Selected from a required list in a program 3 Breadth Requirement 4 Optional

23. Your level of enthusiasm to take this course at the time of initial registration:
1 low **2** medium 3 high

24. Your expected grade in this course:
1 <50 2 50-59 **3** 60-69 4 70-79 5 ≥80

Additional statements or questions which may be supplied in class:

25. 1 2 3 4 5 6 7	28. 1 2 3 4 5 6 7	31. 1 2 3 4 5 6 7	34. 1 2 3 4 5 6 7
26. 1 2 3 4 5 6 7	29. 1 2 3 4 5 6 7	32. 1 2 3 4 5 6 7	35. 1 2 3 4 5 6 7
27. 1 2 3 4 5 6 7	30. 1 2 3 4 5 6 7	33. 1 2 3 4 5 6 7	36. 1 2 3 4 5 6 7

PART II: PLEASE ANSWER ONLY AFTER COMPLETING PART I. Please use the space below to provide supplementary comments on the instructor(s) or course. For example, you may wish to give the reasons for your numerical evaluations or provide specific suggestions for improving the instruction in the course.

- great professor, good enthusiasm, brings up enthusiasm in students
- great examples in class
- good order/organization
- a little slow to start → had to rush at the end
- EXCITED for next algebra course now!! (247!)

END OF SURVEY. THANK YOU FOR YOUR COOPERATION AND PARTICIPATION.

STUDENT SURVEY FORM
UNIVERSITY OF TORONTO

FACULTY OF ARTS & SCIENCE

Note that survey results will be available to the instructor(s) only after final course marks have been submitted.

PART I: INSTRUCTIONS. PLEASE READ FIRST.

Using an HB pencil or a blue or black ball-point pen (but not a felt marking pen), fill completely the numbered oval corresponding to your response for each statement. If using a pen, do not alter original response by making another selection.

Part II *(on the reverse side)* requires a written answer.

Course Identification: Please print course and section you are evaluating

COURSE MAT2404H1F SECTION L101

INSTRUCTOR(S):

A:	Bar-Natan
B:	

1. If evaluating only one instructor, write the name in the upper (A) box. If evaluating two instructors, write their names, one in box A and the other in box B.

DO NOT EVALUATE TEACHING ASSISTANTS ON THIS FORM

Statements about the instructor(s):

Respond to the statements below for instructor A (and instructor B) bearing in mind that there are wide variations in class size and subject matter in Arts and Science.

		extremely poor	very poor	poor	adequate	good	very good	outstanding
2. Communicates goals and requirements of the course clearly and explicitly.	A:	1	2	3	4	5	6	●
	B:	1	2	3	4	5	6	7
3. Uses methods of evaluation (e.g. papers, assignments, tests) that appropriately reflect the subject matter and provide a fair evaluation of student learning.	A:	1	2	3	4	5	●	7
	B:	1	2	3	4	5	6	7
4. Presents material in an organized, well-planned manner.	A:	1	2	3	4	5	6	●
	B:	1	2	3	4	5	6	7
5. Explains concepts clearly with appropriate use of examples.	A:	1	2	3	4	5	6	●
	B:	1	2	3	4	5	6	7
6. Communicates enthusiasm and interest in the course material.	A:	1	2	3	4	5	6	●
	B:	1	2	3	4	5	6	7
7. Attends to students' questions and answers them clearly and effectively.	A:	1	2	3	4	5	●	7
	B:	1	2	3	4	5	6	7
8. Is available for individual consultation, by appointment or stated office hours, to students with problems relating to the course.	A:	1	2	3	4	5	●	7
	B:	1	2	3	4	5	6	7
9. Ensures that student work is graded fairly, with helpful comments and feedback where appropriate.	A:	1	2	3	●	5	6	7
	B:	1	2	3	4	5	6	7
10. Ensures that student work is graded within a reasonable time.	A:	1	2	3	4	5	6	●
	B:	1	2	3	4	5	6	7
11. All things considered, performs effectively as a university teacher.	A:	1	2	3	4	5	6	●
	B:	1	2	3	4	5	6	7

PART I CONTINUES ON THE REVERSE SIDE

Statements about the course: Respond to the statements below, using the following 7-point scale.

SIDE 2

	very low	low	below average	average	above average	high	very high
12. Compared to other courses at the same level (100,200,300,400), the work load is	1	2	3	4	5	●	7
13. Compared to other courses at the same level, the level of difficulty of the material is	1	2	3	4	5	●	7
14. The value of the required reading is	1	2	3	4	5	6	●
15. (If applicable) The value of the tutorials is	●	2	3	4	5	6	7
16. (If applicable) The value of the laboratories is	1	2	3	4	5	6	7
17. (If applicable) The value of the seminars is	1	2	3	4	5	6	7
18. (If applicable) The value of the language conversation classes is	1	2	3	4	5	6	7
19. The value of the overall learning experience is	1	2	3	4	5	6	●

20. Considering your experience with this course, and disregarding your need for it to meet program or degree requirements, would you still have taken this course? ● Yes ○ No

Statements about yourself:

21. Number of full course credits already earned (prior to this session):
① 0-4½ ● 5-9½ ③ 10-14½ ④ 15-19½ ⑤ ≥20

22. Status of the course for you:
● Program Requirement ② Selected from a required list in a program ③ Breadth Requirement ④ Optional

23. Your level of enthusiasm to take this course at the time of initial registration:
① low ② medium ● high

24. Your expected grade in this course:
① <50 ② 50-59 ③ 60-69 ● 70-79 ⑤ ≥80

Additional statements or questions which may be supplied in class:

25. ① ② ③ ④ ⑤ ⑥ ⑦	28. ① ② ③ ④ ⑤ ⑥ ⑦	31. ① ② ③ ④ ⑤ ⑥ ⑦	34. ① ② ③ ④ ⑤ ⑥ ⑦
26. ① ② ③ ④ ⑤ ⑥ ⑦	29. ① ② ③ ④ ⑤ ⑥ ⑦	32. ① ② ③ ④ ⑤ ⑥ ⑦	35. ① ② ③ ④ ⑤ ⑥ ⑦
27. ① ② ③ ④ ⑤ ⑥ ⑦	30. ① ② ③ ④ ⑤ ⑥ ⑦	33. ① ② ③ ④ ⑤ ⑥ ⑦	36. ① ② ③ ④ ⑤ ⑥ ⑦

PART II: PLEASE ANSWER ONLY AFTER COMPLETING PART I. Please use the space below to provide supplementary comments on the instructor(s) or course. For example, you may wish to give the reasons for your numerical evaluations or provide specific suggestions for improving the instruction in the course.

- tutorials were very poorly managed, and not in sync with the course content

- Wiki was helpful, but I don't think "good deed" points would be required to make it helpful

- hints/solutions to assigned problems would be helpful.

END OF SURVEY. THANK YOU FOR YOUR COOPERATION AND PARTICIPATION.

STUDENT SURVEY FORM
UNIVERSITY OF TORONTO

FACULTY OF ARTS & SCIENCE

Note that survey results will be available to the instructor(s) only after final course marks have been submitted.

PART I: INSTRUCTIONS. PLEASE READ FIRST.

Using an HB pencil or a blue or black ball-point pen (but not a felt marking pen), fill completely the numbered oval corresponding to your response for each statement. If using a pen, do not alter original response by making another selection.

Part II *(on the reverse side)* requires a written answer.

Course Identification: Please print course and section you are evaluating

COURSE MAT240H1F SECTION C0101

INSTRUCTOR(S):

A: Dror Bar-Natan

B:

1. If evaluating only one instructor, write the name in the upper (A) box. If evaluating two instructors, write their names, one in box A and the other in box B.

DO NOT EVALUATE TEACHING ASSISTANTS ON THIS FORM

Statements about the instructor(s):

Respond to the statements below for instructor A (and instructor B) bearing in mind that there are wide variations in class size and subject matter in Arts and Science.

		extremely poor	very poor	poor	adequate	good	very good	outstanding
2. Communicates goals and requirements of the course clearly and explicitly.	A:	1	2	3	4	5	6	●
	B:	1	2	3	4	5	6	7
3. Uses methods of evaluation (e.g. papers, assignments, tests) that appropriately reflect the subject matter and provide a fair evaluation of student learning.	A:	1	2	3	●	5	6	7
	B:	1	2	3	4	5	6	7
4. Presents material in an organized, well-planned manner.	A:	1	2	3	4	5	6	●
	B:	1	2	3	4	5	6	7
5. Explains concepts clearly with appropriate use of examples.	A:	1	2	3	4	5	6	●
	B:	1	2	3	4	5	6	7
6. Communicates enthusiasm and interest in the course material.	A:	1	2	3	4	5	6	●
	B:	1	2	3	4	5	6	7
7. Attends to students' questions and answers them clearly and effectively.	A:	1	2	3	4	5	6	●
	B:	1	2	3	4	5	6	7
8. Is available for individual consultation, by appointment or stated office hours, to students with problems relating to the course.	A:	1	2	3	4	5	6	●
	B:	1	2	3	4	5	6	7
9. Ensures that student work is graded fairly, with helpful comments and feedback where appropriate.	A:	1	2	●	4	5	6	7
	B:	1	2	3	4	5	6	7
10. Ensures that student work is graded within a reasonable time.	A:	1	2	3	4	●	6	7
	B:	1	2	3	4	5	6	7
11. All things considered, performs effectively as a university teacher.	A:	1	2	3	4	5	6	●
	B:	1	2	3	4	5	6	7

PART I CONTINUES ON THE REVERSE SIDE

Statements about the course: Respond to the statements below, using the following 7-point scale.

SIDE 2

	very low	low	below average	average	above average	high	very high
12. Compared to other courses at the same level (100,200,300,400), the work load is	1	2	3	4	5	●	7
13. Compared to other courses at the same level, the level of difficulty of the material is	1	2	3	4	5	●	7
14. The value of the required reading is	1	●	3	4	5	6	7
15. (If applicable) The value of the tutorials is	●	2	3	4	5	6	7
16. (If applicable) The value of the laboratories is	1	2	3	4	5	6	7
17. (If applicable) The value of the seminars is	1	2	3	4	5	6	7
18. (If applicable) The value of the language conversation classes is	1	2	3	4	5	6	7
19. The value of the overall learning experience is	1	2	3	4	●	6	7

20. Considering your experience with this course, and disregarding your need for it to meet program or degree requirements, would you still have taken this course? ● Yes ○ No

Statements about yourself:

21. Number of full course credits already earned (prior to this session):
● 0-4½ 2 5-9½ 3 10-14½ 4 15-19½ 5 ≥20

22. Status of the course for you:
1 Program Requirement 2 Selected from a required list in a program 3 Breadth Requirement ● Optional

23. Your level of enthusiasm to take this course at the time of initial registration:
1 low 2 medium ● high

24. Your expected grade in this course:
1 <50 2 50-59 ● 60-69 4 70-79 5 ≥80

Additional statements or questions which may be supplied in class:

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26. 1 2 3 4 5 6 7	29. 1 2 3 4 5 6 7	32. 1 2 3 4 5 6 7	35. 1 2 3 4 5 6 7
27. 1 2 3 4 5 6 7	30. 1 2 3 4 5 6 7	33. 1 2 3 4 5 6 7	36. 1 2 3 4 5 6 7

PART II: PLEASE ANSWER ONLY AFTER COMPLETING PART I. Please use the space below to provide supplementary comments on the instructor(s) or course. For example, you may wish to give the reasons for your numerical evaluations or provide specific suggestions for improving the instruction in the course.

Bar-Nater is the most effective professor I have had. It is unfortunate that the completely useless tutorials damaged my experience in this course (for they were greatly needed and constantly disappointing).

END OF SURVEY. THANK YOU FOR YOUR COOPERATION AND PARTICIPATION.

Faculty of Arts & Science
University of Toronto
Survey Summary Results: FALL, 2006

Course: MAT 240H1F
Section: L0101
Instructor: D. BAR-NATAN

Enrolment: 76
Number of Forms Scanned: 45

STATEMENTS ABOUT THE INSTRUCTOR:

Quest.	% Resp. to Scale Rating							No.	Mean
	1	2	3	4	5	6	7		
2.	0	0	0	4	8	26	60	45	6.4
3.	0	0	2	13	13	40	31	45	5.8
4.	0	0	0	8	2	28	60	45	6.4
5.	0	0	0	6	11	27	54	44	6.3
6.	0	0	0	4	2	20	73	45	6.6
7.	0	0	0	8	4	35	51	45	6.3
8.	0	0	0	11	15	27	45	44	6.1
9.	0	0	4	15	17	35	26	45	5.6
10.	0	0	0	2	24	26	46	45	6.2
11.	0	0	0	4	2	27	65	44	6.5

STATEMENTS ABOUT THE COURSE:

Quest.	% Resp. to Scale Rating							No.	Mean
	1	2	3	4	5	6	7		
12.	0	0	0	46	37	11	4	45	4.7
13.	0	0	2	33	33	24	6	45	5.0
14.	2	6	4	20	32	20	11	43	4.8
15.	21	26	11	21	2	11	4	42	3.1
16.	25	0	0	50	25	0	0	4	3.5
17.	0	0	0	40	0	40	20	5	5.4
18.	0	0	7	28	14	42	7	14	5.1
19.	0	0	2	16	16	32	32	37	5.8
20.	Yes: 87%			No: 12%				40	

OTHER QUESTIONS:

Quest.	% Resp. to Scale Rating							No.	Mean
	1	2	3	4	5	6	7		
25.	0	0	0	0	0	0	0	0	0.0
26.	0	0	0	0	0	0	0	0	0.0
27.	0	0	0	0	0	0	0	0	0.0
28.	0	0	0	0	0	0	0	0	0.0
29.	0	0	0	0	0	0	0	0	0.0
30.	0	0	0	0	0	0	0	0	0.0

Quest.	% Resp. to Scale Rating							No.	Mean
	1	2	3	4	5	6	7		
31.	0	0	0	0	0	0	0	0	0.0
32.	0	0	0	0	0	0	0	0	0.0
33.	0	0	0	0	0	0	0	0	0.0
34.	0	0	0	0	0	0	0	0	0.0
35.	0	0	0	0	0	0	0	0	0.0
36.	0	0	0	0	0	0	0	0	0.0

MEAN RATING ON QUESTION 11 (GLOBAL EVALUATION OF INSTRUCTOR) AS A FUNCTION OF STUDENT INFORMATION:

		No.	Mean Global Eval.
21.	Number of full courses already completed:		
	0- 4.5	27	6.4
	5- 9.5	9	6.9
	10-14.5	4	7.0
	15-19.5	1	6.0
	>=20	1	6.0
22.	Status of the course for the student:		
	Program Requirement	30	6.4
	Selected from a required list in a program	5	6.6
	Breadth requirement	0	-
	Optional	5	7.0
23.	Initial enthusiasm to take course:		
	low	2	6.5
	medium	15	6.6
	high	25	6.5
24.	Expected grade in course:		
	<50	0	-
	50-59	2	6.0
	60-69	6	6.7
	70-79	10	6.4
	80-89	24	6.6
	>=90	0	-