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In[*]:= ω = (a2 + a b + b2)1/2;
λ = FullSimplify[ ( (Sinh[a + b] / (a + b) - ω / Sinh[ω]) / (a b) )
```

Out[*]=

$$-1 + \frac{\sqrt{a^2 + a b + b^2} \operatorname{Csch}\left[\sqrt{a^2 + a b + b^2}\right] \operatorname{Sinh}[a + b]}{a b}$$

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In[*]:= λ /. b -> 0
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Power: Infinite expression $\frac{1}{0}$ encountered.

Out[*]= ComplexInfinity

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In[*]:= Assuming[a > 0, Limit[λ, b -> 0, Direction -> "FromAbove"]]
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Out[*]= \$Aborted

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In[*]:= ser = Collect[
  Normal@Series[λ /. {a -> a ħ, b -> b ħ}, {ħ, 0, 14}],
  ħ, Simplify]
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Out[*]=

$$\frac{1}{6} + \frac{1}{360} (-4 a^2 - a b - 4 b^2) \hbar^2 + \frac{(16 a^4 + 20 a^3 b + 39 a^2 b^2 + 20 a b^3 + 16 b^4) \hbar^4}{15120} +$$

$$\frac{(-192 a^6 - 432 a^5 b - 904 a^4 b^2 - 947 a^3 b^3 - 904 a^2 b^4 - 432 a b^5 - 192 b^6) \hbar^6}{1814400} +$$

$$\frac{1}{119750400} (1280 a^8 + 4160 a^7 b + 10192 a^6 b^2 + 15224 a^5 b^3 + 18379 a^4 b^4 +$$

$$15224 a^3 b^5 + 10192 a^2 b^6 + 4160 a b^7 + 1280 b^8) \hbar^8 + \frac{1}{653837184000}$$

$$(-707584 a^{10} - 3007232 a^9 b - 8642048 a^8 b^2 - 16350800 a^7 b^3 - 24212692 a^6 b^4 - 26994107 a^5 b^5 -$$

$$24212692 a^4 b^6 - 16350800 a^3 b^7 - 8642048 a^2 b^8 - 3007232 a b^9 - 707584 b^{10}) \hbar^{10} +$$

$$\frac{1}{3923023104000} (430080 a^{12} + 2257920 a^{11} b + 7510784 a^{10} b^2 + 17019136 a^9 b^3 +$$

$$29908544 a^8 b^4 + 41063468 a^7 b^5 + 45842287 a^6 b^6 + 41063468 a^5 b^7 +$$

$$29908544 a^4 b^8 + 17019136 a^3 b^9 + 7510784 a^2 b^{10} + 2257920 a b^{11} + 430080 b^{12}) \hbar^{12} +$$

$$\frac{1}{5335311421440000} (-59260928 a^{14} - 370380800 a^{13} b - 1405298688 a^{12} b^2 -$$

$$3691121920 a^{11} b^3 - 7501138048 a^{10} b^4 - 12124407648 a^9 b^5 - 16095995120 a^8 b^6 -$$

$$17633046593 a^7 b^7 - 16095995120 a^6 b^8 - 12124407648 a^5 b^9 - 7501138048 a^4 b^{10} -$$

$$3691121920 a^3 b^{11} - 1405298688 a^2 b^{12} - 370380800 a b^{13} - 59260928 b^{14}) \hbar^{14}$$

In[*]:= ser /. b -> 0

Out[*]=

$$\frac{1}{6} - \frac{a^2 \hbar^2}{90} + \frac{a^4 \hbar^4}{945} - \frac{a^6 \hbar^6}{9450} + \frac{a^8 \hbar^8}{93555} - \frac{691 a^{10} \hbar^{10}}{638512875} + \frac{2 a^{12} \hbar^{12}}{18243225} - \frac{3617 a^{14} \hbar^{14}}{325641566250}$$

In[*]:= Series[Log[$\frac{x}{e^x - e^{-x}}$], {x, 0, 16}]

Out[*]=

$$-\text{Log}[2] - \frac{x^2}{6} + \frac{x^4}{180} - \frac{x^6}{2835} + \frac{x^8}{37800} - \frac{x^{10}}{467775} + \frac{691 x^{12}}{3831077250} - \frac{2 x^{14}}{127702575} + \frac{3617 x^{16}}{2605132530000} + O[x]^{17}$$

In[*]:= 638512875 / 2730

Out[*]=

$$\frac{467775}{2}$$

In[*]:= 2605132530000 / 325641566250

Out[*]=

$$8$$