Peter Lee on October 20, 2009

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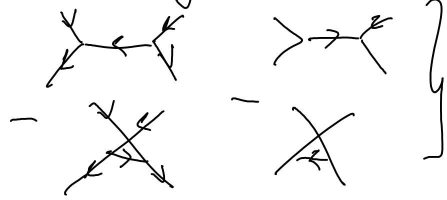
Def An LBA (Lie-Bi-Alg) is a triple $(\mathfrak{g}, [\,,], \delta)$ where $(\mathfrak{g}, [\,])$ is a Lie algebra, $\delta: \mathfrak{g} \to \mathfrak{g} \otimes \mathfrak{g}$ is AS. and satisfies co-Jac, and s.t. together $[\,,]$ satisfy the "co-cycle condition".

Co-Jac: $Alt((\delta \otimes I) \circ \delta) = 0$

Co-Cycle: $\delta([a,b]) = a_{\cdot}\delta(b) - b_{\cdot}\delta(a)$

The action of a on $\Lambda^2 \mathfrak{g}$



careless signs.

Prop If $\mathfrak{g}$ is an LBA, so is $\mathfrak{g}^*$

LA cohomology: $\mathfrak{g}$ a Lie-alg, V - a $\mathfrak{g}$-module.

$$C^m = \mathrm{Hom}_{\mathbb{Q}}(\Lambda^m \mathfrak{g}, V) \qquad (\text{so } C^0 = V)$$

$\partial^n : C^n \to C^{n+1}$ by

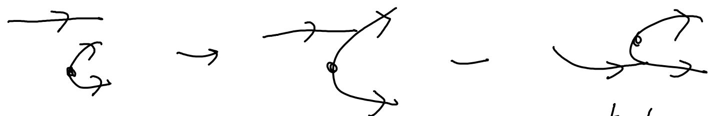
$$(\partial^n f)(x_1 \wedge \dots \wedge x_{n+1})$$
$$= \sum (-1)^{i+1} x_i f(x_1 \wedge \dots \wedge \hat{x}_i \wedge \dots x_{n+1})$$
$$+ \sum_{i<j} (-1)^{i+j} f([x_i, x_j] \wedge x_1 \wedge \dots \hat{x}_i \wedge \dots \hat{x}_j \wedge \dots x_{n+1})$$

Note $H^0 = \ker \partial^0 = V^{\mathfrak{g}}$

Note $f \in C^1$ $\quad \partial f(a \wedge b) = -f([a,b]) + a.f(b) - b.f(a)$

$$\partial f = 0 \iff f([a,b]) = a.f(b) - b.f(a)$$

With $V = \Lambda^2 \mathfrak{g}$. $r \in \Lambda^2 \mathfrak{g}$ is a "co-boundary structure" of an LBA $(\mathfrak{g}, [\,], \delta)$ if $\delta = \partial r$.



take $r = \sum r_1^i \otimes r_2^i$

i.e. $\delta(a) = (\partial r)(a) = a.r = [a, r_1] \otimes r_2 + r_1 \otimes [a, r_2]$

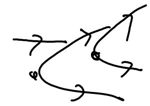
For this (δ) to be a co-bracket, r needs to satisfy a cond. on $[r_{12}, r_{13}] + [r_{12}, r_{23}] + [r_{13}, r_{23}] =: CYB(r)$

In this case, $(g, [\,], \delta_r)$ is called "co-boundary".

GLF: (Great Little Factoid)

$(g, [\,], \partial r)$ is an LBA

Here $V = g \otimes g$.



iff 1. $Sym(r) := r_{12} + r_{21}$ is g-invariant

2. $CYB(r)$ is g-invariant.

key point: $Alt((\partial_r \otimes I) \circ \partial_r)(x) = [CYB(r), x]$

~~Remark 1~~

If r & r' are in $g \otimes g$ and r' is g-invariant,

$$\partial(r + r') = \partial(r)$$

Remark 2 There are four scenarios:

$sym(r) = 0$
$cyb(r) = 0$
"triangular LBA"

$g(sym\, r) = 0$
$cyb(r) = 0$
"quasi-triangular LBA"

$sym(r) = 0$
$g(cyb(r)) = 0$
"No name"

$g(sym\, r) = 0$
$g(cyb(r)) = 0$
"Co-boundary LBA"