Peter Lee on November 23, 2009

November-23-09
9:37 AM

$(\mathfrak{g}, \mathfrak{g}_+, \mathfrak{g}_-)$ is a f.d. Manin triple

1. $U(\mathfrak{g}) \underset{PBW}{\cong} M_+ \otimes M_- \cong U(\mathfrak{g}_-)1_+ \otimes U(\mathfrak{g}_+)1_-$

 by $1 \mapsto 1_+ \otimes 1_-$ (and continue $\mathfrak{g}$-equivariantly)

2. $F = \text{Forget}: \mathcal{M} \to \text{Vect}$ with $\mathcal{M} = \text{Rep}(U(\mathfrak{g}))$

 ${}_{U(\mathfrak{g})}M \mapsto {}_{\mathbb{C}}M$

 $F(M) = \text{Hom}_{\mathfrak{g}}(U(\mathfrak{g}), M)$

 $f(1) \in M \longleftarrow\!\shortmid f$

3. $F(M) = \text{Hom}_{\mathfrak{g}}(M_+ \otimes M_-, M)$

4. Given Φ a Drinfel'd associator, with $R = e^{\hbar t^{12}/2}$, we get a braiding σR ...

<u>Note</u> If $M, N \in \mathcal{M}$, $M \otimes N$ is the usual tensor product; so if $g \in \mathfrak{g}$, $\Delta g = g \otimes 1 + 1 \otimes g$. With $\Phi = 1$ the hexagon is not satisfied as $e^{t^{12}+t^{13}} \neq e^{t^{12}} e^{t^{13}}$.
We will see that the braided monoidal structure on $\mathcal{M}$ induces a new coproduct on $U(\mathfrak{g})$

<u>Recall</u> A tensor structure on a functor $F: \mathcal{C} \to \mathcal{D}$ between two monoidal categories is a family of natural isomorphisms

$$F(X) \otimes F(Y) \xrightarrow{J_{XY}} F(X \otimes Y)$$

$$F(X)\otimes F(Y) \xrightarrow{J_{XY}} F(X\otimes Y)$$

s.t. $\quad J_{X\otimes Y,Z}\circ J_{X,Y} = J_{X,Y\otimes Z}\circ J_{YZ}$

& $\quad J_{X1} = J_{1X} = Id \quad$ (+ an extra requirement related to the braiding)

A tensor structure on $F = \text{Forget} = \text{Hom}_{\mathfrak{g}}(M_+\otimes M_-, -)$:
Let $v\in F(V)$, $w\in F(W)$, we need $J_{V,W}: F(V)\otimes F(W) \to F(V\otimes W)$

$J_{V,W}(v\otimes w): M_+\otimes M_- \longrightarrow V\otimes W$ by

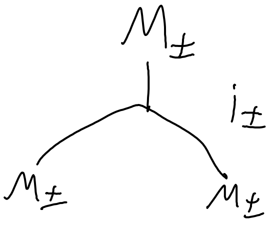
$$M_+\otimes M_- \xmapsto{i_+\otimes i_-} (M_+\otimes M_+)\otimes(M_-\otimes M_-) \xrightarrow[\&\text{ braidings}]{\text{assoc.}} (M_+\otimes M_-)\otimes(M_+\otimes M_-)$$
$$\xmapsto{v\otimes w} V\otimes W$$

$$1_+\otimes 1_- \mapsto (1_+\otimes 1_+)\otimes(1_-\otimes 1_-) \longmapsto \ldots\ldots$$

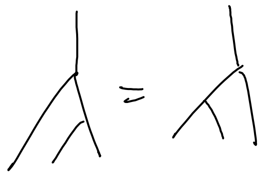
Define $j_{VW}(\hbar) := \underbrace{(\text{assoc.})\circ(1\otimes\beta\otimes 1)\circ(\text{assoc})}_{\text{what appears in the def. of } J_{V,W}} \in U(\mathfrak{g})^{\otimes 4}$
$= 1 + \ldots\ldots$

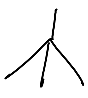
Then $v\otimes w \xmapsto{J_{V,W}} (v\otimes w)\big(j_{V,W}(\hbar)(1_+\otimes 1_-\otimes 1_+\otimes 1_-)\big)$
invertible, so J's invertible.

Coherence (Pictorially)



$i_\pm$; co-associativity is

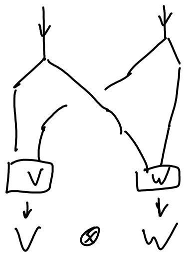


(so we can just write )

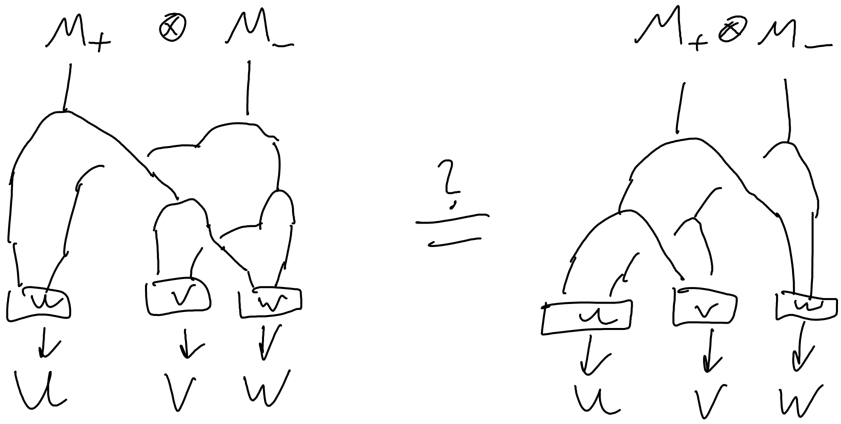
So

$J_{V,W}(v\otimes w)$ is $\qquad M_+ \otimes M_-$

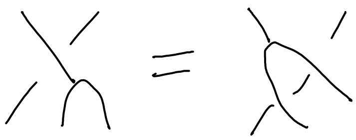
$J_{VW}(V\otimes W)$ is



Coherence $J_{U\otimes V,W}\circ J_{U,V} = J_{U,V\otimes W}\circ J_{V,W}$ becomes:



<u>Note:</u>



by the functoriality of β.

..... Q.E.D.