Peter Lee on November 10, 2009

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LBAs $\leftrightarrow$ Manin triples.

LBA $a \longrightarrow Da = a \oplus a^*$ w/ $\delta = \delta_a - \delta_{a^*}$

If $r = \sum g_i \otimes g^i$ (g_i a basis of a) then $\delta = \partial r$,

$r + r^{21}$ is Da-invt,

$CYB(r) = 0$

$\Rightarrow Da$ is a QTLBA.

Drinfel'd Category. Let (g, g_+, g_-) be a Manin triple. Let M_g be the category with

$$Ob(M_g) = \{g\text{-modules}\}$$

$$Hom_{M_g}(U,V) = [Hom_g(U,V)][[\hbar]]$$

claim M_g is a braided monoidal category:

$\exists$ bifunctor $M \times M \to M$ which is:

braided monoidal / monoidal

1. quasi-associative (... pentagon...)
2. unit.
3. $B_{xy}: X \otimes Y \to Y \otimes X$, ... hexagons ...

.... proof using associators .....

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① A topological $k[[\hbar]]$ module is a $k[[\hbar]]$ module topologized & completed with the $\hbar$-adic topology.

② A topologically free $k[[\hbar]]$-module is a ① which is $\cong V[[\hbar]]$ for some V.

Let A denote the category of top free $k[[\hbar]]$-modules with morphisms being cont. $k[[\hbar]]$ linear maps.

There is a simple-minded tensor structure on $\mathcal{A}$:

$$V[[\hbar]] \otimes W[[\hbar]] := (V \otimes W)[[\hbar]]$$

There is a forgetful functor $F : \mathcal{M}_{\mathfrak{g}} \to \mathcal{A}$

<u>Claim</u> $F(M) = \mathrm{Hom}_{\mathcal{M}}(U(\mathfrak{g}), M)$

<u>Verma modules:</u> Let

$$M_+ := \mathrm{Ind}_{\mathfrak{g}_+}^{\mathfrak{g}} \mathbb{1}_{\mathfrak{g}_+} = U(\mathfrak{g}) \otimes_{\mathfrak{g}_+} \mathbb{1}_{\mathfrak{g}_+} = U(\mathfrak{g})/U(\mathfrak{g}_+)$$

Likewise for M_-.

<u>Claim</u> $U(\mathfrak{g}_\mp) \otimes U(\mathfrak{g}_\pm) \to U(\mathfrak{g})$ are linear isomorphisms.

$\Rightarrow$ $M_+ \cong U(\mathfrak{g}_-)$, $M_- \cong U(\mathfrak{g}_+)$ as v.s.

Therefore there are $i_+ : M_+ \to M_+ \otimes M_+$

$i_- : M_- \to M_- \otimes M_-$

with

$$\begin{tikzcd}
 & M_+ \arrow[dl, "i_+"'] \arrow[dr, "i_+"] & \\
M_+ \otimes M_+ \arrow[d, "i_+ \otimes 1"'] & & M_+ \otimes M_+ \arrow[d, "1 \otimes i_+"] \\
(M_+ \otimes M_+) \otimes M_+ \arrow[rr, "\Phi"] & & M_+ \otimes (M_+ \otimes M_+)
\end{tikzcd}$$

(Follows from $\Phi(1_+ \otimes 1_+ \otimes 1_+) = 1_+ \otimes 1_+ \otimes 1_+$

There are also co-units $\epsilon_\pm : M_\pm \to K$ w/

$$(\epsilon_\pm \otimes 1) \circ i_\pm = 1 = (1 \otimes \epsilon_\pm) \circ i_\pm$$