Top. Hopf Algebra: $A = V[[\hbar]]$ (completed tensor prods).
It is a deformation of A_0 if $A/\hbar A \cong A_0$.
It is a QUEA if $H/\hbar H \cong U(\mathfrak{g})$ for so Lie algebra $\mathfrak{g}$.

Claim Any BiAlg H which is a deformation of a HA H_0 has a unique antipode consistent with the antipode of H_0.

Recall $(\mathfrak{g}, \mathfrak{g}_+, \mathfrak{g}_-)$ a Manin triple, then

$$U(\mathfrak{g}) \xrightarrow[\phi]{\cong} U\mathfrak{g}_- \otimes U\mathfrak{g}_+ = M_+ \otimes M_- \quad \text{where}$$
$$M_\pm := U\mathfrak{g}_\mp 1_\pm$$

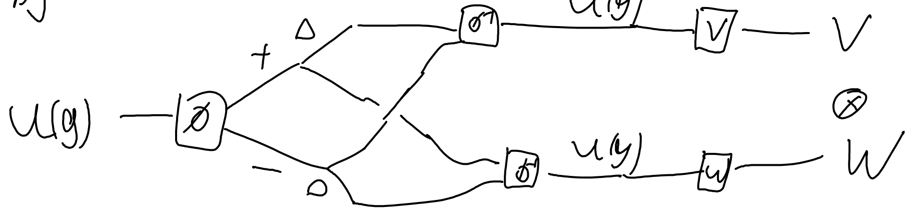
$\mathcal{M}_{\mathfrak{g}} := \mathrm{Rep}\, U\mathfrak{g}$, morphisms are power series of intertwiners.

$F := \mathrm{Forget} : \mathcal{M}_{\mathfrak{g}} \to \mathrm{Vect}[[\hbar]]$

$$F(V) \cong \mathrm{Hom}_{\mathfrak{g}}(U\mathfrak{g}, V)[[\hbar]]$$
$$\cong \mathrm{Hom}_{\mathfrak{g}}(M_+ \otimes M_-, V)[[\hbar]]$$

$$J_{VW} : F(V) \otimes F(W) \to F(V \otimes W) = \left\{ U\mathfrak{g} \xrightarrow{v \otimes w} V \otimes W \right\}$$

by



i.e.,

$$J_{VW}(v \otimes w) : 1_+ \otimes 1_- \xrightarrow{i_+ \otimes i_-} (1_+ \otimes 1_+) \otimes (1_- \otimes 1_-) \xrightarrow{\text{assoc}} M_+ \otimes (M_+ \otimes M_-) \otimes M_-$$
$$\xrightarrow[\text{\& assoc}]{\beta} (M_+ \otimes M_-) \otimes (M_+ \otimes M_-) \xrightarrow{v \otimes w} V \otimes W$$

In detail,

$$J_{VW}(1_+ \otimes 1_-) = (V \otimes W)\left(\Phi^{-1}_{1,2,34}\,\Phi_{2,3,4}\,e^{\hbar\Omega^{32}/2}\,\Phi^{-1}_{3,2,4}\,\Phi_{1,3,24}(1_+ 1_- 1_+ 1_-)\right)$$

set $J_\hbar = (\phi^{-1} \otimes \phi^{-1})(\qquad\qquad)$

So $J_\hbar \in Ug^{\otimes 2}[[\hbar]]$ and $J_{VW}(V \otimes W)(1_+ \otimes 1_-)$

$$= (V \otimes W)(J_\hbar) = J_\hbar (V 1 \otimes W 1)$$

.... $\Delta(\cdot) = J^{-1} \circ \Delta_0(\cdot) \circ J$

<u>Remark</u> Since $J \equiv 1 \mod \hbar$, in deg 0 $\Delta = \Delta_0$, $S = S_0$

So $Ug[[\hbar]]$ is a HA deformation of $U(g)$;

new structure of a

call it $U_\hbar g$

<u>THM</u>
* $U_\hbar g = Ug[[\hbar]]$ as a v.s.
* m, ϵ, η are the same.
* $\Delta = \Delta_0$, $S = S_0 \mod \hbar$
* $\Delta = J^{-1} \Delta_0 J$
* $S = Q S_0 Q^{-1}$ where $Q = m(S_0 \otimes 1) J$
* With $R = (J^{21})^{-1} e^{\hbar\Omega/2} J$, $(U_\hbar g, R)$ is a qTHA & a quantization of (g, r).
* The quasi-classical limit of $U_\hbar g$ is (g, δ).