The Main Theorem: Theorem 1.2 on page 5

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Theorem 1.2.
There exist "universal quantization functors"

(i) $Q : HA_{\langle \Delta - \Delta^{op}, S - S^{-1} \rangle} \to \overline{LBA_{\langle \delta \rangle}}$ *such that for any Lie bialgebra* $\mathfrak{a}$ *over* k $\widehat{Q}(\mathfrak{a}_h) = U_h(\mathfrak{a})$;

(ii) $Q^{qt} : QTHA_{\langle \Delta - \Delta^{op}, R-1, S - S^{-1} \rangle} \to \overline{QTLBA_{\langle r \rangle}}$ *such that for any quasitriangular Lie bialgebra* $\mathfrak{a}$ *over* k $\widehat{Q^{qt}}(\mathfrak{a}_h) = U_h^{qt}(\mathfrak{a})$, *where* $U_h^{qt}(\mathfrak{a})$ *is the quasitriangular quantization defined in Section 6.1;*

(iii) $Q^{YB} : QYBA_{\langle R-1 \rangle} \to CYBA_{\langle r \rangle}$ *such that for any classical Yang-Baxter algebra* (A, r) *over* k *one has* $\widehat{Q^{YB}}(A_h) = (A, R)$, *where* R *is constructed from* r *as explained in Chapter 5.*

What means? What means in topology?

(i) $Q : HA_{\langle \Delta - \Delta^{op}, S - S^{-1} \rangle} \longrightarrow \overline{LBA_{(\delta)}}$

1. Lie bialgebras. In this case the set S consists of two elements of bidegrees (2,1) and (1,2) ("the universal commutator and cocommutator"), and the category $\mathcal{C} = \text{LBA}$ is $\mathcal{F}_S/\mathcal{I}$, where $\mathcal{I}$ is generated by five relations – skew-symmetry and the Jacobi identity for the commutator and cocommutator and the condition that cocommutator is a 1-cocycle. A Lie bialgebra in $\mathcal{N}$ is an object with a linear algebraic structure of type LBA.

Given bra & cobra can write prod & co prod.

3. Hopf algebras. In this case the set S consists of six elements of bidegrees (2,1),(1,2),(0,1),(1,0),(1,1),(1,1) ("the universal product, coproduct, unit, counit, antipode,inverse antipode"), and the category $\mathcal{C} = \text{HA}$ is $\mathcal{F}_S/\mathcal{I}$, where $\mathcal{I}$ is generated by the relations coming from the axioms of a Hopf algebra. A Hopf algebra in $\mathcal{N}$ is an object with a linear algebraic structure of type HA.

(ii) $Q^{qt} : QTHA_{\langle \Delta - \Delta^{op}, R-1, S - S^{-1} \rangle} \longrightarrow \overline{QTLBA_{\langle r \rangle}}$

2. Quasitriangular Lie bialgebras. In this case the set S consists of two elements of bidegrees (2,1) and (0,2) ("the universal commutator and classical r-matrix"), and the category $\mathcal{C} = \text{QTLBA}$ is $\mathcal{F}_S/\mathcal{I}$, where $\mathcal{I}$ is generated by four relations – skew-symmetry and the Jacobi identity for the commutator, invariance of $r + r^{op}$, and the classical Yang-Baxter equation. A quasitriangular Lie bialgebra in $\mathcal{N}$ is an object with a linear algebraic structure of type QTLBA.

4. Quasitriangular Hopf algebras. In this case the set S consists of eight elements of bidegrees (2,1),(1,2),(0,1),(1,0),(1,1),(1,1),(0,2),(0,2) ("the universal product, coproduct, unit, counit, antipode, inverse antipode, R-matrix, inverse R-matrix"), and the category $\mathcal{C} = \text{QTHA}$ is $\mathcal{F}_S/\mathcal{I}$, where $\mathcal{I}$ is generated by the relations coming from the axioms of a quaitriangular Hopf algebra. A quasitriangular Hopf algebra in $\mathcal{N}$ is an object with a linear algebraic structure of type QTHA.

(iii) $Q^{YB} : QYBA_{\langle R-1 \rangle} \longrightarrow \overline{CYBA_{\langle r \rangle}}$

surely the overline was forgotten.

5. Classical Yang-Baxter algebras. In this case the set S consists of three elements of bidegrees (2,1),(0,1),(0,2) ("the universal product, unit, and r-matrix"), and the category $\mathcal{C}$ = CYBA is $\mathcal{F}_S/\mathcal{I}$, where $\mathcal{I}$ is generated by the associativity relation and the classical Yang-Baxter equation. A classical Yang-Baxter algebra in $\mathcal{N}$ is an object with a linear algebraic structure of type CYBA.

6. Quantum Yang-Baxter algebras. In this case the set S consists of three elements of bidegrees (2,1),(0,1),(0,2) ("the universal product, unit, and R-matrix"), and the category $\mathcal{C}$ = QYBA is $\mathcal{F}_S/\mathcal{I}$, where $\mathcal{I}$ is generated by the associativity relation and the quantum Yang-Baxter equation. A quantum Yang-Baxter algebra in $\mathcal{N}$ is an object with a linear algebraic structure of type QYBA.