Pensieve header: Kauffman States for tangles.

In[∘]:=
```
SetDirectory["C:\\drorbn\\AcademicPensieve\\2024-03"];
Once[<< KnotTheory`];
```

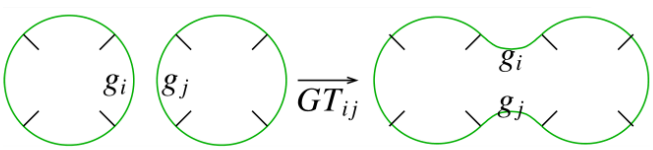
```
Loading KnotTheory` version of February 2, 2020, 10:53:45.2097.
Read more at http://katlas.org/wiki/KnotTheory.
```

In[∘]:=
```
CF[ℰ_] := Expand[ℰ];
```

In[∘]:=
```
SetAttributes[{B, M}, Orderless]; (* B for Boundary, M for Marked *)
CF[b_B] := RotateLeft[#, First@Ordering[#] - 1] & /@ DeleteCases[b, {}];
```

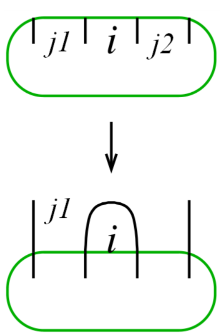
In[∘]:=
```
CF[G_b_[f_]] := G_CF[b][CF[f]]
```

In[∘]:=
```
G_b1_[f1_] ⊕ G_b2_[f2_] ^:=
  CF@G_Join[b1,b2][Expand[f1 f2] /. {M[m1___] M[m2___] :→ M[m1, m2], M[]^2 → M[]}];
```



GT for Gap Touch:

In[∘]:=
```
GT_{i_,j_}@G_B[{li___,i_,ri___},{lj___,j_,rj___},bs___][f_] := CF@G_B[{ri,li,j,rj,lj,i},bs][f /. {
    M[i, j, ___] → 0, M[i | j, ms___] :→ M[i, j, ms], M[ms___] :→ M[i, ms] + M[j, ms]
  }]
```



cor·don (kôr′dn)

n.

1. A line of people, military posts, or ships stationed around an area to enclose or guard it: *a police cordon.*

2. A rope, line, tape, or similar border stretched around an area, usually by the police, indicating that access is restricted.

```
In[∘]:= Cordon_i_@G_B[{li___,i_,ri___},bs___][f_] := Module[{j1, j2},
          {j1, j2} = {First@{ri, li}, Last@{ri, li}};
          CF@G_B[Most@{ri,li},bs][f
               /. {M[i, ms___] :> M[ms], M[___] → 0}
             /. {M[j1, j2, ___] → 0, M[j2, ms___] :> M[j1, ms]}
           ]
         ]
```

Strand Operations. c for contract, mc for magnetic contract:

```
In[∘]:= c_i_,j_@t : G_B[{li___,i_,ri___},{___,j_,___},___][__] := t // GT_j,First@{ri,li} // Cordon_j
```

```
In[∘]:= c_i_,j_@t : G_B[{___,i_,j_,___},___][__] := Cordon_j@t
        c_i_,j_@t : G_B[{j_,___,i_},___][__] := Cordon_j@t
        c_i_,j_@t : G_B[{___,j_,i_,___},___][__] := Cordon_i@t
        c_i_,j_@t : G_B[{i_,___,j_},___][__] := Cordon_i@t
```

```
In[∘]:= mc[ℰ_] := ℰ //.
          t : G_B[{___,i_,___},{___,j_,___},___][__] | G_B[{___,i_,j_,___},___][__] | G_B[{j_,___,i_},___][__] /;
            i + j == 0 :> c_i,j@t
```

“KSI” for Kauffman States Invariant.

```
In[∘]:= KSI@P_i_,j_ := CF@G_B[{i,j}][M[]];
        KSI[x : X[i_, j_, k_, l_]] := KSI@If[PositiveQ[x], X_-i,j,k,-l, X̄_-j,k,l,-i];
        KSI[X_i_,j_,k_,l_] := CF@G_B[{i,j,k,l}][μ T^-1 M[i] + T M[k] + M[l] + M[j]];
        KSI[X̄_i_,j_,k_,l_] := CF@G_B[{i,j,k,l}][μ T M[i] + T^-1 M[k] + M[l] + M[j]];
        KSI[K_] := Fold[mc[#1 ⊕ #2] &, G_B[][1], List @@ (KSI /@ PD@K)];
```

Knots

```
In[∘]:= Cut[pd_PD] := Module[{n = Length[pd]},
           pd /. {X[2 n, i_, 1, j_] :> X[2 n, i, 2 n + 1, j],
             X[i_, 1, j_, 2 n] :> X[i, 2 n + 1, j, 2 n], X[i_, 2 n, j_, 1] :> X[i, 2 n, j, 2 n + 1]}
          ];
        KSIK[K_] := KSI[Cut@PD@K][[1]] /. {M[] → 1, T → T^(1/2), μ → -1}
```

```
In[∘]:= Collect[KSI[Cut@PD@Knot[8, 17]][[1]] /. M[] → 1, μ, Expand]
```

Out[∘]=

$$3 + \left(\frac{4}{T^2} + 4\,T^2\right)\mu + \left(5 + \frac{3}{T^4} + 3\,T^4\right)\mu^2 + \left(\frac{1}{T^6} + \frac{3}{T^2} + 3\,T^2 + T^6\right)\mu^3 + \left(2 + \frac{1}{T^4} + T^4\right)\mu^4 + \left(\frac{1}{T^2} + T^2\right)\mu^5 + \mu^6$$

In[∘]:= **KSIK[Knot[8, 17]]**

Out[∘]=

$$11 - \frac{1}{T^3} + \frac{4}{T^2} - \frac{8}{T} - 8\,T + 4\,T^2 - T^3$$

In[∘]:= **Alexander[Knot[8, 17]][T]**

Out[∘]=

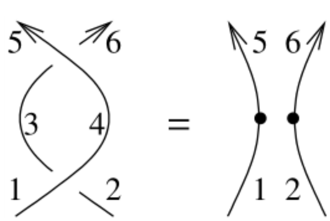
$$11 - \frac{1}{T^3} + \frac{4}{T^2} - \frac{8}{T} - 8\,T + 4\,T^2 - T^3$$

In[∘]:= **Monitor[Timing@Table[res = (K → KSIK[K] / Alexander[K][T]), {K, AllKnots[{3, 9}]}], res]**

Out[∘]=

{26.8438, {Knot[3, 1] → 1, Knot[4, 1] → 1, Knot[5, 1] → 1, Knot[5, 2] → 1,
Knot[6, 1] → 1, Knot[6, 2] → 1, Knot[6, 3] → 1, Knot[7, 1] → 1, Knot[7, 2] → 1,
Knot[7, 3] → 1, Knot[7, 4] → 1, Knot[7, 5] → 1, Knot[7, 6] → 1, Knot[7, 7] → 1,
Knot[8, 1] → 1, Knot[8, 2] → 1, Knot[8, 3] → 1, Knot[8, 4] → 1, Knot[8, 5] → 1,
Knot[8, 6] → 1, Knot[8, 7] → 1, Knot[8, 8] → 1, Knot[8, 9] → 1, Knot[8, 10] → 1,
Knot[8, 11] → 1, Knot[8, 12] → 1, Knot[8, 13] → 1, Knot[8, 14] → 1, Knot[8, 15] → 1,
Knot[8, 16] → 1, Knot[8, 17] → 1, Knot[8, 18] → 1, Knot[8, 19] → 1, Knot[8, 20] → 1,
Knot[8, 21] → 1, Knot[9, 1] → 1, Knot[9, 2] → 1, Knot[9, 3] → 1, Knot[9, 4] → 1,
Knot[9, 5] → 1, Knot[9, 6] → 1, Knot[9, 7] → 1, Knot[9, 8] → 1, Knot[9, 9] → 1,
Knot[9, 10] → 1, Knot[9, 11] → 1, Knot[9, 12] → 1, Knot[9, 13] → 1, Knot[9, 14] → 1,
Knot[9, 15] → 1, Knot[9, 16] → 1, Knot[9, 17] → 1, Knot[9, 18] → 1, Knot[9, 19] → 1,
Knot[9, 20] → 1, Knot[9, 21] → 1, Knot[9, 22] → 1, Knot[9, 23] → 1, Knot[9, 24] → 1,
Knot[9, 25] → 1, Knot[9, 26] → 1, Knot[9, 27] → 1, Knot[9, 28] → 1, Knot[9, 29] → 1,
Knot[9, 30] → 1, Knot[9, 31] → 1, Knot[9, 32] → 1, Knot[9, 33] → 1, Knot[9, 34] → 1,
Knot[9, 35] → 1, Knot[9, 36] → 1, Knot[9, 37] → 1, Knot[9, 38] → 1, Knot[9, 39] → 1,
Knot[9, 40] → 1, Knot[9, 41] → 1, Knot[9, 42] → 1, Knot[9, 43] → 1, Knot[9, 44] → 1,
Knot[9, 45] → 1, Knot[9, 46] → 1, Knot[9, 47] → 1, Knot[9, 48] → 1, Knot[9, 49] → 1}}

Reidemeister 2



In[∘]:= **lhs = CF[KSI@PD[$X_{-2,4,3,-1}$, $\overline{X}_{-4,6,5,-3}$]]**

Out[∘]=

$G_{B[\{-2,6,5,-1\}]}[\mu^2\, M[-2] + T\, M[-1] + T\,\mu\, M[-1] + M[5] + T\, M[6] + T\,\mu\, M[6]]$

In[∘]:= **rhs = $GT_{5,-2}$@KSI@PD[$P_{-1,5}$, $P_{-2,6}$]**

Out[∘]=

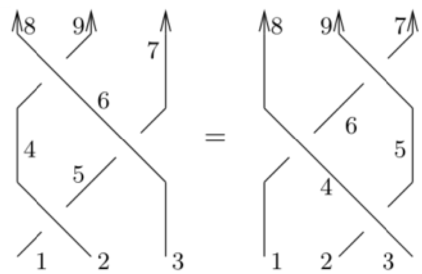
$G_{B[\{-2,6,5,-1\}]}[M[-2]+M[5]]$

In[∘]:= **lhs⟦1⟧ - rhs⟦1⟧ /. μ → -1**

Out[∘]=

0

Reidemeister 3



In[∘]:= **lhs = KSI[PD[$X_{-2,5,4,-1}$, $X_{-3,7,6,-5}$, $X_{-6,9,8,-4}$]]**

Out[∘]=

$$G_{B[\{-3,7,9,8,-1,-2\}]}\Big[\frac{\mu^3 M[-3,-2]}{T^3}+M[-3,-1]+\mu M[-3,-1]+\frac{\mu^2 M[-3,-1]}{T^2}+\frac{\mu M[-3,7]}{T}+T M[-3,8]+T\mu M[-3,8]+M[-3,9]+2\mu M[-3,9]+\frac{\mu M[-2,-1]}{T}+\frac{\mu^2 M[-2,7]}{T^2}+\mu M[-2,8]+\frac{\mu M[-2,9]}{T}+\frac{\mu^2 M[-2,9]}{T}+T M[-1,7]+\frac{\mu M[-1,7]}{T}+T M[-1,8]+M[-1,9]+T^2 M[-1,9]+\mu M[-1,9]+T^2 M[7,8]+T M[7,9]+T^3 M[8,9]\Big]$$

In[∘]:= **rhs = KSI[PD[$X_{-3,5,4,-2}$, $X_{-4,6,8,-1}$, $X_{-5,7,9,-6}$]]**

Out[∘]=

$$G_{B[\{-3,7,9,8,-1,-2\}]}\Big[\frac{\mu^3 M[-3,-2]}{T^3}+\frac{\mu^2 M[-3,-1]}{T^2}+\frac{\mu M[-3,7]}{T}+\frac{\mu M[-3,8]}{T}+\frac{\mu^2 M[-3,8]}{T}+\mu M[-3,9]+\frac{\mu M[-2,-1]}{T}+M[-2,7]+\mu M[-2,7]+\frac{\mu^2 M[-2,7]}{T^2}+M[-2,8]+2\mu M[-2,8]+T M[-2,9]+T\mu M[-2,9]+T M[-1,7]+\frac{\mu M[-1,7]}{T}+T M[-1,8]+T^2 M[-1,9]+M[7,8]+T^2 M[7,8]+\mu M[7,8]+T M[7,9]+T^3 M[8,9]\Big]$$

In[∘]:=
```
Collect[lhs[[1]] - rhs[[1]], μ]
```

Out[∘]=

$$M[-3,-1]+T\,M[-3,8]+M[-3,9]-M[-2,7]-M[-2,8]-T\,M[-2,9]+\mu^2\left(-\frac{M[-3,8]}{T}+\frac{M[-2,9]}{T}\right)+M[-1,9]+\mu\left(M[-3,-1]-\frac{M[-3,8]}{T}+T\,M[-3,8]+M[-3,9]-M[-2,7]-M[-2,8]+\frac{M[-2,9]}{T}-T\,M[-2,9]+M[-1,9]-M[7,8]\right)-M[7,8]$$

In[∘]:=
```
lhs[[1]] - rhs[[1]] /. μ → -1
```

Out[∘]=

0

Tree Counts

In[∘]:=
```
TC[K_] := KSI[Cut@PD@K][[1]] /. {M[] → 1, T → 1, μ → 1}
```

In[∘]:=
```
TC[Knot[3, 1]]
```

Out[∘]=

3

In[∘]:=
```
TC /@ AllKnots[{3, 9}]
```

Out[∘]=

{3, 5, 5, 7, 9, 11, 13, 7, 11, 13, 15, 17, 19, 21, 13, 17, 17, 19, 21, 23, 23, 25, 25, 27, 27, 29, 29, 31, 33, 35, 37, 45, 27, 33, 33, 9, 15, 19, 21, 23, 27, 29, 31, 31, 33, 33, 35, 37, 37, 39, 39, 39, 41, 41, 41, 43, 43, 45, 45, 47, 47, 49, 51, 51, 53, 55, 59, 61, 69, 27, 37, 45, 57, 55, 75, 49, 43, 53, 53, 53, 45, 69, 45, 49}

In[∘]:=
```
MaximalBy[AllKnots[{3, 9}], TC]
```

Out[∘]=

{Knot[9, 40]}

In[∘]:=
```
MaximalBy[AllKnots[{3, 10}], TC]
```

Out[∘]=

{Knot[10, 123]}

In[∘]:=
```
TC[Knot[10, 123]]
```

Out[∘]=

121

In[∘]:=
```
TC[TorusKnot[5, 3]]
```

Out[∘]=

121

In[∘]:=
```
TC[TorusKnot[7, 3]]
```

Out[∘]=

841

In[∘]:= **TC[TorusKnot[8, 3]]**

Out[∘]=

2205

In[∘]:= **KSIK[TorusKnot[8, 3]]**

Out[∘]=

$$-1+\frac{1}{T^7}-\frac{1}{T^6}+\frac{1}{T^4}-\frac{1}{T^3}+\frac{1}{T}+T-T^3+T^4-T^6+T^7$$

In[∘]:= **TC[TorusKnot[7, 4]]**

Out[∘]=

35 287

In[∘]:= **KSIK[TorusKnot[7, 4]]**

Out[∘]=

$$-1+\frac{1}{T^9}-\frac{1}{T^8}+\frac{1}{T^5}-\frac{1}{T^4}+\frac{1}{T^2}+T^2-T^4+T^5-T^8+T^9$$