Pensieve header: Kauffman States for tangles.

```
In[•]:= SetDirectory["C:\\drorbn\\AcademicPensieve\\2024-03"];
        Once[<< KnotTheory`];
        Cut[pd_PD] := Module[{n = Length[pd]},
          pd /. {X[2 n, i_, 1, j_] :> X[2 n, i, 2 n + 1, j],
            X[i_, 1, j_, 2 n] :> X[i, 2 n + 1, j, 2 n], X[i_, 2 n, j_, 1] :> X[i, 2 n, j, 2 n + 1]}
         ]
```

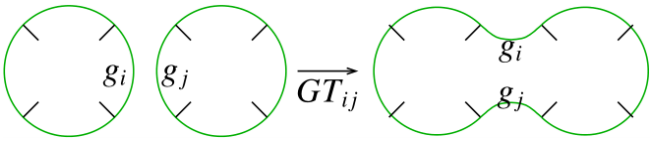
```
Loading KnotTheory` version of February 2, 2020, 10:53:45.2097.
Read more at http://katlas.org/wiki/KnotTheory.
```

```
In[•]:= CF[ℰ_] := Expand[ℰ];
```

```
In[•]:= SetAttributes[B, Orderless];
        CF[b_B] := RotateLeft[#, First@Ordering[#] - 1] & /@ DeleteCases[b, {}];
```

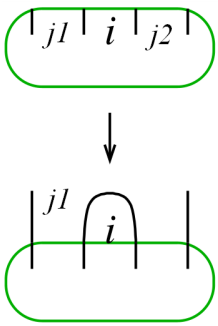
```
In[•]:= CF[G_b_[f_]] := G_CF[b][CF[f]]
```

```
In[•]:= G_b1_[f1_] ⊕ G_b2_[f2_] ^:= CF@G_Join[b1,b2][f1 f2];
```



GT for Gap Touch:

```
In[•]:= GT_i_,j_@G_B[{li___,i_,ri___},{lj___,j_,rj___},bs___][f_] := CF@G_B[{ri,li,j,rj,lj,i},bs][f /. {
          Y_i[l1___] Y_j[l2___] :> Sort@Y_i[j, l1, l2] + Sort@Y_j[i, l1, l2],
          Y_i[l1___] Y_k_[ll___, j, rl___] :> Sort@Y_k[l1, ll, rl, i, j],
          Y_i[l1___] Y[ll___, j, rl___] :> Sort@Y[l1, ll, rl, i, j],
          Y_j[l1___] Y_k_[ll___, i, rl___] :> Sort@Y_k[l1, ll, rl, i, j],
          Y_j[l1___] Y[ll___, i, rl___] :> Sort@Y[l1, ll, rl, i, j],
          Y_[___, i, ___] Y_[___, j, ___] → 0,
          Y_[___, i, ___] Y[___, j, ___] → 0,
          Y_[___, j, ___] Y[___, i, ___] → 0,
          Y[___, i, ___] Y[___, j, ___] → 0
         }]
```



cor·don (kôr′dn)

n.

1. A line of people, military posts, or ships stationed around an area to enclose or guard it: *a police cordon.*

2. A rope, line, tape, or similar border stretched around an area, usually by the police, indicating that access is restricted.

In[•]:=
```
Cordon_i_@G_B[{li___,i_,ri___},bs___][f_] := Module[{j1, j2},
  {j1, j2} = {First@{ri, li}, Last@{ri, li}};
  CF@G_B[Most@{ri,li},bs][f /. {
       Y_i[l___] → 0,
       Y_k_[ll___, i, rl___] :↦ Y_k[ll, rl],
       Y[ll___, i, rl___] :↦ Y[ll, rl]
      } /. {
      Y_j1[l1___] Y_j2[l2___] :↦ Sort@Y_j1[l1, l2],
      Y_j1[___, j2, ___] → 0,
      Y_j2[___, j1, ___] → 0,
      Y_j1[ls___] Y_k_[ll___, j2, rl___] :↦ Sort@Y_k[j1, ls, ll, rl],
      Y_j2[ls___] Y_k_[ll___, j1, rl___] :↦ Sort@Y_k[j1, ls, ll, rl],
      Y_j1[ls___] Y[ll___, j2, rl___] :↦ Sort@Y[j1, ls, ll, rl],
      Y_j2[ls___] Y[ll___, j1, rl___] :↦ Sort@Y[j1, ls, ll, rl],
      Y_[___, j1, ___] Y_[___, j2, ___] → 0,
      Y_[___, j2, ___] Y[___, j1, ___] → 0,
      Y[___, j1, ___] Y[___, j2, ___] → 0,
      Y_[___, j1, ___, j2, ___] → 0,
      Y_[___, j2, ___, j1, ___] → 0,
      Y[___, j1, ___, j2, ___] → 0,
      Y[___, j2, ___, j1, ___] → 0
     }]
 ]
```

Strand Operations. c for contract, mc for magnetic contract:

In[•]:=
```
c_i_,j_@t : G_B[{li___,i_,ri___},{___,j_,___},___][__] := t // GT_j,First@{ri,li} // Cordon_j
```

In[•]:=
```
c_i_,j_@t : G_B[{___,i_,j_,___},___][__] := Cordon_j@t
c_i_,j_@t : G_B[{j_,___,i_},___][__] := Cordon_j@t
c_i_,j_@t : G_B[{___,j_,i_,___},___][__] := Cordon_i@t
c_i_,j_@t : G_B[{i_,___,j_},___][__] := Cordon_i@t
```

```
In[•]:= mc[ℰ_] := ℰ //.
  t : G_B[{___,i_,___},{___,j_,___},___][__] | G_B[{___,i_,j_,___},___][__] | G_B[{j_,___,i_},___][__] /;
    i + j == 0 :> c_{i,j}@t
```

"KSI" for Kauffman States Invariant.

```
In[•]:= KSI@P_{i_,j_} := CF@G_B[{i,j}][Y_i[] Y_j[]];
KSI[x : X[i_, j_, k_, l_]] := KSI@If[PositiveQ[x], X_{-i,j,k,-l}, X̄_{-j,k,l,-i}];
KSI[X_{i_,j_,k_,l_}] :=
  CF@G_B[{i,j,k,l}][m T^-1 Y_k[i] Y_j[] Y_l[] + T Y_i[k] Y_j[] Y_l[] + Y_j[l] Y_i[] Y_k[] + Y_l[j] Y_i[] Y_k[]];
KSI[X̄_{i_,j_,k_,l_}] :=
  CF@G_B[{i,j,k,l}][m T Y_k[i] Y_j[] Y_l[] + T^-1 Y_i[k] Y_j[] Y_l[] + Y_j[l] Y_i[] Y_k[] + Y_l[j] Y_i[] Y_k[]];
KSI[K_] := Fold[mc[#1 ⊕ #2] &, G_B[][1], List @@ (KSI /@ PD@K)];
```

Knots

```
In[•]:= KSIK[K_] := KSI[Cut@PD@K][[1]] /. {Y_[] → 1, T → T^(1/2), m → -1}
```

```
In[•]:= KSIK[Knot[8, 17]]
```

```
Out[•]=
```

$$11 - \frac{1}{T^3} + \frac{4}{T^2} - \frac{8}{T} - 8\,T + 4\,T^2 - T^3$$

```
In[•]:= Alexander[Knot[8, 17]][T]
```

```
Out[•]=
```

$$11 - \frac{1}{T^3} + \frac{4}{T^2} - \frac{8}{T} - 8\,T + 4\,T^2 - T^3$$

```
In[•]:= Monitor[
  Timing@Table[res = (K → KSIK[K] / Alexander[K][T]), {K, AllKnots[{3, 10}]}],
  res]
```

```
Out[•]=
{1442.59, {Knot[3, 1] → 1, Knot[4, 1] → 1, Knot[5, 1] → 1, Knot[5, 2] → 1, Knot[6, 1] → 1,
  Knot[6, 2] → 1, Knot[6, 3] → 1, Knot[7, 1] → 1, Knot[7, 2] → 1, Knot[7, 3] → 1,
  Knot[7, 4] → 1, Knot[7, 5] → 1, Knot[7, 6] → 1, Knot[7, 7] → 1, Knot[8, 1] → 1,
  Knot[8, 2] → 1, Knot[8, 3] → 1, Knot[8, 4] → 1, Knot[8, 5] → 1, Knot[8, 6] → 1,
  Knot[8, 7] → 1, Knot[8, 8] → 1, Knot[8, 9] → 1, Knot[8, 10] → 1, Knot[8, 11] → 1,
  Knot[8, 12] → 1, Knot[8, 13] → 1, Knot[8, 14] → 1, Knot[8, 15] → 1, Knot[8, 16] → 1,
  Knot[8, 17] → 1, Knot[8, 18] → 1, Knot[8, 19] → 1, Knot[8, 20] → 1, Knot[8, 21] → 1,
  Knot[9, 1] → 1, Knot[9, 2] → 1, Knot[9, 3] → 1, Knot[9, 4] → 1, Knot[9, 5] → 1,
  Knot[9, 6] → 1, Knot[9, 7] → 1, Knot[9, 8] → 1, Knot[9, 9] → 1, Knot[9, 10] → 1,
  Knot[9, 11] → 1, Knot[9, 12] → 1, Knot[9, 13] → 1, Knot[9, 14] → 1, Knot[9, 15] → 1,
  Knot[9, 16] → 1, Knot[9, 17] → 1, Knot[9, 18] → 1, Knot[9, 19] → 1, Knot[9, 20] → 1,
```

```
Knot[9, 21] → 1, Knot[9, 22] → 1, Knot[9, 23] → 1, Knot[9, 24] → 1, Knot[9, 25] → 1,
Knot[9, 26] → 1, Knot[9, 27] → 1, Knot[9, 28] → 1, Knot[9, 29] → 1, Knot[9, 30] → 1,
Knot[9, 31] → 1, Knot[9, 32] → 1, Knot[9, 33] → 1, Knot[9, 34] → 1, Knot[9, 35] → 1,
Knot[9, 36] → 1, Knot[9, 37] → 1, Knot[9, 38] → 1, Knot[9, 39] → 1, Knot[9, 40] → 1,
Knot[9, 41] → 1, Knot[9, 42] → 1, Knot[9, 43] → 1, Knot[9, 44] → 1, Knot[9, 45] → 1,
Knot[9, 46] → 1, Knot[9, 47] → 1, Knot[9, 48] → 1, Knot[9, 49] → 1, Knot[10, 1] → 1,
Knot[10, 2] → 1, Knot[10, 3] → 1, Knot[10, 4] → 1, Knot[10, 5] → 1, Knot[10, 6] → 1,
Knot[10, 7] → 1, Knot[10, 8] → 1, Knot[10, 9] → 1, Knot[10, 10] → 1, Knot[10, 11] → 1,
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Knot[10, 17] → 1, Knot[10, 18] → 1, Knot[10, 19] → 1, Knot[10, 20] → 1, Knot[10, 21] → 1,
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Knot[10, 37] → 1, Knot[10, 38] → 1, Knot[10, 39] → 1, Knot[10, 40] → 1, Knot[10, 41] → 1,
Knot[10, 42] → 1, Knot[10, 43] → 1, Knot[10, 44] → 1, Knot[10, 45] → 1, Knot[10, 46] → 1,
Knot[10, 47] → 1, Knot[10, 48] → 1, Knot[10, 49] → 1, Knot[10, 50] → 1, Knot[10, 51] → 1,
Knot[10, 52] → 1, Knot[10, 53] → 1, Knot[10, 54] → 1, Knot[10, 55] → 1, Knot[10, 56] → 1,
Knot[10, 57] → 1, Knot[10, 58] → 1, Knot[10, 59] → 1, Knot[10, 60] → 1, Knot[10, 61] → 1,
Knot[10, 62] → 1, Knot[10, 63] → 1, Knot[10, 64] → 1, Knot[10, 65] → 1, Knot[10, 66] → 1,
Knot[10, 67] → 1, Knot[10, 68] → 1, Knot[10, 69] → 1, Knot[10, 70] → 1, Knot[10, 71] → 1,
Knot[10, 72] → 1, Knot[10, 73] → 1, Knot[10, 74] → 1, Knot[10, 75] → 1, Knot[10, 76] → 1,
Knot[10, 77] → 1, Knot[10, 78] → 1, Knot[10, 79] → 1, Knot[10, 80] → 1, Knot[10, 81] → 1,
Knot[10, 82] → 1, Knot[10, 83] → 1, Knot[10, 84] → 1, Knot[10, 85] → 1, Knot[10, 86] → 1,
Knot[10, 87] → 1, Knot[10, 88] → 1, Knot[10, 89] → 1, Knot[10, 90] → 1, Knot[10, 91] → 1,
Knot[10, 92] → 1, Knot[10, 93] → 1, Knot[10, 94] → 1, Knot[10, 95] → 1, Knot[10, 96] → 1,
Knot[10, 97] → 1, Knot[10, 98] → 1, Knot[10, 99] → 1, Knot[10, 100] → 1, Knot[10, 101] → 1,
Knot[10, 102] → 1, Knot[10, 103] → 1, Knot[10, 104] → 1, Knot[10, 105] → 1,
Knot[10, 106] → 1, Knot[10, 107] → 1, Knot[10, 108] → 1, Knot[10, 109] → 1,
Knot[10, 110] → 1, Knot[10, 111] → 1, Knot[10, 112] → 1, Knot[10, 113] → 1,
Knot[10, 114] → 1, Knot[10, 115] → 1, Knot[10, 116] → 1, Knot[10, 117] → 1,
Knot[10, 118] → 1, Knot[10, 119] → 1, Knot[10, 120] → 1, Knot[10, 121] → 1,
Knot[10, 122] → 1, Knot[10, 123] → 1, Knot[10, 124] → 1, Knot[10, 125] → 1,
Knot[10, 126] → 1, Knot[10, 127] → 1, Knot[10, 128] → 1, Knot[10, 129] → 1,
Knot[10, 130] → 1, Knot[10, 131] → 1, Knot[10, 132] → 1, Knot[10, 133] → 1,
Knot[10, 134] → 1, Knot[10, 135] → 1, Knot[10, 136] → 1, Knot[10, 137] → 1,
Knot[10, 138] → 1, Knot[10, 139] → 1, Knot[10, 140] → 1, Knot[10, 141] → 1,
Knot[10, 142] → 1, Knot[10, 143] → 1, Knot[10, 144] → 1, Knot[10, 145] → 1,
Knot[10, 146] → 1, Knot[10, 147] → 1, Knot[10, 148] → 1, Knot[10, 149] → 1,
Knot[10, 150] → 1, Knot[10, 151] → 1, Knot[10, 152] → 1, Knot[10, 153] → 1,
Knot[10, 154] → 1, Knot[10, 155] → 1, Knot[10, 156] → 1, Knot[10, 157] → 1,
Knot[10, 158] → 1, Knot[10, 159] → 1, Knot[10, 160] → 1, Knot[10, 161] → 1,
Knot[10, 162] → 1, Knot[10, 163] → 1, Knot[10, 164] → 1, Knot[10, 165] → 1}}
```
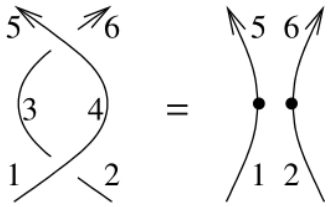

Reidemeister 2



In[•]:=
```
CF[KSI@PD[X_{-2,4,3,-1}, X̄_{-4,6,5,-3}] /. m → -1]
```

Out[•]=

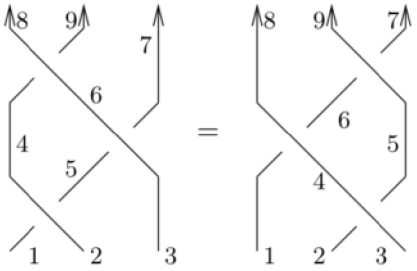
$$G_{B[\{-2,6,5,-1\}]}[Y_{-2}[5]\,Y_{-1}[]\,Y_6[] + Y_{-1}[]\,Y_5[-2]\,Y_6[]]$$

In[•]:=
```
GT_{5,-2}@KSI@PD[P_{-1,5}, P_{-2,6}]
```

Out[•]=

$$G_{B[\{-2,6,5,-1\}]}[Y_{-2}[5]\,Y_{-1}[]\,Y_6[] + Y_{-1}[]\,Y_5[-2]\,Y_6[]]$$

Reidemeister 3



In[•]:=
```
lhs = KSI[PD[X_{-2,5,4,-1}, X_{-3,7,6,-5}, X_{-6,9,8,-4}]] /. m → -1
```

Out[•]=

$$G_{B[\{-3,7,9,8,-1,-2\}]}\Big[$$
$$T^3\,Y_{-3}[9]\,Y_{-2}[8]\,Y_{-1}[]\,Y_7[] + T\,Y_{-3}[]\,Y_{-2}[7]\,Y_{-1}[9]\,Y_8[] + T^2\,Y_{-3}[-1,9]\,Y_{-2}[]\,Y_7[]\,Y_8[] -$$
$$Y_{-2}[]\,Y_{-1}[-3,9]\,Y_7[]\,Y_8[] - \frac{Y_{-3}[]\,Y_{-1}[9]\,Y_7[-2]\,Y_8[]}{T} + \frac{Y_{-3}[9]\,Y_{-1}[]\,Y_7[]\,Y_8[-2]}{T} +$$
$$T^2\,Y_{-3}[]\,Y_{-2}[7,8]\,Y_{-1}[]\,Y_9[] + T\,Y_{-3}[-1]\,Y_{-2}[]\,Y_7[8]\,Y_9[] + T\,Y_{-2}[]\,Y_{-1}[-3]\,Y_7[8]\,Y_9[] -$$
$$Y_{-3}[]\,Y_{-1}[]\,Y_7[-2,8]\,Y_9[] - \frac{Y_{-3}[-1]\,Y_{-2}[]\,Y_8[7]\,Y_9[]}{T} - \frac{Y_{-2}[]\,Y_{-1}[-3]\,Y_8[7]\,Y_9[]}{T} +$$
$$\frac{Y_{-3}[]\,Y_{-1}[]\,Y_8[-2,7]\,Y_9[]}{T^2} - T\,Y_{-2}[8]\,Y_{-1}[]\,Y_7[]\,Y_9[-3] - \frac{Y_{-1}[]\,Y_7[]\,Y_8[-2]\,Y_9[-3]}{T^3} +$$
$$T\,Y_{-3}[]\,Y_{-2}[7]\,Y_8[]\,Y_9[-1] - \frac{Y_{-3}[]\,Y_7[-2]\,Y_8[]\,Y_9[-1]}{T} + \frac{Y_{-2}[]\,Y_7[]\,Y_8[]\,Y_9[-3,-1]}{T^2}\Big]$$

In[•]:=
```
rhs = KSI[PD[X_{-3,5,4,-2}, X_{-4,6,8,-1}, X_{-5,7,9,-6}]] /. m → -1
```

Out[•]=

$$G_{B[\{-3,7,9,8,-1,-2\}]}\Big[T^3 Y_{-3}[9]\,Y_{-2}[8]\,Y_{-1}[]\,Y_7[] + T\,Y_{-3}[]\,Y_{-2}[7]\,Y_{-1}[9]\,Y_8[] + T^2\,Y_{-3}[-1,9]\,Y_{-2}[]\,Y_7[]\,Y_8[] - Y_{-2}[]\,Y_{-1}[-3,9]\,Y_7[]\,Y_8[] + T\,Y_{-3}[]\,Y_{-1}[9]\,Y_7[-2]\,Y_8[] - T\,Y_{-3}[9]\,Y_{-1}[]\,Y_7[]\,Y_8[-2] + T^2\,Y_{-3}[]\,Y_{-2}[7,8]\,Y_{-1}[]\,Y_9[] + T\,Y_{-3}[-1]\,Y_{-2}[]\,Y_7[8]\,Y_9[] - \frac{Y_{-2}[]\,Y_{-1}[-3]\,Y_7[8]\,Y_9[]}{T} - Y_{-3}[]\,Y_{-1}[]\,Y_7[-2,8]\,Y_9[] + T\,Y_{-3}[-1]\,Y_{-2}[]\,Y_8[7]\,Y_9[] - \frac{Y_{-2}[]\,Y_{-1}[-3]\,Y_8[7]\,Y_9[]}{T} + \frac{Y_{-3}[]\,Y_{-1}[]\,Y_8[-2,7]\,Y_9[]}{T^2} + \frac{Y_{-2}[8]\,Y_{-1}[]\,Y_7[]\,Y_9[-3]}{T} - \frac{Y_{-1}[]\,Y_7[]\,Y_8[-2]\,Y_9[-3]}{T^3} - \frac{Y_{-3}[]\,Y_{-2}[7]\,Y_8[]\,Y_9[-1]}{T} - \frac{Y_{-3}[]\,Y_7[-2]\,Y_8[]\,Y_9[-1]}{T} + \frac{Y_{-2}[]\,Y_7[]\,Y_8[]\,Y_9[-3,-1]}{T^2} \Big]$$

In[•]:=
```
lhs[[1]] - rhs[[1]]
```

Out[•]=

$$-\frac{Y_{-3}[]\,Y_{-1}[9]\,Y_7[-2]\,Y_8[]}{T} - T\,Y_{-3}[]\,Y_{-1}[9]\,Y_7[-2]\,Y_8[] + \frac{Y_{-3}[9]\,Y_{-1}[]\,Y_7[]\,Y_8[-2]}{T} + T\,Y_{-3}[9]\,Y_{-1}[]\,Y_7[]\,Y_8[-2] + \frac{Y_{-2}[]\,Y_{-1}[-3]\,Y_7[8]\,Y_9[]}{T} + T\,Y_{-2}[]\,Y_{-1}[-3]\,Y_7[8]\,Y_9[] - \frac{Y_{-3}[-1]\,Y_{-2}[]\,Y_8[7]\,Y_9[]}{T} - T\,Y_{-3}[-1]\,Y_{-2}[]\,Y_8[7]\,Y_9[] - \frac{Y_{-2}[8]\,Y_{-1}[]\,Y_7[]\,Y_9[-3]}{T} - T\,Y_{-2}[8]\,Y_{-1}[]\,Y_7[]\,Y_9[-3] + \frac{Y_{-3}[]\,Y_{-2}[7]\,Y_8[]\,Y_9[-1]}{T} + T\,Y_{-3}[]\,Y_{-2}[7]\,Y_8[]\,Y_9[-1]$$