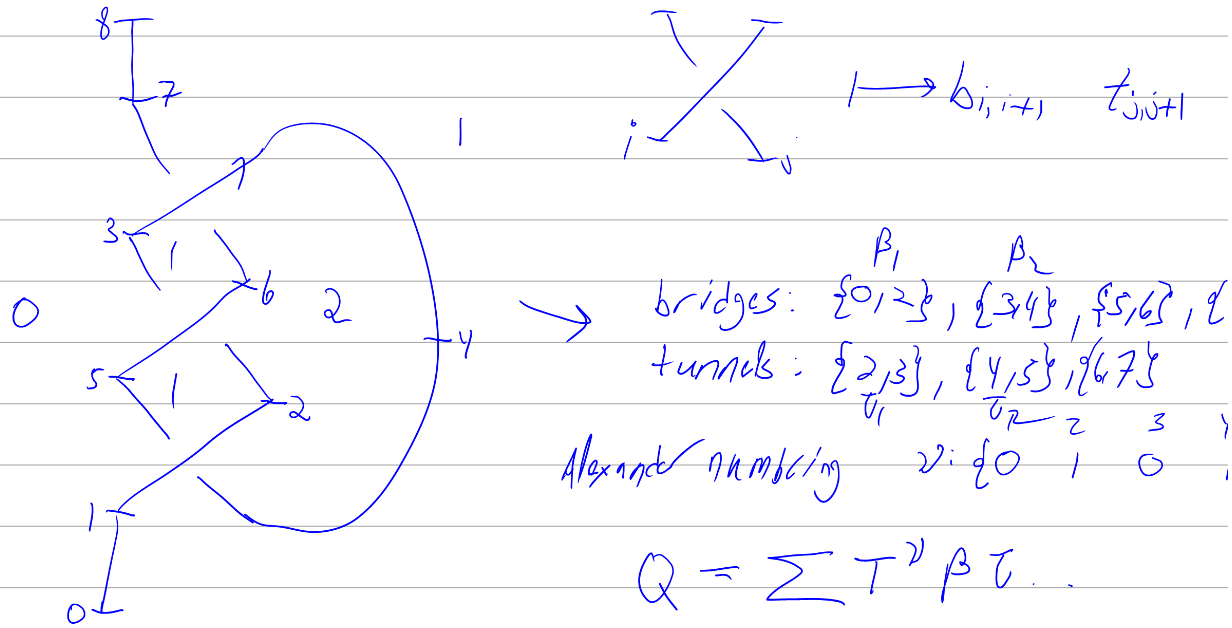
Pensieve header: The Alexander polynomial using bridges, tunnels, and Alexander numbering.

In[ ]:=
```
SetDirectory["C:\\drorbn\\AcademicPensieve\\2023-12"];
Once[
 << KnotTheory`;
 << "../Talks/Oaxaca-2210/Rot.m"
]
```

```
Loading KnotTheory` version of February 2, 2020, 10:53:45.2097.
Read more at http://katlas.org/wiki/KnotTheory.
```

```
Loading Rot.m from http://drorbn.net/la22/ap to compute rotation numbers.
```

In[ ]:=
```
K1 = EPD[X1,2];
K3 = EPD[X1,4, X5,2, X3,6];
K8 = Knot[8, 17];
K10 = Knot[10, 165];
```



In[•]:=
```
mat[K_, flip_] := Module[{},
   {Cs, φ} = Rot[K] /. {s_Integer, i_, j_} /; flip ⧴ {s, j, i};
   n = Length[Cs]; bridges = tunnels = {}; bn = tn = 0; (* completed features *)
   cfb = ¬ flip; (* current feautre is bridge *);
   cfs = 0; (* current feature start *)
   tνut = 0;
   (* Total ν at up transitions *) tνdt = 0;
   (* Total ν at down transitions *)
   ν = {lν = 0}; ϕ = {}; Q = 0;
   For[k = 1, k ≤ 2 n, ++k,
    Cs /. {
      {s_, k, j_} ⧴ (
        AppendTo[ϕ, bn + 1];
        If[¬ cfb, AppendTo[tunnels, {cfs, k}]; ++tn;
         cfb = True;
         cfs = k; tνut += lν; Q += T^lν (T - 1) β_(bn+1) τ_tn];
        AppendTo[ν, lν += s];
       ),
      {s_, i_, k} ⧴ (
        AppendTo[ϕ, tn + 1];
        If[cfb, AppendTo[bridges, {cfs, k}]; ++bn;
         cfb = False;
         cfs = k; tνdt += lν; Q += T^lν (1 - T) β_bn τ_(tn+1) ];
        AppendTo[ν, lν -= s];
       )
     };
    If[k == 1, ffb = cfb];
    Cs /. {
      {s_, k, j_} /; k > j ⧴ (Q += T^ν⟦k⟧ (T - 1) (T^s - 1) β_ϕ⟦k⟧ τ_ϕ⟦j⟧),
      {s_, i_, k} /; k > i ⧴ (Q += T^ν⟦i⟧ (T - 1) (T^s - 1) β_ϕ⟦i⟧ τ_ϕ⟦k⟧)
     }
    ];
   Factor@Table[∂_(β_i, τ_j) Q / (T - 1), {i, Max[bn, tn]}, {j, Max[bn, tn]}]
  ];
mat[K_] := mat[K, False];
mat[Flip@K_] := mat[K, True];
```

In[•]:=
```
K = K3
```

Out[•]=

$\mathrm{EPD}[X_{1,4}, X_{5,2}, X_{3,6}]$

In[∘]:= `mat[K] // MatrixForm`

Out[∘]//MatrixForm=

$$\begin{pmatrix} -T & -1+T & 0 \\ 1 & -T & -1+T \\ -1+T & 1 & -T \end{pmatrix}$$

In[∘]:= `mat[Flip@K] // MatrixForm`

Out[∘]//MatrixForm=

$$\begin{pmatrix} \frac{1}{T} & -1 & \frac{-1+T}{T} \\ \frac{-1+T}{T} & \frac{1}{T} & -1 \\ 0 & \frac{-1+T}{T} & \frac{1}{T} \end{pmatrix}$$

In[∘]:= $\text{Factor}\left[\text{mat}[K] + \left(\text{mat}[\text{Flip@K}]^{T} \text{ /. } T \to T^{-1}\right)\right]$

Out[∘]=

```
{{0, 0, 0}, {0, 0, 0}, {0, 0, 0}}
```

In[∘]:= $\text{Select}\left[\text{AllKnots}[\{3, 11\}],\ \text{Union@Flatten@Factor}\left[\text{mat}[\#] + \left(\text{mat}[\text{Flip@}\#]^{T} \text{ /. } T \to T^{-1}\right)\right] \text{ =!= } \{0\}\ \&\right]$

Out[∘]=

```
{}
```

```
Reverse@Sort[Reverse /@ Tally[
    Union@Table[A = mat[K];
      u = Factor[ (T^(-Total[φ]/2) (T^(-1/2))^(tvut+tvdt) Det[A]) / Alexander[K][T] (-1)^(Last@ϕ) If[cfb, -1, 1]];
      {(*Total[First/@Cs],*)
       (*Total[v],*) (*Total[Cs/.{s_Integer,i_,j_}:>s v[[i]]],*)
       u},
      {K, AllKnots[{3, 11}]}],
    Most[#1] === Most[#2] &]]
```

Out[∘]=

```
{{1, {1}}}
```

In[∘]:=

```
Total@Table[A = mat[K];
  Simplify[(-1)^(Last@ϕ) If[cfb, -1, 1] Alexander[K][T] == T^(-Total[φ]/2) (T^(-1/2))^(tvut+tvdt) Det[A]],
  {K, AllKnots[{3, 11}]}]
```

Out[∘]=

801 True

In[∘]:= `Total@Table[A = mat[K]; Total[First /@ Cs] == -tvut + tvdt, {K, AllKnots[{3, 10}]}]`

Out[∘]=

249 True