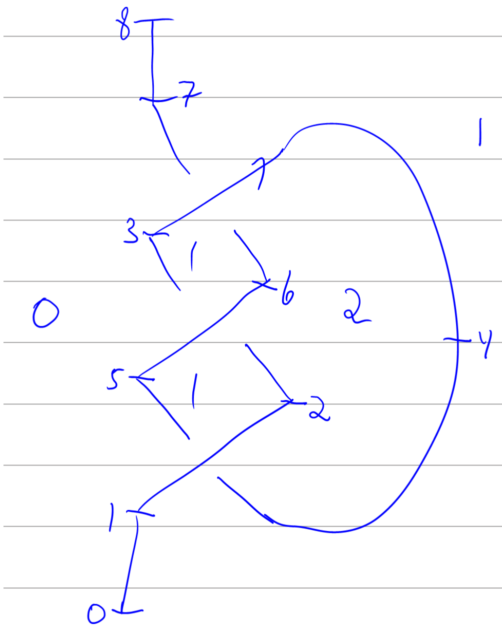
Pensieve header: The Alexander polynomial using bridges, tunnels, and Alexander numbering.

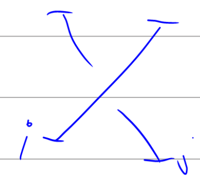
In[∘]:=
```
SetDirectory["C:\\drorbn\\AcademicPensieve\\2023-12"];
Once[
 << KnotTheory`;
 << "../Talks/Oaxaca-2210/Rot.m"
]
```

```
Loading KnotTheory` version of February 2, 2020, 10:53:45.2097.
Read more at http://katlas.org/wiki/KnotTheory.
```

```
Loading Rot.m from http://drorbn.net/la22/ap to compute rotation numbers.
```

In[∘]:=
```
K1 = EPD[X1,2];
K3 = EPD[X1,4, X5,2, X3,6];
K8 = Knot[8, 17];
K10 = Knot[10, 165];
```





$\mapsto b_{i,i+1} \quad t_{j,j+1}$

$\qquad\quad \beta_1 \quad \beta_2$

bridges: $\{0,1,2\}, \{3,4\}, \{5,6\}, \{$

tunnels: $\{2,3\}, \{4,5\}, \{6,7\}$

$\qquad\quad \tau_1 \quad \tau_2$

Alexander numbering $\nu: \{0 \quad 1 \quad 0$

$Q = \sum T^\nu \beta \tau \ldots$

In[ ]:=
```
mat[K_, flip_] := Module[{},
    {Cs, φ} = Rot[K] /. {s_, i_, j_} /; flip ⧴ {s, j, i};
    n = Length[Cs];
    bridges = tunnels = {};
    bn = tn = 0; (* completed features *)
    cfb = ¬ flip; (* current feautre is bridge *);
    cfs = 0; (* current feature start *)
    ν = {lν = 0};
    ϕ = {};
    Q = 0;
    For[k = 1, k ≤ 2 n, ++k,
     Cs /. {
       {s_, k, j_} ⧴ (
         AppendTo[ϕ, bn + 1];
         If[¬ cfb, AppendTo[tunnels, {cfs, k}]; ++tn;
          cfb = True;
          cfs = k; Q += T^lν (T - 1) β_(bn+1) τ_tn];
         AppendTo[ν, lν += s];
        ),
       {s_, i_, k} ⧴ (
         AppendTo[ϕ, tn + 1];
         If[cfb, AppendTo[bridges, {cfs, k}]; ++bn;
          cfb = False;
          cfs = k; Q += T^lν (1 - T) β_bn τ_(tn+1) ];
         AppendTo[ν, lν -= s];
        )
      };
     Cs /. {
       {s_, k, j_} /; k > j ⧴ (Q += T^ν⟦k⟧ (T - 1) (T^s - 1) β_ϕ⟦k⟧ τ_ϕ⟦j⟧),
       {s_, i_, k} /; k > i ⧴ (Q += T^ν⟦i⟧ (T - 1) (T^s - 1) β_ϕ⟦i⟧ τ_ϕ⟦k⟧)
      }
    ];
    Factor@Table[∂_(β_i, τ_j) Q / (T - 1), {i, 1, bn + If[flip, 1, 0]}, {j, 1, tn + If[flip, 0, 1]}]
   ];
mat[K_] := mat[K, False];
mat[Flip@K_] := mat[K, True];
```

In[ ]:=
```
K = Knot[10, 165]
```

Out[ ]=
```
Knot[10, 165]
```

In[∘]:=
```
mat[K] // MatrixForm
```

Out[∘]//MatrixForm=

$$\begin{pmatrix} -1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ T & -T^2 & 0 & 0 & 0 & 0 & 0 & (-1+T)\,T & 0 \\ 0 & T & -T^2 & 0 & 0 & (-1+T)\,T & 0 & 0 & 0 \\ 1-T & 0 & T & -1 & 0 & 0 & 0 & 0 & 0 \\ 0 & (-1+T)\,T & 0 & T & -T^2 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & T & -T^2 & (-1+T)\,T & 0 & 0 \\ 0 & 0 & (-1+T)\,T & 0 & 0 & 1 & -T^2 & 0 & -1+T \\ 0 & 0 & 0 & 0 & (-1+T)\,T & 0 & T & -T^2 & 0 \\ 0 & 0 & 0 & 1-T & 0 & -1+T & 0 & T & -T \end{pmatrix}$$

In[∘]:=
```
mat[Flip@K] // MatrixForm
```

Out[∘]//MatrixForm=

$$\begin{pmatrix} 1 & -\frac{1}{T} & 0 & -\frac{-1+T}{T} & 0 & 0 & 0 & 0 & 0 \\ 0 & \frac{1}{T^2} & -\frac{1}{T} & 0 & \frac{-1+T}{T^2} & 0 & 0 & 0 & 0 \\ 0 & 0 & \frac{1}{T^2} & -\frac{1}{T} & 0 & 0 & \frac{-1+T}{T^2} & 0 & 0 \\ 0 & 0 & 0 & 1 & -\frac{1}{T} & 0 & 0 & 0 & -\frac{-1+T}{T} \\ 0 & 0 & 0 & 0 & \frac{1}{T^2} & -\frac{1}{T} & 0 & \frac{-1+T}{T^2} & 0 \\ 0 & 0 & \frac{-1+T}{T^2} & 0 & 0 & \frac{1}{T^2} & -1 & 0 & \frac{-1+T}{T} \\ 0 & 0 & 0 & 0 & 0 & \frac{-1+T}{T^2} & \frac{1}{T^2} & -\frac{1}{T} & 0 \\ 0 & \frac{-1+T}{T^2} & 0 & 0 & 0 & 0 & 0 & \frac{1}{T^2} & -\frac{1}{T} \\ 0 & 0 & 0 & 0 & 0 & 0 & \frac{-1+T}{T} & 0 & \frac{1}{T} \end{pmatrix}$$

In[∘]:=
```
Factor[mat[K] + (mat[Flip@K]ᵀ /. T → T⁻¹)]
```

Out[∘]=

{{0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0},
{0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0},
{0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0}, {0, 0, 0, 0, 0, 0, 0, 0, 0}}

In[∘]:=
```
Det[mat[K]]
```

Out[∘]=

$$2\,T^7 - 10\,T^8 + 15\,T^9 - 10\,T^{10} + 2\,T^{11}$$

In[∘]:=
```
Alexander[K][T]
```

Out[∘]=

$$-15 - \frac{2}{T^2} + \frac{10}{T} + 10\,T - 2\,T^2$$