Let H be a genus g handlebody in $\mathbb{R}^3$, let $\Sigma = \partial H$, and let $p \in \Sigma$ be a basepoint (think "H is a tubular neighborhood of a pinched tangle T"). Let $C = (\operatorname{int} H)^c$ and let $\tau\colon \pi_1(C) \to H^1(H;\mathbb{Z}) \cong \mathbb{Z}^g$ be induced by Alexander (?) duality. Let $\tilde\Sigma \subset \tilde C$ be the τ-covers of $\Sigma \subset C$, respectively, with covering projections ϕ. Let $\tilde p = \phi^{-1}(p)$; it is a copy of $\mathbb{Z}^g$. Let $R \coloneqq \mathbb{Z}H^1(H;\mathbb{Z}) \cong \mathbb{Z}[T_i^{\pm 1}]_{i=1}^g$ be the group ring of $H^1(H;\mathbb{Z})$, the ring of Laurent polynomials in variables $T_1,\dots,T_g$, and note that $\Omega_g \coloneqq H_1(\tilde\Sigma,\tilde p)$ and $H_1(\tilde C,\tilde p)$ are R-modules and that Ω_g does not depend on the embedding of H.

Definition 1. $A(H) \coloneqq H_1(\tilde C)$, *the Alexander module of* H, *and* $AL(H) \coloneqq \ker i_*\colon \Omega_g \to H_1(\tilde C,\tilde p)$, *the Alexander Lagrangian of* H.

Note that if H is a tubular neighborhood of a knot K then $A(H)$ is the standard Alexander module of K. Note that i_* is always surjective so we have the exact sequence

$$0 \longrightarrow AL(H) \longrightarrow \Omega_g \xrightarrow{\ i_*\ } H_1(\tilde C,\tilde p) \longrightarrow 0.$$

The following, coming from $\tilde p \to \tilde C \to (\tilde C,\tilde p)$, is also exact, and non-canonically split:

$$\begin{array}{ccccccccc} 0 \to & H_1(\tilde C) & \to & H_1(\tilde C,\tilde p) & \to & H_0(\tilde p) & \to & H_0(\tilde C) & \to 0 \\ & \| & & & & \| & & \| & \\ & A(H) & & & & R & \xrightarrow{\ \epsilon\ } & \mathbb{Z} & \end{array}$$

where ϵ is the augmentation map. And so with $R_0 = \ker\epsilon$, $A(H)\oplus R_0 \cong H_1(\tilde C,\tilde p) \cong \Omega_g/AL(H)$, where the first isomorphism is non-canonical. In the knot case the isomorphism is canonical.