

$$\begin{aligned}
 \text{In}[*]:= \text{del} &= \frac{1}{\sqrt{x^2+y^2+z^2}} - \left(\frac{1}{\sqrt{(x+1)^2+y^2+z^2}} + \frac{1}{\sqrt{(x-1)^2+y^2+z^2}} + \frac{1}{\sqrt{x^2+(y+1)^2+z^2}} + \right. \\
 &\quad \left. \frac{1}{\sqrt{x^2+(y-1)^2+z^2}} + \frac{1}{\sqrt{x^2+y^2+(z+1)^2}} + \frac{1}{\sqrt{x^2+y^2+(z-1)^2}} \right) / 6 \\
 \text{Out}[*]:= &\frac{1}{\sqrt{x^2+y^2+z^2}} + \frac{1}{6} \left(-\frac{1}{\sqrt{x^2+y^2+(-1+z)^2}} - \frac{1}{\sqrt{x^2+(-1+y)^2+z^2}} - \right. \\
 &\quad \left. \frac{1}{\sqrt{(-1+x)^2+y^2+z^2}} - \frac{1}{\sqrt{(1+x)^2+y^2+z^2}} - \frac{1}{\sqrt{x^2+(1+y)^2+z^2}} - \frac{1}{\sqrt{x^2+y^2+(1+z)^2}} \right)
 \end{aligned}$$

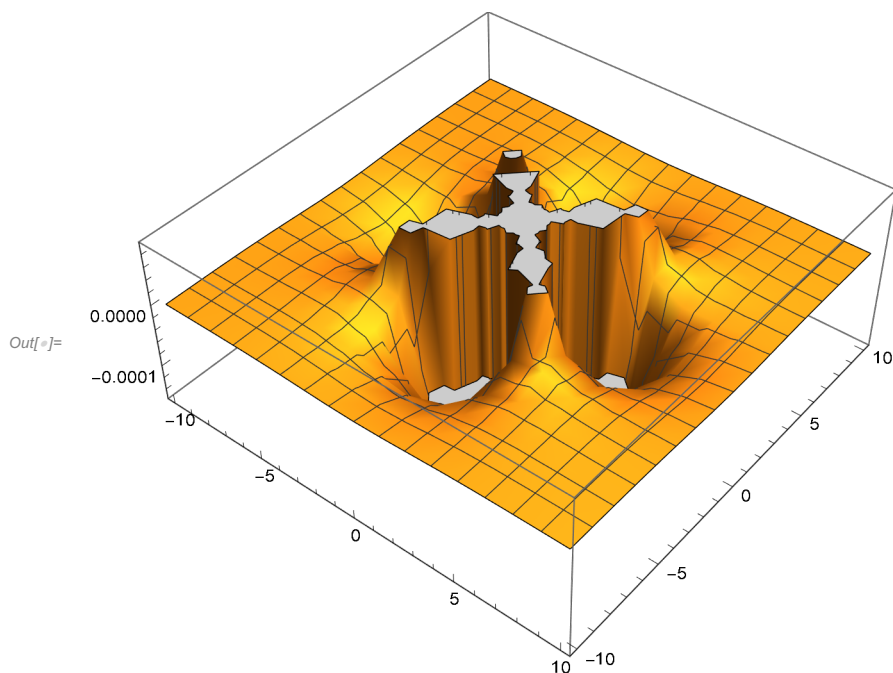
In[*]:= Simplify[del]

$$\begin{aligned}
 \text{Out}[*]:= &\frac{1}{\sqrt{x^2+y^2+z^2}} + \frac{1}{6} \left(-\frac{1}{\sqrt{x^2+y^2+(-1+z)^2}} - \frac{1}{\sqrt{x^2+(-1+y)^2+z^2}} - \right. \\
 &\quad \left. \frac{1}{\sqrt{(-1+x)^2+y^2+z^2}} - \frac{1}{\sqrt{(1+x)^2+y^2+z^2}} - \frac{1}{\sqrt{x^2+(1+y)^2+z^2}} - \frac{1}{\sqrt{x^2+y^2+(1+z)^2}} \right)
 \end{aligned}$$

In[*]:= del /. Thread[{x, y, z} -> RandomReal[{-100, 100}, 3]]

Out[*]:= 1.33542 × 10⁻¹⁰

In[*]:= Plot3D[del /. z -> 0, {x, -10, 10}, {y, -10, 10}]



In[*]:= Δ[f_][x_, y_, z_] :=
 f[x, y, z] - (f[x + 1, y, z] + f[x - 1, y, z] + f[x, y + 1, z] + f[x, y - 1, z] +
 f[x, y, z + 1] + f[x, y, z - 1]) / 6

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In[*]:= f[x_, y_, z_] := 1 / (1 + Total[Abs /@ {x, y, z}]);
Table[Δ[f][x, y, 1], {x, 0, 10}, {y, 0, 10}] // MatrixForm
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Out[*]//MatrixForm=

$\frac{1}{18}$	0	$\frac{1}{180}$	$\frac{1}{180}$	$\frac{1}{210}$	$\frac{1}{252}$	$\frac{5}{1512}$	$\frac{1}{360}$	$\frac{7}{2970}$	$\frac{1}{495}$	$\frac{1}{572}$
0	$\frac{1}{60}$	$\frac{1}{120}$	$\frac{1}{210}$	$\frac{1}{336}$	$\frac{1}{504}$	$\frac{1}{720}$	$\frac{1}{990}$	$\frac{1}{1320}$	$\frac{1}{1716}$	$\frac{1}{2184}$
$\frac{1}{180}$	$\frac{1}{120}$	$\frac{1}{210}$	$\frac{1}{336}$	$\frac{1}{504}$	$\frac{1}{720}$	$\frac{1}{990}$	$\frac{1}{1320}$	$\frac{1}{1716}$	$\frac{1}{2184}$	$\frac{1}{2730}$
$\frac{1}{180}$	$\frac{1}{210}$	$\frac{1}{336}$	$\frac{1}{504}$	$\frac{1}{720}$	$\frac{1}{990}$	$\frac{1}{1320}$	$\frac{1}{1716}$	$\frac{1}{2184}$	$\frac{1}{2730}$	$\frac{1}{3360}$
$\frac{1}{210}$	$\frac{1}{336}$	$\frac{1}{504}$	$\frac{1}{720}$	$\frac{1}{990}$	$\frac{1}{1320}$	$\frac{1}{1716}$	$\frac{1}{2184}$	$\frac{1}{2730}$	$\frac{1}{3360}$	$\frac{1}{4080}$
$\frac{1}{252}$	$\frac{1}{504}$	$\frac{1}{720}$	$\frac{1}{990}$	$\frac{1}{1320}$	$\frac{1}{1716}$	$\frac{1}{2184}$	$\frac{1}{2730}$	$\frac{1}{3360}$	$\frac{1}{4080}$	$\frac{1}{4896}$
$\frac{5}{1512}$	$\frac{1}{720}$	$\frac{1}{990}$	$\frac{1}{1320}$	$\frac{1}{1716}$	$\frac{1}{2184}$	$\frac{1}{2730}$	$\frac{1}{3360}$	$\frac{1}{4080}$	$\frac{1}{4896}$	$\frac{1}{5814}$
$\frac{1}{360}$	$\frac{1}{990}$	$\frac{1}{1320}$	$\frac{1}{1716}$	$\frac{1}{2184}$	$\frac{1}{2730}$	$\frac{1}{3360}$	$\frac{1}{4080}$	$\frac{1}{4896}$	$\frac{1}{5814}$	$\frac{1}{6840}$
$\frac{7}{2970}$	$\frac{1}{1320}$	$\frac{1}{1716}$	$\frac{1}{2184}$	$\frac{1}{2730}$	$\frac{1}{3360}$	$\frac{1}{4080}$	$\frac{1}{4896}$	$\frac{1}{5814}$	$\frac{1}{6840}$	$\frac{1}{7980}$
$\frac{1}{495}$	$\frac{1}{1716}$	$\frac{1}{2184}$	$\frac{1}{2730}$	$\frac{1}{3360}$	$\frac{1}{4080}$	$\frac{1}{4896}$	$\frac{1}{5814}$	$\frac{1}{6840}$	$\frac{1}{7980}$	$\frac{1}{9240}$
$\frac{1}{572}$	$\frac{1}{2184}$	$\frac{1}{2730}$	$\frac{1}{3360}$	$\frac{1}{4080}$	$\frac{1}{4896}$	$\frac{1}{5814}$	$\frac{1}{6840}$	$\frac{1}{7980}$	$\frac{1}{9240}$	$\frac{1}{10626}$