

$$(Alt) In[] := Simplify[Inverse[$\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} - \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix}$] == $\begin{pmatrix} \frac{1}{1 - (a_{11} + a_{12} \frac{1}{1 - a_{22}} a_{21})} & \frac{1}{1 - a_{11}} a_{12} \frac{1}{1 - (a_{22} + a_{21} \frac{1}{1 - a_{11}} a_{12})} \\ \frac{1}{1 - a_{22}} a_{21} \frac{1}{1 - (a_{11} + a_{12} \frac{1}{1 - a_{22}} a_{21})} & \frac{1}{1 - (a_{22} + a_{21} \frac{1}{1 - a_{11}} a_{12})} \end{pmatrix}$]$$

(Alt) Out[] = True

$$(Alt) In[] := Simplify[$\frac{1}{1 - a_{11}} a_{12} \frac{1}{1 - (a_{22} + a_{21} \frac{1}{1 - a_{11}} a_{12})}$ == $\frac{1}{1 - (a_{11} + a_{12} \frac{1}{1 - a_{22}} a_{21})} a_{12} \frac{1}{1 - a_{22}}$]$$

(Alt) Out[] = True

$$(Alt) In[] := \left\{ \frac{1}{1 - \frac{1}{1-x}}, \frac{1}{1 - \frac{x}{1-x}} \right\} // Simplify$$

$$(Alt) Out[] = \left\{ \frac{-1+x}{x}, \frac{-1+x}{-1+2x} \right\}$$

(Alt) In[] := **PQR = PolynomialQuotientRemainder;**

$$(Alt) In[] := det = Det[B = IdentityMatrix[3] - (A = Table[a₁₀ i+j, {i, 3}, {j, 3}])]$$

$$(Alt) Out[] = 1 - a_{11} - a_{12} a_{21} - a_{22} + a_{11} a_{22} - a_{13} a_{31} + a_{13} a_{22} a_{31} - a_{12} a_{23} a_{31} - a_{13} a_{21} a_{32} - a_{23} a_{32} + a_{11} a_{23} a_{32} - a_{33} + a_{11} a_{33} + a_{12} a_{21} a_{33} + a_{22} a_{33} - a_{11} a_{22} a_{33}$$

$$(Alt) In[] := d1 = 1 - a_{11}; PQR[det, d1, a_{11}]$$

$$(Alt) Out[] = \{1 - a_{22} - a_{23} a_{32} - a_{33} + a_{22} a_{33}, -a_{12} a_{21} - a_{13} a_{31} + a_{13} a_{22} a_{31} - a_{12} a_{23} a_{31} - a_{13} a_{21} a_{32} + a_{12} a_{21} a_{33}\}$$

$$(Alt) In[] := PQR[1 - a_{22} - a_{23} a_{32} - a_{33} + a_{22} a_{33}, 1 - a_{22}, a_{22}]$$

$$(Alt) Out[] = \{1 - a_{33}, -a_{23} a_{32}\}$$

$$(Alt) In[] := -a_{12} a_{21} - a_{13} a_{31} + a_{13} a_{22} a_{31} - a_{12} a_{23} a_{31} - a_{13} a_{21} a_{32} + a_{12} a_{21} a_{33} // Simplify$$

$$(Alt) Out[] = a_{13} ((-1 + a_{22}) a_{31} - a_{21} a_{32}) + a_{12} (-a_{23} a_{31} + a_{21} (-1 + a_{33}))$$

$$(Alt) In[] := Simplify[det == (1 - a_{11})$$

$$\left(1 - a_{22} - a_{23} a_{32} - a_{33} + a_{22} a_{33} + \frac{-a_{12} a_{21} - a_{13} a_{31} + a_{13} a_{22} a_{31} - a_{12} a_{23} a_{31} - a_{13} a_{21} a_{32} + a_{12} a_{21} a_{33}}{1 - a_{11}} \right)]$$

(Alt) Out[] = True

$$(Alt) In[] := Simplify\left[\frac{1}{det} == 1 / \left((1 - a_{11}) \left(1 - a_{22} - a_{23} a_{32} - a_{33} + a_{22} a_{33} + \frac{-a_{12} a_{21} - a_{13} a_{31} + a_{13} a_{22} a_{31} - a_{12} a_{23} a_{31} - a_{13} a_{21} a_{32} + a_{12} a_{21} a_{33}}{1 - a_{11}} \right) \right)\right]$$

(Alt) Out[] = True

(Alt) In[]:= **Simplify** $\left[\frac{1}{\det} == \frac{1}{(1 - a_{11}) \left((1 - a_{22}) \left(1 - a_{33} - \frac{a_{23} a_{32}}{1 - a_{22}} \right) + \frac{-a_{12} a_{21} - a_{13} a_{31} + a_{13} a_{22} a_{31} - a_{12} a_{23} a_{31} - a_{13} a_{21} a_{32} + a_{12} a_{21} a_{33}}{1 - a_{11}} \right)} \right]$

(Alt) Out[]:= True

(Alt) In[]:= **Simplify** $\left[\frac{1}{\det} == \frac{1}{(1 - a_{11}) \left(1 - \left(a_{22} + a_{23} a_{32} + a_{33} - a_{22} a_{33} - \frac{-a_{12} a_{21} - a_{13} a_{31} + a_{13} a_{22} a_{31} - a_{12} a_{23} a_{31} - a_{13} a_{21} a_{32} + a_{12} a_{21} a_{33}}{1 - a_{11}} \right) \right)} \right]$

(Alt) Out[]:= True

(Alt) In[]:= **Simplify** $\left[\frac{1}{\det} == \frac{1}{1 - a_{11}} \frac{1}{1 - \left(a_{22} + a_{23} a_{32} + a_{33} - a_{22} a_{33} - \frac{-a_{12} a_{21} - a_{13} a_{31} + a_{13} a_{22} a_{31} - a_{12} a_{23} a_{31} - a_{13} a_{21} a_{32} + a_{12} a_{21} a_{33}}{1 - a_{11}} \right)} \right]$

(Alt) Out[]:= True

(Alt) In[]:= **i11 = Inverse[B][[1, 1]]**

(Alt) Out[]:= $\frac{(1 - a_{22} - a_{23} a_{32} - a_{33} + a_{22} a_{33})}{(1 - a_{11} - a_{12} a_{21} - a_{22} + a_{11} a_{22} - a_{13} a_{31} + a_{13} a_{22} a_{31} - a_{12} a_{23} a_{31} - a_{13} a_{21} a_{32} - a_{23} a_{32} + a_{11} a_{23} a_{32} - a_{33} + a_{11} a_{33} + a_{12} a_{21} a_{33} + a_{22} a_{33} - a_{11} a_{22} a_{33})}$

(Alt) In[]:= **PQR** $[1 - a_{11} - a_{12} a_{21} - a_{22} + a_{11} a_{22} - a_{13} a_{31} + a_{13} a_{22} a_{31} - a_{12} a_{23} a_{31} - a_{13} a_{21} a_{32} - a_{23} a_{32} + a_{11} a_{23} a_{32} - a_{33} + a_{11} a_{33} + a_{12} a_{21} a_{33} + a_{22} a_{33} - a_{11} a_{22} a_{33}, 1 - a_{11}, a_{11}]$

(Alt) Out[]:= $\{1 - a_{22} - a_{23} a_{32} - a_{33} + a_{22} a_{33}, -a_{12} a_{21} - a_{13} a_{31} + a_{13} a_{22} a_{31} - a_{12} a_{23} a_{31} - a_{13} a_{21} a_{32} + a_{12} a_{21} a_{33}\}$

(Alt) In[]:= **Simplify** $\left[i11 == \frac{1}{1 - a_{11}} \frac{1 - a_{22} - a_{23} a_{32} - a_{33} + a_{22} a_{33}}{1 - \left(a_{22} + a_{23} a_{32} + a_{33} - a_{22} a_{33} - \frac{-a_{12} a_{21} - a_{13} a_{31} + a_{13} a_{22} a_{31} - a_{12} a_{23} a_{31} - a_{13} a_{21} a_{32} + a_{12} a_{21} a_{33}}{1 - a_{11}} \right)} \right]$

(Alt) Out[]:= True

(Alt) In[]:= **Together** $\left[a + f \frac{1}{c} g + \left(d + f \frac{1}{c} n \right) \frac{1}{\left(b + m \frac{1}{c} n \right)} \left(e + m \frac{1}{c} g \right) \right]$

(Alt) Out[]:= $\frac{a b c^2 + c^2 d e + b c f g + c d g m + c e f n + a c m n + 2 f g m n}{c (b c + m n)}$

(Alt) In[]:= **Simplify** $\left[a + f \frac{1}{1 - c} g + \left(d + f \frac{1}{1 - c} n \right) \frac{1}{1 - \left(b + m \frac{1}{1 - c} n \right)} \left(e + m \frac{1}{1 - c} g \right) == a + \frac{(1 - c) d e + (1 - b) f g + d g m + e f n}{(1 - b) (1 - c) - m n} \right]$

(Alt) Out[]:= True

$$(Alt) In[] := \text{Together} \left[a + f \frac{1}{1-c} g + \left(d + f \frac{1}{1-c} n \right) \frac{1}{1 - \left(b + m \frac{1}{1-c} n \right)} \left(e + m \frac{1}{1-c} g \right) \right]$$

$$(Alt) Out[] := \frac{a - ab - ac + abc + de - cde + fg - bfg + dgm + efn - amn}{1 - b - c + bc - mn}$$

$$(Alt) In[] := \text{Simplify}[de - cde + fg - bfg + dgm + efn]$$

$$(Alt) Out[] := d(e - ce + gm) + f(g - bg + en)$$

$$(Alt) In[] := \text{Factor} \left[a + f \frac{1}{1-c} g + \left(d + f \frac{1}{1-c} n \right) \frac{1}{1 - \left(b + m \frac{1}{1-c} n \right)} \left(e + m \frac{1}{1-c} g \right) \right]$$

$$(Alt) Out[] := \frac{a - ab - ac + abc + de - cde + fg - bfg + dgm + efn - amn}{1 - b - c + bc - mn}$$

$$(Alt) In[] := \text{Factor} \left[\frac{1}{1 - \left(b + m \frac{1}{1-c} n \right)} \right]$$

$$(Alt) Out[] := -\frac{-1 + c}{1 - b - c + bc - mn}$$

$$(Alt) In[] := \text{Simplify}[1 + b(-1 + c) - c - mn == (1 - b)(1 - c) - mn]$$

$$(Alt) Out[] := \text{True}$$

$$(Alt) In[] := \text{Simplify}[1 - b - c + bc - mn]$$

$$(Alt) Out[] := 1 + b(-1 + c) - c - mn$$

$$(Alt) In[] := \text{Simplify} \left[f \frac{cc}{\omega} g + \left(d + f \frac{cc}{\omega} n \right) \frac{1}{1 - \left(b + m \frac{cc}{\omega} n \right)} \left(e + m \frac{cc}{\omega} g \right) \right]$$

$$(Alt) Out[] := -\frac{cc d gm + cc f(g - bg + en) + de \omega}{cc mn + (-1 + b) \omega}$$

$$(Alt) In[] := \text{Simplify} \left[a + f \frac{1}{1-c} g + \left(d + f \frac{1}{1-c} n \right) \frac{1}{1 - \left(b + m \frac{1}{1-c} n \right)} \left(e + m \frac{1}{1-c} g \right) == \right.$$

$$\left. a + \frac{(1-c)de + (1-b)fg + dgm + efn}{(1-b)(1-c) - mn} \right]$$

$$(Alt) In[] := \text{FullSimplify} \left[\frac{1}{1 - \left(d + c \frac{1}{1-a} b \right)} == \frac{1}{1-d} c \frac{1}{1 - \left(a + b \frac{1}{1-d} c \right)} b \frac{1}{1-d} + \frac{1}{1-d} \right]$$

$$(Alt) Out[] := \text{True}$$

$$\text{(Alt) In[*]} := \partial_x \left(\frac{1}{1-x} \right)$$

$$\text{(Alt) Out[*]} := \frac{1}{(1-x)^2}$$