

Pensieve header: Kauffman @ A cubic root of -1.

```
In[ ]:= << KnotTheory`
```

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Loading KnotTheory` version of February 2, 2020, 10:53:45.2097.
Read more at http://katlas.org/wiki/KnotTheory.
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```
In[ ]:= KB[pd_PD] := Module[{p, t1, t2, t3, t4, B, d, KB, todo, front, x, v},
  SetAttributes[p, Orderless];
  KB = 1;
  todo = List@@pd;
  front = {};
  v[x_X] := Length[front ∩ (List@@x)];
  While[Length[todo] > 0,
    x = RandomChoice[MaximalBy[todo, v]];
    todo = DeleteCases[todo, x];
    t1 = KB (x /. X[i_, j_, k_, l_] → A * p[i, j] * p[k, l] + B * p[i, l] * p[j, k]);
    t2 = Expand[t1];
    t3 = t2 /. {p[i_, j_] * p[j_, k_] → p[i, k]};
    t4 = t3 /. {p[i_, i_] → d, p[i_, j_]^2 → d};
    KB = Expand[t4 /. {B → 1/A, d → -A^2 - 1/A^2}];
    front = Complement[front ∪ (List@@x), front ∩ (List@@x)];
  ];
  KB
];
KB[K_] := KB[PD[K]]
```

```
In[ ]:= Short[tabK = KB /@ AllKnots[{3, 8}]]
```

$$\text{Out[ ]//Short} = \left\{ \frac{1}{A^7} + \frac{1}{A^3} + A - A^9, -\frac{1}{A^{10}} - A^{10}, \frac{1}{A^9} + \frac{1}{A^5} + \frac{1}{A} - A^{15}, \right. \\ \left. \ll 30 \gg, -1 + \frac{1}{A^{12}} - \frac{1}{A^8} - \frac{1}{A^4} - A^4 + A^{16}, -\frac{2}{A^{10}} - \frac{1}{A^2} + A^6 + A^{14} - A^{18} \right\}$$

```
In[ ]:= Union@Simplify[tabK /. A → e^{π i/3}]
```

```
Out[ ]:= {1}
```

```
In[ ]:= Short[tabL = KB /@ AllLinks[{2, 10}]]
```

$$\text{Out[ ]//Short} = \left\{ \frac{1}{A^6} + \frac{1}{A^2} + A^2 + A^6, 1 + \frac{1}{A^{12}} + A^4 + A^8, \ll 414 \gg, -5 - \frac{1}{A^{20}} - \frac{4}{A^{12}} - \frac{4}{A^8} - \frac{4}{A^4} - 5A^4 - 6A^8 - A^{12} - A^{16} - A^{20} \right\}$$

```
In[ ]:= Union@Simplify[tabL /. A → e^{π i/3}]
```

```
Out[ ]:= {1}
```

```
In[ ]:= (e^{π i/3})^3
```

```
Out[ ]:= -1
```

```
In[ ]:= (e^{π i/3})^{-4}
```

```
Out[ ]:= e^{2 i π / 3}
```

$$\text{In}[*]:= (e^{\pi i/3})^{-4} / (e^{\pi i/3})^4$$

$$\text{Out}[*]= e^{-\frac{2i\pi}{3}}$$

$$\text{In}[*]:= \mathbf{N} [ (-\mathbf{A}^2 - \mathbf{A}^{-2}) / . \mathbf{A} \rightarrow e^{\pi i/3} ]$$

$$\text{Out}[*]= 1. + 0. i$$