In[∘]:= `Expand[Binomial[z + 3, z]]`

Out[∘]= $1+\frac{11 z}{6}+z^2+\frac{z^3}{6}$

In[∘]:= `A[ℰ_] := Expand[ℰ /. z_n_ :> FunctionExpand@Binomial[z + n, n]]`

In[∘]:= `Table[A[z_n], {n, 0, 10}]`

Out[∘]= $\left\{1,\ 1+z,\ 1+\frac{3 z}{2}+\frac{z^2}{2},\ 1+\frac{11 z}{6}+z^2+\frac{z^3}{6},\ 1+\frac{25 z}{12}+\frac{35 z^2}{24}+\frac{5 z^3}{12}+\frac{z^4}{24},\right.$
$1+\frac{137 z}{60}+\frac{15 z^2}{8}+\frac{17 z^3}{24}+\frac{z^4}{8}+\frac{z^5}{120},\ 1+\frac{49 z}{20}+\frac{203 z^2}{90}+\frac{49 z^3}{48}+\frac{35 z^4}{144}+\frac{7 z^5}{240}+\frac{z^6}{720},$
$1+\frac{363 z}{140}+\frac{469 z^2}{180}+\frac{967 z^3}{720}+\frac{7 z^4}{18}+\frac{23 z^5}{360}+\frac{z^6}{180}+\frac{z^7}{5040},$
$1+\frac{761 z}{280}+\frac{29531 z^2}{10080}+\frac{267 z^3}{160}+\frac{1069 z^4}{1920}+\frac{9 z^5}{80}+\frac{13 z^6}{960}+\frac{z^7}{1120}+\frac{z^8}{40320},$
$1+\frac{7129 z}{2520}+\frac{6515 z^2}{2016}+\frac{4523 z^3}{2268}+\frac{95 z^4}{128}+\frac{3013 z^5}{17280}+\frac{5 z^6}{192}+\frac{29 z^7}{12096}+\frac{z^8}{8064}+\frac{z^9}{362880},\ 1+\frac{7381 z}{2520}+$
$\left.\frac{177133 z^2}{50400}+\frac{84095 z^3}{36288}+\frac{341693 z^4}{362880}+\frac{8591 z^5}{34560}+\frac{7513 z^6}{172800}+\frac{121 z^7}{24192}+\frac{11 z^8}{30240}+\frac{11 z^9}{725760}+\frac{z^{10}}{3628800}\right\}$

In[∘]:=
```
B[1] = 1;
B[z] := z_1 - 1;
B[ℰ_] := Expand@Sum[Coefficient[ℰ, z, n] * B[z^n], {n, 0, Exponent[ℰ, z]}];
B[z^n_] := B[z^n] = Expand[n! z_n + B[z^n - n! A[z_n]]]
```

In[∘]:= `B[1]`

Out[∘]= 1

In[∘]:= `B[z] // A`

Out[∘]= z

In[∘]:= `B[z^5] // A`

Out[∘]= z^5

In[∘]:= `Table[B[z^n], {n, 0, 5}]`

Out[∘]= $\{1,\ -1+z_1,\ 1-3 z_1+2 z_2,\ -1+7 z_1-12 z_2+6 z_3,$
$1-15 z_1+50 z_2-60 z_3+24 z_4,\ -1+31 z_1-180 z_2+390 z_3-360 z_4+120 z_5\}$

In[∘]:= `B[Sum[ζ^n z^n / n!, {n, 0, 5}]]`

Out[∘]= $1-\zeta+\frac{\zeta^2}{2}-\frac{\zeta^3}{6}+\frac{\zeta^4}{24}-\frac{\zeta^5}{120}+\zeta z_1-\frac{3 \zeta^2 z_1}{2}+\frac{7 \zeta^3 z_1}{6}-\frac{5 \zeta^4 z_1}{8}+\frac{31 \zeta^5 z_1}{120}+$
$\zeta^2 z_2-2 \zeta^3 z_2+\frac{25 \zeta^4 z_2}{12}-\frac{3 \zeta^5 z_2}{2}+\zeta^3 z_3-\frac{5 \zeta^4 z_3}{2}+\frac{13 \zeta^5 z_3}{4}+\zeta^4 z_4-3 \zeta^5 z_4+\zeta^5 z_5$

```
n1 = 30;
F = B[Sum[ζ^n z^n / n!, {n, 0, n1}]];
Table[Simplify@
  FindGeneratingFunction[Table[Coefficient[Coefficient[F, z_n], ζ, k], {k, n - 1, n1}],
   ζ, FunctionSpace → All], {n, 1, 2}]
```

Out[∘]= $Aborted

In[∘]:=

```
Series[-e^(-2 x) + e^(-x), {x, 0, 10}]
```

Out[∘]=

$$x-\frac{3x^2}{2}+\frac{7x^3}{6}-\frac{5x^4}{8}+\frac{31x^5}{120}-\frac{7x^6}{80}+\frac{127x^7}{5040}-\frac{17x^8}{2688}+\frac{73x^9}{51\,840}-\frac{341x^{10}}{1\,209\,600}+O[x]^{11}$$

In[∘]:=

```
n1 = 30;
F = B[Sum[ζ^n z^n / n!, {n, 0, n1}]];
Table[
 FullSimplify[
  FindSequenceFunction[Table[k! Coefficient[Coefficient[F, z_n], ζ, k], {k, 1, n1}]][ζ]],
 {n,
  1,
  7}]
```

Out[∘]=

$$\Big\{-(-2)^{\zeta}+(-1)^{\zeta},\ (-3)^{\zeta}+(-2)^{1+\zeta}+(-1)^{\zeta},\ -(-4)^{\zeta}-3(-2)^{\zeta}+(-1)^{\zeta}+(-1)^{\zeta}3^{1+\zeta},$$
$$(-5)^{\zeta}+(-4)^{1+\zeta}+(-1)^{\zeta}+(-1)^{1+\zeta}2^{2+\zeta}+2(-1)^{\zeta}3^{1+\zeta},\ \left(1-5\times2^{\zeta}-5\times2^{1+2\zeta}+10\times3^{\zeta}+5^{1+\zeta}-6^{\zeta}\right)e^{i\pi\zeta},$$
$$\left(1-3\times2^{1+\zeta}+5\times3^{1+\zeta}-5\times4^{1+\zeta}+3\times5^{1+\zeta}-6^{1+\zeta}+7^{\zeta}\right)e^{i\pi\zeta},$$
$$\left(1-7\times2^{\zeta}+7\times3^{1+\zeta}-7\times2^{\zeta}\times3^{1+\zeta}-35\times4^{\zeta}+7\times5^{1+\zeta}+7^{1+\zeta}-8^{\zeta}\right)e^{i\pi\zeta}\Big\}$$