

Pensieve header: A combinatorial riddle from Assaf: Prove  $\sum_{k=0}^n \binom{2(n-k)}{n-k} \binom{2k}{k} = 4^n$ .

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In[*]:= AS[n_] := Sum[Binomial[2 (n - k), n - k] Binomial[2 k, k], {k, 0, n}]
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In[*]:= AS /@ Range[8]
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Out[*]:= {4, 16, 64, 256, 1024, 4096, 16384, 65536}
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In[*]:= DSolve[x f'[x] == 4 x f[x] + 4 x^2 f'[x] - 2 x f[x] ^ f[0] == 1, f[x], x]
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Out[*]:= {{f[x] ->  $\frac{1}{\sqrt{1-4x}}$ }}
```