

Pensieve header: Stephen's frog hopping problem.

A frog hops across a pond over  $n$  water lilies, at each step making a random jump forward. How many steps is it expected to take?

```
In[*]:= f[1] = 1;
f[n_] /; n > 1 := f[n] = 1 + Sum[f[k], {k, 1, n - 1}] / n;
```

```
In[*]:= Table[f[n], {n, 1, 20}]
```

```
Out[*]:= {1,  $\frac{3}{2}$ ,  $\frac{11}{6}$ ,  $\frac{25}{12}$ ,  $\frac{137}{60}$ ,  $\frac{49}{20}$ ,  $\frac{363}{140}$ ,  $\frac{761}{280}$ ,  $\frac{7129}{2520}$ ,  $\frac{7381}{2520}$ ,  $\frac{83711}{27720}$ ,  $\frac{86021}{27720}$ ,  $\frac{1145993}{360360}$ ,
 $\frac{1171733}{360360}$ ,  $\frac{1195757}{360360}$ ,  $\frac{2436559}{720720}$ ,  $\frac{42142223}{12252240}$ ,  $\frac{14274301}{4084080}$ ,  $\frac{275295799}{77597520}$ ,  $\frac{55835135}{15519504}$ }
```

```
In[*]:= Table[f[n], {n, 1, 20}] // N
```

```
Out[*]:= {1., 1.5, 1.83333, 2.08333, 2.28333, 2.45, 2.59286, 2.71786, 2.82897, 2.92897, 3.01988,
3.10321, 3.18013, 3.25156, 3.31823, 3.38073, 3.43955, 3.49511, 3.54774, 3.59774}
```

```
In[*]:= g[1] = 1.;
g[n_] /; n > 1 := g[n] = 1 + Sum[g[k], {k, 1, n - 1}] / n;
```

```
In[*]:= Table[g[n], {n, 1, 20}]
```

```
Out[*]:= {1., 1.5, 1.83333, 2.08333, 2.28333, 2.45, 2.59286, 2.71786, 2.82897, 2.92897, 3.01988,
3.10321, 3.18013, 3.25156, 3.31823, 3.38073, 3.43955, 3.49511, 3.54774, 3.59774}
```

```
In[*]:= Table[g[e^n // Round], {n, 10}] // Timing
```

```
Out[*]:= {120.75,
{1.83333, 2.59286, 3.59774, 4.59361, 5.5778, 6.57739, 7.57801, 8.5774, 9.57727, 10.5772}}
```

```
In[*]:= s[0] = 0; s[1] = 1.;
s[n_] /; n > 1 := s[n] =  $\frac{n}{n-1}$  s[n-1] + 1;
h[n_] := s[n] - s[n-1]
```

```
In[*]:= Table[h[n], {n, 20}]
```

```
Out[*]:= {1., 2., 2.5, 2.83333, 3.08333, 3.28333, 3.45, 3.59286, 3.71786, 3.82897, 3.92897,
4.01988, 4.10321, 4.18013, 4.25156, 4.31823, 4.38073, 4.43955, 4.49511, 4.54774}
```

```
In[*]:= e10 // Round
```

```
Out[*]:= 22026
```

```
In[*]:= Table[h[n], {n, e10 // Round}] // Last // Timing
```

```
Out[*]:= {0.140625, 11.5772}
```

```
In[*]:= Table[h[e^n // Round], {n, 10}]
```

```
Out[*]:= {2.5, 3.45, 4.54774, 5.57543, 6.57105, 7.57491, 8.57709, 9.57706, 10.5771, 11.5772}
```