EDDO at k=0

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Compute $[e^{F(z)\zeta}]_\lambda := e^{\lambda\partial_z\partial_\zeta} e^{F(z)\zeta} =: e^{W_\lambda + F_\lambda\zeta}$

$$(\partial_\lambda W + \zeta\partial_\lambda F)e^{W_\lambda+F_\lambda\zeta} = \partial_\lambda e^{W+F\zeta} =$$

W, F depend on z, λ but not on ζ

$$= \partial_\lambda e^{\lambda\partial_z\partial_\zeta} e^{F(z)\zeta} = \partial_z\partial_\zeta e^{\lambda\partial_z\partial_\zeta} e^{F(z)\zeta}$$

$$= \partial_z\partial_\zeta e^{W+F\zeta}$$

$$\boxed{F(\lambda=0)=F}$$

initial condition

$$= \partial_z(F e^{W+F\zeta})$$

$$= (\partial_z F + F\partial_z W + F\zeta\partial_z F)e^{W+F_\lambda\zeta}$$

So $\partial_\lambda W + \zeta\partial_\lambda F = \partial_z F + F\partial_z W + F\zeta\partial_z F$

So $\boxed{\partial_\lambda F = F\partial_z F}$ *

and $\partial_\lambda W = \partial_z F + F\partial_z W$

characteristics for *:

$$\frac{d\lambda}{dt} = 1 \qquad \frac{dz}{dt} = -\phi \qquad \frac{d\phi}{dt} = 0$$

$$\lambda = t+a \qquad z = -\phi t + b \qquad \phi = c$$

$F(\lambda, z) =$
$F(0, \phi\lambda + z)$

take $a=\lambda$, $b=z$
$t=-\lambda$

If $F = f(\lambda)z$, get

$f'z = fzf$

$f' = f^2$

$f = \frac{1}{1-\lambda} \qquad f' = \frac{1}{(1-\lambda)^2}$

$\partial_\lambda W = \frac{1}{1-\lambda} + \frac{z}{1-\lambda}\partial_z W$

$W = \log(1-\lambda)^{-1}$