In[∘]:= $$\begin{pmatrix}1 & b\,d^{-1}\\ 0 & 1\end{pmatrix}.\begin{pmatrix}a-b\,d^{-1}\,c & 0\\ 0 & d\end{pmatrix}.\begin{pmatrix}1 & 0\\ d^{-1}\,c & 1\end{pmatrix}$$

Out[∘]= `{{a, b}, {c, d}}`

In[∘]:= $$\text{Simplify}\left[\text{Inverse}\left[\begin{pmatrix}a & b\\ c & d\end{pmatrix}\right] == \begin{pmatrix}\left(a-b\,d^{-1}\,c\right)^{-1} & -\left(a-b\,d^{-1}\,c\right)^{-1}\,b\,d^{-1}\\ -d^{-1}\,c\,\left(a-b\,d^{-1}\,c\right)^{-1} & d^{-1}+d^{-1}\,c\,\left(a-b\,d^{-1}\,c\right)^{-1}\,b\,d^{-1}\end{pmatrix}\right]$$

Out[∘]= True

In[∘]:= $$\text{Simplify}\left[d^{-1}+d^{-1}\,c\,\left(a-b\,d^{-1}\,c\right)^{-1}\,b\,d^{-1}\right]$$

Out[∘]= $$\frac{a}{-b\,c+a\,d}$$

In[∘]:= $$\begin{pmatrix}1 & b\,\lambda\,d^{-1}\\ 0 & 1\end{pmatrix}.\begin{pmatrix}a-b\,\lambda\,d^{-1}\,c & 0\\ 0 & d\end{pmatrix}.\begin{pmatrix}1 & 0\\ \lambda\,d^{-1}\,c & 1\end{pmatrix}\ \text{// Simplify // MatrixForm}$$

Out[∘]//MatrixForm=
$$\begin{pmatrix}a+\frac{b\,c\,(-1+\lambda)\,\lambda}{d} & b\,\lambda\\ c\,\lambda & d\end{pmatrix}$$

In[∘]:= $$\begin{pmatrix}1 & b\,\lambda\,d^{-1}\\ 0 & 1\end{pmatrix}.\begin{pmatrix}a-b\,\lambda\,d^{-1}\,c & 0\\ 0 & \lambda^{-1}\,d\end{pmatrix}.\begin{pmatrix}1 & 0\\ \lambda\,d^{-1}\,c & 1\end{pmatrix}\ \text{// Simplify // MatrixForm}$$

Out[∘]//MatrixForm=
$$\begin{pmatrix}a & b\\ c & \frac{d}{\lambda}\end{pmatrix}$$

In[∘]:= $A = \text{Table}[a_{i,j}, \{i, 4\}, \{j, 4\}]$

Out[∘]= $\{\{a_{1,1}, a_{1,2}, a_{1,3}, a_{1,4}\}, \{a_{2,1}, a_{2,2}, a_{2,3}, a_{2,4}\}, \{a_{3,1}, a_{3,2}, a_{3,3}, a_{3,4}\}, \{a_{4,1}, a_{4,2}, a_{4,3}, a_{4,4}\}\}$

In[∘]:= `Minors[Det[A] Inverse[A], 2] / Det[A] // Together`

Out[∘]= $\{\{-a_{3,4}\,a_{4,3}+a_{3,3}\,a_{4,4},\ a_{2,4}\,a_{4,3}-a_{2,3}\,a_{4,4},\ -a_{2,4}\,a_{3,3}+a_{2,3}\,a_{3,4},\ -a_{1,4}\,a_{4,3}+a_{1,3}\,a_{4,4},$
$a_{1,4}\,a_{3,3}-a_{1,3}\,a_{3,4},\ -a_{1,4}\,a_{2,3}+a_{1,3}\,a_{2,4}\},\ \{a_{3,4}\,a_{4,2}-a_{3,2}\,a_{4,4},\ -a_{2,4}\,a_{4,2}+a_{2,2}\,a_{4,4},$
$a_{2,4}\,a_{3,2}-a_{2,2}\,a_{3,4},\ a_{1,4}\,a_{4,2}-a_{1,2}\,a_{4,4},\ -a_{1,4}\,a_{3,2}+a_{1,2}\,a_{3,4},\ a_{1,4}\,a_{2,2}-a_{1,2}\,a_{2,4}\},$
$\{-a_{3,3}\,a_{4,2}+a_{3,2}\,a_{4,3},\ a_{2,3}\,a_{4,2}-a_{2,2}\,a_{4,3},\ -a_{2,3}\,a_{3,2}+a_{2,2}\,a_{3,3},\ -a_{1,3}\,a_{4,2}+a_{1,2}\,a_{4,3},$
$a_{1,3}\,a_{3,2}-a_{1,2}\,a_{3,3},\ -a_{1,3}\,a_{2,2}+a_{1,2}\,a_{2,3}\},\ \{-a_{3,4}\,a_{4,1}+a_{3,1}\,a_{4,4},\ a_{2,4}\,a_{4,1}-a_{2,1}\,a_{4,4},$
$-a_{2,4}\,a_{3,1}+a_{2,1}\,a_{3,4},\ -a_{1,4}\,a_{4,1}+a_{1,1}\,a_{4,4},\ a_{1,4}\,a_{3,1}-a_{1,1}\,a_{3,4},\ -a_{1,4}\,a_{2,1}+a_{1,1}\,a_{2,4}\},$
$\{a_{3,3}\,a_{4,1}-a_{3,1}\,a_{4,3},\ -a_{2,3}\,a_{4,1}+a_{2,1}\,a_{4,3},\ a_{2,3}\,a_{3,1}-a_{2,1}\,a_{3,3},\ a_{1,3}\,a_{4,1}-a_{1,1}\,a_{4,3},$
$-a_{1,3}\,a_{3,1}+a_{1,1}\,a_{3,3},\ a_{1,3}\,a_{2,1}-a_{1,1}\,a_{2,3}\},\ \{-a_{3,2}\,a_{4,1}+a_{3,1}\,a_{4,2},\ a_{2,2}\,a_{4,1}-a_{2,1}\,a_{4,2},$
$-a_{2,2}\,a_{3,1}+a_{2,1}\,a_{3,2},\ -a_{1,2}\,a_{4,1}+a_{1,1}\,a_{4,2},\ a_{1,2}\,a_{3,1}-a_{1,1}\,a_{3,2},\ -a_{1,2}\,a_{2,1}+a_{1,1}\,a_{2,2}\}\}$

In[∘]:=
```
p = 3; q = 2;
n = p + q;
A = Table[α_(10 i+j), {i, n}, {j, n}];
Map[MatrixForm, ( a = A[[1 ;; p, 1 ;; p]]       b = A[[1 ;; p, p + 1 ;; n]]
                  c = A[[p + 1 ;; n, 1 ;; p]]   d = A[[p + 1 ;; n, p + 1 ;; n]] ), {2}] // MatrixForm
```

Out[∘]//MatrixForm=
$$\begin{pmatrix}\begin{pmatrix}\alpha_{11} & \alpha_{12} & \alpha_{13}\\ \alpha_{21} & \alpha_{22} & \alpha_{23}\\ \alpha_{31} & \alpha_{32} & \alpha_{33}\end{pmatrix} & \begin{pmatrix}\alpha_{14} & \alpha_{15}\\ \alpha_{24} & \alpha_{25}\\ \alpha_{34} & \alpha_{35}\end{pmatrix}\\ \begin{pmatrix}\alpha_{41} & \alpha_{42} & \alpha_{43}\\ \alpha_{51} & \alpha_{52} & \alpha_{53}\end{pmatrix} & \begin{pmatrix}\alpha_{44} & \alpha_{45}\\ \alpha_{54} & \alpha_{55}\end{pmatrix}\end{pmatrix}$$

In[∘]:= `Simplify[Det[A] * Inverse[a - b.Inverse[d].c]] // MatrixForm`

Out[∘]//MatrixForm=

$$\begin{pmatrix} \alpha_{23}\,\alpha_{35}\,\alpha_{44}\,\alpha_{52} - \alpha_{23}\,\alpha_{34}\,\alpha_{45}\,\alpha_{52} - \alpha_{22}\,\alpha_{35}\,\alpha_{44}\,\alpha_{53} + \alpha_{22}\,\alpha_{34}\,\alpha_{45}\,\alpha_{53} - \alpha_{23}\,\alpha_{35}\,\alpha_{42}\,\alpha_{54} + \alpha_{22}\,\alpha_{35}\,\alpha_{43}\,\alpha_{54} + \alpha_{23}\,\alpha_{32}\,\alpha_{4} \\ -\alpha_{23}\,\alpha_{35}\,\alpha_{44}\,\alpha_{51} + \alpha_{23}\,\alpha_{34}\,\alpha_{45}\,\alpha_{51} + \alpha_{21}\,\alpha_{35}\,\alpha_{44}\,\alpha_{53} - \alpha_{21}\,\alpha_{34}\,\alpha_{45}\,\alpha_{53} + \alpha_{23}\,\alpha_{35}\,\alpha_{41}\,\alpha_{54} - \alpha_{21}\,\alpha_{35}\,\alpha_{43}\,\alpha_{54} - \alpha_{23}\,\alpha_{31}\,\alpha \\ \alpha_{22}\,\alpha_{35}\,\alpha_{44}\,\alpha_{51} - \alpha_{22}\,\alpha_{34}\,\alpha_{45}\,\alpha_{51} - \alpha_{21}\,\alpha_{35}\,\alpha_{44}\,\alpha_{52} + \alpha_{21}\,\alpha_{34}\,\alpha_{45}\,\alpha_{52} - \alpha_{22}\,\alpha_{35}\,\alpha_{41}\,\alpha_{54} + \alpha_{21}\,\alpha_{35}\,\alpha_{42}\,\alpha_{54} + \alpha_{22}\,\alpha_{31}\,\alpha_{4} \end{pmatrix}$$

In[∘]:=

```
Ai = Simplify[Inverse[A]];
di = Simplify[Inverse[d]];
api = Simplify[Inverse[a - b.di.c]];
Simplify[
 ( Ai[[1 ;; p, 1 ;; p]]        Ai[[1 ;; p, p + 1 ;; n]]     ) == (  api          -api.b.di        )]
 ( Ai[[p + 1 ;; n, 1 ;; p]]    Ai[[p + 1 ;; n, p + 1 ;; n]] )    ( -di.c.api    di + di.c.api.b.di )
```

Out[∘]= True