

```
In[*]:= KD /: KDi,j := If[i === j, 1, 0];
{z*, x*, y*} = {ξ, ξ, η}; {ξ*, ξ*, η*} = {z, x, y};
(u-i)* := (u*)i;
CF[ε-] := Module[
  {vs = Cases[ε, (z | ξ | y | η)-, ∞] ∪ {z, ξ, y, η}},
  Total[CoefficientRules[Expand[ε], vs] /. (ps- → c-) ⇒ Simplify[c] (Times @@ vsps)
];
E /: E[ω1-, Q1-] ≡ E[ω2-, Q2-] := CF[ω1 == ω2] ∧ CF[Q1 == Q2];
```

```
In[*]:= E /: Zipξs_List@E[ω-, Q-] := Module[{ξ, z, zs, c, ys, ηs, qt, zrule, ξrule},
  zs = Table[ξ*, {ξ, ξs}];
  c = Q /. Alternatives @@ (ξs ∪ zs) → 0;
  ys = Table[∂ξ (Q /. Alternatives @@ zs → 0), {ξ, ξs}];
  ηs = Table[∂z (Q /. Alternatives @@ ξs → 0), {z, zs}];
  qt = Inverse@Table[KDz,ξ* - ∂z,ξQ, {ξ, ξs}, {z, zs}];
  zrule = Thread[zs → qt.(zs + ys)];
  ξrule = Thread[ξs → ξs + ηs.qt];
  CF /@ E[ω Det[qt], c + ηs.qt.ys];
```

In[*]:= E1 =

$$\text{CF} /@ \text{E} \left[1, \text{Tr} \left[\left(\begin{array}{ccc} \theta_1 & \theta_2 & \theta_3 \end{array} \right) \cdot \begin{pmatrix} z_1 \\ z_2 \\ z_3 \end{pmatrix} + \left(\begin{array}{ccc} \xi_1 & \xi_2 & \xi_3 \end{array} \right) \cdot \begin{pmatrix} \phi_1 \\ \phi_2 \\ \phi_3 \end{pmatrix} + \left(\begin{array}{ccc} \xi_1 & \xi_2 & \xi_3 \end{array} \right) \cdot \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \cdot \begin{pmatrix} z_1 \\ z_2 \\ z_3 \end{pmatrix} \right] \right]$$

$$\text{Out[*]} = \text{E} \left[1, a_{11} z_1 \xi_1 + a_{12} z_2 \xi_1 + a_{13} z_3 \xi_1 + a_{21} z_1 \xi_2 + a_{22} z_2 \xi_2 + a_{23} z_3 \xi_2 + a_{31} z_1 \xi_3 + a_{32} z_2 \xi_3 + a_{33} z_3 \xi_3 + z_1 \theta_1 + z_2 \theta_2 + z_3 \theta_3 + \xi_1 \phi_1 + \xi_2 \phi_2 + \xi_3 \phi_3 \right]$$

In[*]:= E2 = Zip_{ξs}[E1]

$$\text{Out[*]} = \text{E} \left[\frac{1}{1 - a_{33}}, \left(a_{11} + \frac{a_{13} a_{31}}{1 - a_{33}} \right) z_1 \xi_1 + \left(a_{12} + \frac{a_{13} a_{32}}{1 - a_{33}} \right) z_2 \xi_1 + \left(a_{21} + \frac{a_{23} a_{31}}{1 - a_{33}} \right) z_1 \xi_2 + \left(a_{22} + \frac{a_{23} a_{32}}{1 - a_{33}} \right) z_2 \xi_2 + z_1 \left(\theta_1 + \frac{a_{31} \theta_3}{1 - a_{33}} \right) + z_2 \left(\theta_2 + \frac{a_{32} \theta_3}{1 - a_{33}} \right) + \frac{\theta_3 \phi_3}{1 - a_{33}} + \xi_1 \left(\phi_1 + \frac{a_{13} \phi_3}{1 - a_{33}} \right) + \xi_2 \left(\phi_2 + \frac{a_{23} \phi_3}{1 - a_{33}} \right) \right]$$

In[*]:= E3 = Zip_{ξ2, ξ3}[E1]

$$\text{Out[*]} = \text{E} \left[\frac{1}{1 - a_{23} a_{32} + a_{22} (-1 + a_{33}) - a_{33}}, \left(a_{13} (-(-1 + a_{22}) a_{31} + a_{21} a_{32}) + a_{12} (a_{23} a_{31} - a_{21} (-1 + a_{33})) + a_{11} (1 - a_{23} a_{32} + a_{22} (-1 + a_{33}) - a_{33}) \right) z_1 \xi_1 / \left(1 - a_{23} a_{32} + a_{22} (-1 + a_{33}) - a_{33} \right) + \left(z_1 \left((-1 + a_{23} a_{32} - a_{22} (-1 + a_{33}) + a_{33}) \theta_1 - a_{31} (a_{23} \theta_2 - (-1 + a_{22}) \theta_3) + a_{21} ((-1 + a_{33}) \theta_2 - a_{32} \theta_3) \right) \right) / \left((-1 + a_{23} a_{32} - a_{22} (-1 + a_{33}) + a_{33}) \right) + \left(\xi_1 \left((-1 + a_{23} a_{32} - a_{22} (-1 + a_{33}) + a_{33}) \phi_1 - a_{13} (a_{32} \phi_2 - (-1 + a_{22}) \phi_3) + a_{12} ((-1 + a_{33}) \phi_2 - a_{23} \phi_3) \right) \right) / \left((-1 + a_{23} a_{32} - a_{22} (-1 + a_{33}) + a_{33}) \right) + \left(\theta_3 (a_{32} \phi_2 - (-1 + a_{22}) \phi_3) + \theta_2 (-(-1 + a_{33}) \phi_2 + a_{23} \phi_3) \right) / \left(1 - a_{23} a_{32} + a_{22} (-1 + a_{33}) - a_{33} \right) \right]$$

In[*]:= **E4 = Zip**_{ξ₂} [E2]

$$\text{Out[*]} = \mathbb{E} \left[\frac{1}{1 - a_{23} a_{32} + a_{22} (-1 + a_{33}) - a_{33}}, \right. \\ \left. \left(\frac{a_{13} (-(-1 + a_{22}) a_{31} + a_{21} a_{32}) + a_{12} (a_{23} a_{31} - a_{21} (-1 + a_{33})) + a_{11} (1 - a_{23} a_{32} + a_{22} (-1 + a_{33}) - a_{33})}{z_1 \xi_1} \right) / \left(1 - a_{23} a_{32} + a_{22} (-1 + a_{33}) - a_{33} \right) + \right. \\ \left. \left(z_1 \left(\frac{(-1 + a_{23} a_{32} - a_{22} (-1 + a_{33}) + a_{33}) \theta_1 - a_{31} (a_{23} \theta_2 - (-1 + a_{22}) \theta_3) + a_{21} ((-1 + a_{33}) \theta_2 - a_{32} \theta_3)}{(-1 + a_{23} a_{32} - a_{22} (-1 + a_{33}) + a_{33})} \right) \right) / \left(-1 + a_{23} a_{32} - a_{22} (-1 + a_{33}) + a_{33} \right) + \right. \\ \left. \left(\xi_1 \left(\frac{(-1 + a_{23} a_{32} - a_{22} (-1 + a_{33}) + a_{33}) \phi_1 - a_{13} (a_{32} \phi_2 - (-1 + a_{22}) \phi_3) + a_{12} ((-1 + a_{33}) \phi_2 - a_{23} \phi_3)}{(-1 + a_{23} a_{32} - a_{22} (-1 + a_{33}) + a_{33})} \right) \right) / \left(-1 + a_{23} a_{32} - a_{22} (-1 + a_{33}) + a_{33} \right) + \right. \\ \left. \left(\theta_3 (a_{32} \phi_2 - (-1 + a_{22}) \phi_3) + \theta_2 (-(-1 + a_{33}) \phi_2 + a_{23} \phi_3) \right) / \left(1 - a_{23} a_{32} + a_{22} (-1 + a_{33}) - a_{33} \right) \right]$$

In[*]:= **E3 == E4**

Out[*]:= True

Convention: $\begin{pmatrix} \gamma & \theta \\ \phi & a \end{pmatrix}$ stands for $\text{Det}[\gamma]^{-1} e^{\theta z + \zeta \phi + \zeta a z}$.

$$\begin{pmatrix} 1 & \theta_1 & \theta_2 & \theta_3 \\ \phi_1 & a_{11} & a_{12} & a_{13} \\ \phi_2 & a_{21} & a_{22} & a_{23} \\ \phi_3 & a_{31} & a_{32} & a_{33} \end{pmatrix} \rightarrow \begin{pmatrix} (1 - a_{33}) & \theta_1 + \theta_3 (1 - a_{33})^{-1} a_{31} & \theta_2 + \theta_3 (1 - a_{33})^{-1} a_{31} & \theta_3 \\ \phi_1 + a_{13} (1 - a_{33})^{-1} \phi_3 & a_{11} + a_{13} (1 - a_{33})^{-1} a_{31} & a_{12} + a_{13} (1 - a_{33})^{-1} a_{31} & a_{13} \\ \phi_2 + a_{23} (1 - a_{33})^{-1} \phi_3 & a_{21} + a_{23} (1 - a_{33})^{-1} a_{31} & a_{22} + a_{23} (1 - a_{33})^{-1} a_{31} & a_{23} \\ \phi_3 & a_{31} & a_{32} & a_{33} \end{pmatrix} \\ \downarrow \qquad \qquad \qquad \downarrow \\ \left(\begin{pmatrix} 1 - \begin{pmatrix} a_{22} & a_{23} \\ a_{32} & a_{33} \end{pmatrix} \end{pmatrix} \right) \square \stackrel{?}{=} \left((1 - a_{33}) \oplus \begin{pmatrix} 1 - \begin{pmatrix} a_{22} + a_{23} (1 - a_{33})^{-1} a_{32} \end{pmatrix} \end{pmatrix} \right) \square \\ \square \qquad \qquad \qquad \square \qquad \qquad \qquad \square \qquad \qquad \qquad \square$$

In[*]:= **F1 =**

$$\text{CF} / @ \mathbb{E} \left[1, \text{Tr} \left[\left(\eta_1 \ \eta_2 \ \eta_3 \right) \cdot \begin{pmatrix} z_1 \\ z_2 \\ z_3 \end{pmatrix} + \left(\xi_1 \ \xi_2 \ \xi_3 \right) \cdot \begin{pmatrix} y_1 \\ y_2 \\ y_3 \end{pmatrix} + \left(\zeta_1 \ \zeta_2 \ \zeta_3 \right) \cdot \begin{pmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{pmatrix} \cdot \begin{pmatrix} z_1 \\ z_2 \\ z_3 \end{pmatrix} \right] \right]$$

$$\text{Out[*]} = \mathbb{E} \left[1, y_1 \xi_1 + a_{11} z_1 \xi_1 + a_{12} z_2 \xi_1 + a_{13} z_3 \xi_1 + y_2 \xi_2 + a_{21} z_1 \xi_2 + a_{22} z_2 \xi_2 + a_{23} z_3 \xi_2 + y_3 \xi_3 + a_{31} z_1 \xi_3 + a_{32} z_2 \xi_3 + a_{33} z_3 \xi_3 + z_1 \eta_1 + z_2 \eta_2 + z_3 \eta_3 \right]$$

In[*]:= **F2 = Zip**_{ξ₃} [F1]

$$\text{Out[*]} = \mathbb{E} \left[\frac{1}{1 - a_{33}}, y_1 \xi_1 + \frac{a_{13} y_3 \xi_1}{1 - a_{33}} + \left(a_{11} + \frac{a_{13} a_{31}}{1 - a_{33}} \right) z_1 \xi_1 + \left(a_{12} + \frac{a_{13} a_{32}}{1 - a_{33}} \right) z_2 \xi_1 + y_2 \xi_2 + \frac{a_{23} y_3 \xi_2}{1 - a_{33}} + \right. \\ \left. \left(a_{21} + \frac{a_{23} a_{31}}{1 - a_{33}} \right) z_1 \xi_2 + \left(a_{22} + \frac{a_{23} a_{32}}{1 - a_{33}} \right) z_2 \xi_2 + z_1 \eta_1 + z_2 \eta_2 + \frac{y_3 \eta_3}{1 - a_{33}} + \frac{a_{31} z_1 \eta_3}{1 - a_{33}} + \frac{a_{32} z_2 \eta_3}{1 - a_{33}} \right]$$

In[*]:= **F3 = Zip**_{ξ₂, ξ₃} [**F1**]

$$\text{Out[*]} = \mathbb{E} \left[\frac{1}{1 - a_{23} a_{32} + a_{22} (-1 + a_{33}) - a_{33}}, \right. \\ \left. y_1 \xi_1 + \frac{(-a_{13} a_{32} + a_{12} (-1 + a_{33})) y_2 \xi_1}{-1 + a_{23} a_{32} - a_{22} (-1 + a_{33}) + a_{33}} + \frac{(-a_{13} (-1 + a_{22}) + a_{12} a_{23}) y_3 \xi_1}{1 - a_{23} a_{32} + a_{22} (-1 + a_{33}) - a_{33}} + \right. \\ \left. \left(\frac{(a_{13} (-(-1 + a_{22}) a_{31} + a_{21} a_{32}) + a_{12} (a_{23} a_{31} - a_{21} (-1 + a_{33})) + a_{11}}{(1 - a_{23} a_{32} + a_{22} (-1 + a_{33}) - a_{33})) z_1 \xi_1}{(1 - a_{23} a_{32} + a_{22} (-1 + a_{33}) - a_{33})} + \right. \right. \\ \left. \frac{z_1 \eta_1 + \frac{(1 - a_{33}) y_2 \eta_2}{1 - a_{23} a_{32} + a_{22} (-1 + a_{33}) - a_{33}} + \frac{a_{23} y_3 \eta_2}{1 - a_{23} a_{32} + a_{22} (-1 + a_{33}) - a_{33}}}{1 - a_{23} a_{32} + a_{22} (-1 + a_{33}) - a_{33}} + \right. \\ \left. \frac{(-a_{23} a_{31} + a_{21} (-1 + a_{33})) z_1 \eta_2}{-1 + a_{23} a_{32} - a_{22} (-1 + a_{33}) + a_{33}} + \frac{a_{32} y_2 \eta_3}{1 - a_{23} a_{32} + a_{22} (-1 + a_{33}) - a_{33}} + \right. \\ \left. \frac{(1 - a_{22}) y_3 \eta_3}{1 - a_{23} a_{32} + a_{22} (-1 + a_{33}) - a_{33}} + \frac{(-(-1 + a_{22}) a_{31} + a_{21} a_{32}) z_1 \eta_3}{1 - a_{23} a_{32} + a_{22} (-1 + a_{33}) - a_{33}} \right]$$

In[*]:= **F4 = Zip**_{ξ₂} [**F2**]

$$\text{Out[*]} = \mathbb{E} \left[\frac{1}{1 - a_{23} a_{32} + a_{22} (-1 + a_{33}) - a_{33}}, \right. \\ \left. y_1 \xi_1 + \frac{(-a_{13} a_{32} + a_{12} (-1 + a_{33})) y_2 \xi_1}{-1 + a_{23} a_{32} - a_{22} (-1 + a_{33}) + a_{33}} + \frac{(-a_{13} (-1 + a_{22}) + a_{12} a_{23}) y_3 \xi_1}{1 - a_{23} a_{32} + a_{22} (-1 + a_{33}) - a_{33}} + \right. \\ \left. \left(\frac{(a_{13} (-(-1 + a_{22}) a_{31} + a_{21} a_{32}) + a_{12} (a_{23} a_{31} - a_{21} (-1 + a_{33})) + a_{11}}{(1 - a_{23} a_{32} + a_{22} (-1 + a_{33}) - a_{33})) z_1 \xi_1}{(1 - a_{23} a_{32} + a_{22} (-1 + a_{33}) - a_{33})} + z_1 \eta_1 + \right. \right. \\ \left. \frac{y_2 \eta_2}{1 - a_{22} + \frac{a_{23} a_{32}}{-1 + a_{33}}} + \frac{a_{23} y_3 \eta_2}{1 - a_{23} a_{32} + a_{22} (-1 + a_{33}) - a_{33}} + \frac{(-a_{23} a_{31} + a_{21} (-1 + a_{33})) z_1 \eta_2}{-1 + a_{23} a_{32} - a_{22} (-1 + a_{33}) + a_{33}} + \right. \\ \left. \frac{a_{32} y_2 \eta_3}{1 - a_{23} a_{32} + a_{22} (-1 + a_{33}) - a_{33}} + \frac{(-1 + a_{22}) y_3 \eta_3}{-1 + a_{23} a_{32} - a_{22} (-1 + a_{33}) + a_{33}} + \right. \\ \left. \frac{(-(-1 + a_{22}) a_{31} + a_{21} a_{32}) z_1 \eta_3}{1 - a_{23} a_{32} + a_{22} (-1 + a_{33}) - a_{33}} \right]$$

In[*]:= **F3 ≡ F4**

Out[*]:= True

$$\text{In[*]} = \left(\mathbf{G1} = \mathbf{CF} / @ \mathbb{E} \left[\mathbf{1}, \text{Tr} \left[\left(\eta_1 \ \eta_2 \right) \cdot \begin{pmatrix} z_1 \\ z_2 \end{pmatrix} + \left(\xi_1 \ \xi_2 \right) \cdot \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} + \left(\xi_1 \ \xi_2 \right) \cdot \begin{pmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{pmatrix} \cdot \begin{pmatrix} z_1 \\ z_2 \end{pmatrix} \right] \right] \right] \equiv \\ \mathbb{E} \left[\mathbf{1}, \text{Tr} \left[\left(\eta_1 \ \eta_2 \ \xi_1 \ \xi_2 \right) \cdot \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & 0 & a_{11} & a_{12} \\ 0 & 1 & a_{21} & a_{22} \end{pmatrix} \cdot \begin{pmatrix} y_1 \\ y_2 \\ z_1 \\ z_2 \end{pmatrix} \right] \right]$$

Out[*]:= True

$$\text{In[*]:= } (\mathbf{G2} = \text{Zip}_{\{\xi_2\}}[\mathbf{G1}]) \equiv \mathbb{E} \left[(\mathbf{1} - \mathbf{a}_{22})^{-1}, \text{Tr} \left[(\eta_1 \ \eta_2 \ \xi_1) \cdot \begin{pmatrix} \mathbf{0} & \mathbf{0} & \mathbf{1} \\ \mathbf{0} & (\mathbf{1} - \mathbf{a}_{22})^{-1} & (\mathbf{1} - \mathbf{a}_{22})^{-1} \mathbf{a}_{21} \\ \mathbf{1} & \mathbf{a}_{12} (\mathbf{1} - \mathbf{a}_{22})^{-1} & \mathbf{a}_{11} + \mathbf{a}_{12} (\mathbf{1} - \mathbf{a}_{22})^{-1} \mathbf{a}_{21} \end{pmatrix} \cdot \begin{pmatrix} \mathbf{y}_1 \\ \mathbf{y}_2 \\ \mathbf{z}_1 \end{pmatrix} \right] \right]$$

Out[*]= True

$$\text{In[*]:= } \begin{pmatrix} (\mathbf{1} - \mathbf{a}_{22})^{-1} & (\mathbf{1} - \mathbf{a}_{22})^{-1} \mathbf{a}_{21} \\ \mathbf{a}_{12} (\mathbf{1} - \mathbf{a}_{22})^{-1} & \mathbf{a}_{11} + \mathbf{a}_{12} (\mathbf{1} - \mathbf{a}_{22})^{-1} \mathbf{a}_{21} \end{pmatrix} == \begin{pmatrix} \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \mathbf{a}_{11} \end{pmatrix} + \begin{pmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{a}_{12} \end{pmatrix} \cdot \left((\mathbf{1} - \mathbf{a}_{22})^{-1} \begin{pmatrix} \mathbf{1} & \mathbf{1} \\ \mathbf{1} & \mathbf{1} \end{pmatrix} \right) \cdot \begin{pmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{a}_{21} \end{pmatrix}$$

Out[*]= True

$$\text{In[*]:= } (\mathbf{G3} = \text{Zip}_{\{\xi_1, \xi_2\}}[\mathbf{G1}]) \equiv \mathbb{E} \left[\text{Det} \left[\begin{pmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} \end{pmatrix} - \begin{pmatrix} \mathbf{a}_{11} & \mathbf{a}_{12} \\ \mathbf{a}_{21} & \mathbf{a}_{22} \end{pmatrix} \right]^{-1}, \text{Tr} \left[(\eta_1 \ \eta_2) \cdot \begin{pmatrix} \frac{(\mathbf{1} - \mathbf{a}_{22})}{\text{Det} \left[\begin{pmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} \end{pmatrix} - \begin{pmatrix} \mathbf{a}_{11} & \mathbf{a}_{12} \\ \mathbf{a}_{21} & \mathbf{a}_{22} \end{pmatrix} \right]} & \frac{\mathbf{a}_{12}}{\text{Det} \left[\begin{pmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} \end{pmatrix} - \begin{pmatrix} \mathbf{a}_{11} & \mathbf{a}_{12} \\ \mathbf{a}_{21} & \mathbf{a}_{22} \end{pmatrix} \right]} \\ \frac{\mathbf{a}_{21}}{\text{Det} \left[\begin{pmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} \end{pmatrix} - \begin{pmatrix} \mathbf{a}_{11} & \mathbf{a}_{12} \\ \mathbf{a}_{21} & \mathbf{a}_{22} \end{pmatrix} \right]} & \frac{(\mathbf{1} - \mathbf{a}_{11})}{\text{Det} \left[\begin{pmatrix} \mathbf{1} & \mathbf{0} \\ \mathbf{0} & \mathbf{1} \end{pmatrix} - \begin{pmatrix} \mathbf{a}_{11} & \mathbf{a}_{12} \\ \mathbf{a}_{21} & \mathbf{a}_{22} \end{pmatrix} \right]} \end{pmatrix} \cdot \begin{pmatrix} \mathbf{y}_1 \\ \mathbf{y}_2 \end{pmatrix} \right] \right]$$

Out[*]= True

$$\text{In[*]:= } (\mathbf{G4} = \text{Zip}_{\{\xi_1\}}[\mathbf{G2}]) \equiv \mathbb{E} \left[(\mathbf{1} - \mathbf{a}_{22})^{-1} \left(\mathbf{1} - \left(\mathbf{a}_{11} + \mathbf{a}_{12} (\mathbf{1} - \mathbf{a}_{22})^{-1} \mathbf{a}_{21} \right) \right)^{-1}, \text{Tr} \left[(\eta_1 \ \eta_2) \cdot \begin{pmatrix} \left(\mathbf{1} - \left(\mathbf{a}_{11} + \mathbf{a}_{12} (\mathbf{1} - \mathbf{a}_{22})^{-1} \mathbf{a}_{21} \right) \right)^{-1} & \left(\mathbf{1} - \left(\mathbf{a}_{11} + \mathbf{a}_{12} (\mathbf{1} - \mathbf{a}_{22})^{-1} \mathbf{a}_{21} \right) \right)^{-1} \mathbf{a}_{12} \\ \left(\mathbf{1} - \mathbf{a}_{22} \right)^{-1} \mathbf{a}_{21} \left(\mathbf{1} - \left(\mathbf{a}_{11} + \mathbf{a}_{12} (\mathbf{1} - \mathbf{a}_{22})^{-1} \mathbf{a}_{21} \right) \right)^{-1} & \left(\mathbf{1} - \mathbf{a}_{22} \right)^{-1} + \left(\mathbf{1} - \mathbf{a}_{22} \right)^{-1} \mathbf{a}_{21} \left(\mathbf{1} - \left(\mathbf{a}_{11} + \mathbf{a}_{12} (\mathbf{1} - \mathbf{a}_{22})^{-1} \mathbf{a}_{21} \right) \right)^{-1} \end{pmatrix} \cdot \begin{pmatrix} \mathbf{y}_1 \\ \mathbf{y}_2 \end{pmatrix} \right] \right]$$

Out[*]= True

$$\text{In[*]:= } \text{Together} /@ \left((\mathbf{1} - \mathbf{a}_{22})^{-1} \mathbf{a}_{21} \left(\mathbf{1} - \left(\mathbf{a}_{11} + \mathbf{a}_{12} (\mathbf{1} - \mathbf{a}_{22})^{-1} \mathbf{a}_{21} \right) \right)^{-1} \mathbf{a}_{12} (\mathbf{1} - \mathbf{a}_{22})^{-1} + \{ \mathbf{0}, (\mathbf{1} - \mathbf{a}_{22})^{-1} \} \right)$$

$$\text{Out[*]:= } \left\{ -\frac{\mathbf{a}_{12} \mathbf{a}_{21}}{(-\mathbf{1} + \mathbf{a}_{22}) (\mathbf{1} - \mathbf{a}_{11} - \mathbf{a}_{12} \mathbf{a}_{21} - \mathbf{a}_{22} + \mathbf{a}_{11} \mathbf{a}_{22})}, \frac{-\mathbf{1} + \mathbf{a}_{11}}{-\mathbf{1} + \mathbf{a}_{11} + \mathbf{a}_{12} \mathbf{a}_{21} + \mathbf{a}_{22} - \mathbf{a}_{11} \mathbf{a}_{22}} \right\}$$

$$\text{In[*]:= } \text{Simplify} [\{ \mathbf{0}, \omega^{-1} \} + \omega^{-1} \mathbf{a}_{21} \left(\mathbf{1} - \left(\mathbf{a}_{11} + \mathbf{a}_{12} \omega^{-1} \mathbf{a}_{21} \right) \right)^{-1} \mathbf{a}_{12} \omega^{-1}]$$

$$\text{Out[*]:= } \left\{ \frac{\mathbf{a}_{12} \mathbf{a}_{21}}{\omega^2 - \omega^2 \mathbf{a}_{11} - \omega \mathbf{a}_{12} \mathbf{a}_{21}}, \frac{-\mathbf{1} + \mathbf{a}_{11}}{-\omega + \omega \mathbf{a}_{11} + \mathbf{a}_{12} \mathbf{a}_{21}} \right\}$$

$$\text{In[*]:= } \mathbf{G3} \equiv \mathbf{G4}$$

Out[*]= True

$$\text{In[*]:= } \text{Simplify} \left[(\mathbf{1} - \mathbf{a}_{22})^{-1} \left(\mathbf{1} - \left(\mathbf{a}_{11} + \mathbf{a}_{12} (\mathbf{1} - \mathbf{a}_{22})^{-1} \mathbf{a}_{21} \right) \right)^{-1} == (\mathbf{1} - \mathbf{a}_{11})^{-1} \left(\mathbf{1} - \left(\mathbf{a}_{22} + \mathbf{a}_{21} (\mathbf{1} - \mathbf{a}_{11})^{-1} \mathbf{a}_{12} \right) \right)^{-1} \right]$$

Out[*]= True

$$\text{In[*]:= } \text{Together} \left[\left(\mathbf{1} - \left(\mathbf{a}_{11} + \mathbf{a}_{12} (\mathbf{1} - \mathbf{a}_{22})^{-1} \mathbf{a}_{21} \right) \right)^{-1} \right]$$

$$\text{Out[*]:= } \frac{\mathbf{1} - \mathbf{a}_{22}}{\mathbf{1} - \mathbf{a}_{11} - \mathbf{a}_{12} \mathbf{a}_{21} - \mathbf{a}_{22} + \mathbf{a}_{11} \mathbf{a}_{22}}$$