Pensieve header: Experiments with connectedness and exponentiation.

```
In[•]:= Unprotect[SeriesData];
        SeriesData /: Expand[sd_SeriesData] := MapAt[Expand, sd, 3];
        Protect[SeriesData];

In[•]:= $k = 3;

In[•]:= Derivative[m_, n_][G_k_] /; Max[m, n] > k + 1 := 0 &

In[•]:= {D[G_2[ζ, z], z, z, ζ, z], D[G_2[ζ, z], z, z, ζ, z, z]}
Out[•]= {G_2^(1,3)[ζ, z], 0}

In[•]:= NilZip[F_] := Module[{s = F, t = F, n = 0},
          While[t =!= 0, s += (t = Expand[∂_ζ ∂_z t / (++n))]];
          Expand@s ]

In[•]:= NilZip[G_4[ζ, z]]
Out[•]= G_4[ζ, z] + G_4^(1,1)[ζ, z] + 1/2 G_4^(2,2)[ζ, z] + 1/6 G_4^(3,3)[ζ, z] + 1/24 G_4^(4,4)[ζ, z] + 1/120 G_4^(5,5)[ζ, z]

In[•]:= F = Sum[ϵ^k G_k[ζ, z], {k, $k}] + O[ϵ]^($k+1)
Out[•]= G_1[ζ, z] ϵ + G_2[ζ, z] ϵ^2 + G_3[ζ, z] ϵ^3 + O[ϵ]^4
```

In[•]:=
```
Expand@Log[NilZip[ⅇ^F]]
```

Out[•]=

$$\left(G_1[\zeta, z] + G_1^{(1,1)}[\zeta, z] + \frac{1}{2} G_1^{(2,2)}[\zeta, z]\right) \epsilon +$$
$$\left(G_2[\zeta, z] + G_1^{(0,1)}[\zeta, z]\, G_1^{(1,0)}[\zeta, z] + \frac{1}{2} G_1^{(1,1)}[\zeta, z]^2 +\right.$$
$$G_2^{(1,1)}[\zeta, z] + G_1^{(1,0)}[\zeta, z]\, G_1^{(1,2)}[\zeta, z] + \frac{1}{2} G_1^{(0,2)}[\zeta, z]\, G_1^{(2,0)}[\zeta, z] +$$
$$G_1^{(0,1)}[\zeta, z]\, G_1^{(2,1)}[\zeta, z] + \frac{3}{2} G_1^{(1,2)}[\zeta, z]\, G_1^{(2,1)}[\zeta, z] +$$
$$\left.G_1^{(1,1)}[\zeta, z]\, G_1^{(2,2)}[\zeta, z] + \frac{5}{8} G_1^{(2,2)}[\zeta, z]^2 + \frac{1}{2} G_2^{(2,2)}[\zeta, z] + \frac{1}{6} G_2^{(3,3)}[\zeta, z]\right) \epsilon^2 +$$
$$\left(G_3[\zeta, z] + G_2^{(0,1)}[\zeta, z]\, G_1^{(1,0)}[\zeta, z] + \frac{1}{2} G_1^{(0,2)}[\zeta, z]\, G_1^{(1,0)}[\zeta, z]^2 + G_1^{(0,1)}[\zeta, z]\, G_2^{(1,0)}[\zeta, z] +\right.$$
$$G_1^{(0,1)}[\zeta, z]\, G_1^{(1,0)}[\zeta, z]\, G_1^{(1,1)}[\zeta, z] + \frac{1}{3} G_1^{(1,1)}[\zeta, z]^3 + G_1^{(1,1)}[\zeta, z]\, G_2^{(1,1)}[\zeta, z] +$$
$$G_3^{(1,1)}[\zeta, z] + G_2^{(1,0)}[\zeta, z]\, G_1^{(1,2)}[\zeta, z] + 2\, G_1^{(1,0)}[\zeta, z]\, G_1^{(1,1)}[\zeta, z]\, G_1^{(1,2)}[\zeta, z] +$$
$$G_1^{(1,0)}[\zeta, z]\, G_2^{(1,2)}[\zeta, z] + \frac{1}{2} G_1^{(0,1)}[\zeta, z]^2\, G_1^{(2,0)}[\zeta, z] +$$
$$\frac{1}{2} G_2^{(0,2)}[\zeta, z]\, G_1^{(2,0)}[\zeta, z] + G_1^{(0,2)}[\zeta, z]\, G_1^{(1,1)}[\zeta, z]\, G_1^{(2,0)}[\zeta, z] +$$
$$\frac{3}{2} G_1^{(0,1)}[\zeta, z]\, G_1^{(1,2)}[\zeta, z]\, G_1^{(2,0)}[\zeta, z] + \frac{3}{2} G_1^{(1,2)}[\zeta, z]^2\, G_1^{(2,0)}[\zeta, z] +$$
$$\frac{1}{2} G_2^{(1,3)}[\zeta, z]\, G_1^{(2,0)}[\zeta, z] + \frac{1}{2} G_1^{(0,2)}[\zeta, z]\, G_2^{(2,0)}[\zeta, z] + G_2^{(0,1)}[\zeta, z]\, G_1^{(2,1)}[\zeta, z] +$$
$$\frac{3}{2} G_1^{(0,2)}[\zeta, z]\, G_1^{(1,0)}[\zeta, z]\, G_1^{(2,1)}[\zeta, z] + 2\, G_1^{(0,1)}[\zeta, z]\, G_1^{(1,1)}[\zeta, z]\, G_1^{(2,1)}[\zeta, z] +$$
$$\frac{9}{2} G_1^{(1,1)}[\zeta, z]\, G_1^{(1,2)}[\zeta, z]\, G_1^{(2,1)}[\zeta, z] + \frac{3}{2} G_2^{(1,2)}[\zeta, z]\, G_1^{(2,1)}[\zeta, z] +$$
$$\frac{3}{2} G_1^{(0,2)}[\zeta, z]\, G_1^{(2,1)}[\zeta, z]^2 + G_1^{(0,1)}[\zeta, z]\, G_2^{(2,1)}[\zeta, z] + \frac{3}{2} G_1^{(1,2)}[\zeta, z]\, G_2^{(2,1)}[\zeta, z] +$$
$$G_1^{(0,1)}[\zeta, z]\, G_1^{(1,0)}[\zeta, z]\, G_1^{(2,2)}[\zeta, z] + \frac{3}{2} G_1^{(1,1)}[\zeta, z]^2\, G_1^{(2,2)}[\zeta, z] +$$
$$G_2^{(1,1)}[\zeta, z]\, G_1^{(2,2)}[\zeta, z] + \frac{5}{2} G_1^{(1,0)}[\zeta, z]\, G_1^{(1,2)}[\zeta, z]\, G_1^{(2,2)}[\zeta, z] +$$
$$\frac{5}{4} G_1^{(0,2)}[\zeta, z]\, G_1^{(2,0)}[\zeta, z]\, G_1^{(2,2)}[\zeta, z] + \frac{5}{2} G_1^{(0,1)}[\zeta, z]\, G_1^{(2,1)}[\zeta, z]\, G_1^{(2,2)}[\zeta, z] +$$
$$\frac{27}{4} G_1^{(1,2)}[\zeta, z]\, G_1^{(2,1)}[\zeta, z]\, G_1^{(2,2)}[\zeta, z] + \frac{5}{2} G_1^{(1,1)}[\zeta, z]\, G_1^{(2,2)}[\zeta, z]^2 +$$
$$\frac{37}{24} G_1^{(2,2)}[\zeta, z]^3 + G_1^{(1,1)}[\zeta, z]\, G_2^{(2,2)}[\zeta, z] + \frac{5}{4} G_1^{(2,2)}[\zeta, z]\, G_2^{(2,2)}[\zeta, z] + \frac{1}{2} G_3^{(2,2)}[\zeta, z] +$$
$$\frac{1}{2} G_1^{(1,0)}[\zeta, z]\, G_2^{(2,3)}[\zeta, z] + G_1^{(2,1)}[\zeta, z]\, G_2^{(2,3)}[\zeta, z] + \frac{1}{2} G_1^{(0,2)}[\zeta, z]\, G_2^{(3,1)}[\zeta, z] +$$
$$\frac{1}{2} G_1^{(0,1)}[\zeta, z]\, G_2^{(3,2)}[\zeta, z] + G_1^{(1,2)}[\zeta, z]\, G_2^{(3,2)}[\zeta, z] + \frac{1}{2} G_1^{(1,1)}[\zeta, z]\, G_2^{(3,3)}[\zeta, z] +$$
$$\left.\frac{3}{4} G_1^{(2,2)}[\zeta, z]\, G_2^{(3,3)}[\zeta, z] + \frac{1}{6} G_3^{(3,3)}[\zeta, z] + \frac{1}{24} G_3^{(4,4)}[\zeta, z]\right) \epsilon^3 + O[\epsilon]^4$$

```
In[•]:= NilBra_λ_[F_] := Module[{s = F, t = F, n = 0},
          While[t =!= 0, s += (t = Expand[λ ∂_ζ ∂_z t / (++n))]];
          Expand@s ]

In[•]:= GG = Expand@Log[NilBra_λ[ℯ^F]]
```

Out[•]=

$$\left(G_1[\zeta,z]+\lambda\,G_1^{(1,1)}[\zeta,z]+\frac{1}{2}\lambda^2\,G_1^{(2,2)}[\zeta,z]\right)\epsilon+$$
$$\left(G_2[\zeta,z]+\lambda\,G_1^{(0,1)}[\zeta,z]\,G_1^{(1,0)}[\zeta,z]+\frac{1}{2}\lambda^2\,G_1^{(1,1)}[\zeta,z]^2+\right.$$
$$\lambda\,G_2^{(1,1)}[\zeta,z]+\lambda^2\,G_1^{(1,0)}[\zeta,z]\,G_1^{(1,2)}[\zeta,z]+\frac{1}{2}\lambda^2\,G_1^{(0,2)}[\zeta,z]\,G_1^{(2,0)}[\zeta,z]+$$
$$\lambda^2\,G_1^{(0,1)}[\zeta,z]\,G_1^{(2,1)}[\zeta,z]+\frac{3}{2}\lambda^3\,G_1^{(1,2)}[\zeta,z]\,G_1^{(2,1)}[\zeta,z]+\lambda^3\,G_1^{(1,1)}[\zeta,z]\,G_1^{(2,2)}[\zeta,z]+$$
$$\left.\frac{5}{8}\lambda^4\,G_1^{(2,2)}[\zeta,z]^2+\frac{1}{2}\lambda^2\,G_2^{(2,2)}[\zeta,z]+\frac{1}{6}\lambda^3\,G_2^{(3,3)}[\zeta,z]\right)\epsilon^2+$$
$$\left(G_3[\zeta,z]+\lambda\,G_2^{(0,1)}[\zeta,z]\,G_1^{(1,0)}[\zeta,z]+\frac{1}{2}\lambda^2\,G_1^{(0,2)}[\zeta,z]\,G_1^{(1,0)}[\zeta,z]^2+\right.$$
$$\lambda\,G_1^{(0,1)}[\zeta,z]\,G_2^{(1,0)}[\zeta,z]+\lambda^2\,G_1^{(0,1)}[\zeta,z]\,G_1^{(1,0)}[\zeta,z]\,G_1^{(1,1)}[\zeta,z]+$$
$$\frac{1}{3}\lambda^3\,G_1^{(1,1)}[\zeta,z]^3+\lambda^2\,G_1^{(1,1)}[\zeta,z]\,G_2^{(1,1)}[\zeta,z]+\lambda\,G_3^{(1,1)}[\zeta,z]+$$
$$\lambda^2\,G_2^{(1,0)}[\zeta,z]\,G_1^{(1,2)}[\zeta,z]+2\lambda^3\,G_1^{(1,0)}[\zeta,z]\,G_1^{(1,1)}[\zeta,z]\,G_1^{(1,2)}[\zeta,z]+$$
$$\lambda^2\,G_1^{(1,0)}[\zeta,z]\,G_2^{(1,2)}[\zeta,z]+\frac{1}{2}\lambda^2\,G_1^{(0,1)}[\zeta,z]^2\,G_1^{(2,0)}[\zeta,z]+$$
$$\frac{1}{2}\lambda^2\,G_2^{(0,2)}[\zeta,z]\,G_1^{(2,0)}[\zeta,z]+\lambda^3\,G_1^{(0,2)}[\zeta,z]\,G_1^{(1,1)}[\zeta,z]\,G_1^{(2,0)}[\zeta,z]+$$
$$\frac{3}{2}\lambda^3\,G_1^{(0,1)}[\zeta,z]\,G_1^{(1,2)}[\zeta,z]\,G_1^{(2,0)}[\zeta,z]+\frac{3}{2}\lambda^4\,G_1^{(1,2)}[\zeta,z]^2\,G_1^{(2,0)}[\zeta,z]+$$
$$\frac{1}{2}\lambda^3\,G_2^{(1,3)}[\zeta,z]\,G_1^{(2,0)}[\zeta,z]+\frac{1}{2}\lambda^2\,G_1^{(0,2)}[\zeta,z]\,G_2^{(2,0)}[\zeta,z]+$$
$$\lambda^2\,G_2^{(0,1)}[\zeta,z]\,G_1^{(2,1)}[\zeta,z]+\frac{3}{2}\lambda^3\,G_1^{(0,2)}[\zeta,z]\,G_1^{(1,0)}[\zeta,z]\,G_1^{(2,1)}[\zeta,z]+$$
$$2\lambda^3\,G_1^{(0,1)}[\zeta,z]\,G_1^{(1,1)}[\zeta,z]\,G_1^{(2,1)}[\zeta,z]+\frac{9}{2}\lambda^4\,G_1^{(1,1)}[\zeta,z]\,G_1^{(1,2)}[\zeta,z]\,G_1^{(2,1)}[\zeta,z]+$$
$$\frac{3}{2}\lambda^3\,G_2^{(1,2)}[\zeta,z]\,G_1^{(2,1)}[\zeta,z]+\frac{3}{2}\lambda^4\,G_1^{(0,2)}[\zeta,z]\,G_1^{(2,1)}[\zeta,z]^2+$$
$$\lambda^2\,G_1^{(0,1)}[\zeta,z]\,G_2^{(2,1)}[\zeta,z]+\frac{3}{2}\lambda^3\,G_1^{(1,2)}[\zeta,z]\,G_2^{(2,1)}[\zeta,z]+$$
$$\lambda^3\,G_1^{(0,1)}[\zeta,z]\,G_1^{(1,0)}[\zeta,z]\,G_1^{(2,2)}[\zeta,z]+\frac{3}{2}\lambda^4\,G_1^{(1,1)}[\zeta,z]^2\,G_1^{(2,2)}[\zeta,z]+$$
$$\lambda^3\,G_2^{(1,1)}[\zeta,z]\,G_1^{(2,2)}[\zeta,z]+\frac{5}{2}\lambda^4\,G_1^{(1,0)}[\zeta,z]\,G_1^{(1,2)}[\zeta,z]\,G_1^{(2,2)}[\zeta,z]+$$
$$\frac{5}{4}\lambda^4\,G_1^{(0,2)}[\zeta,z]\,G_1^{(2,0)}[\zeta,z]\,G_1^{(2,2)}[\zeta,z]+\frac{5}{2}\lambda^4\,G_1^{(0,1)}[\zeta,z]\,G_1^{(2,1)}[\zeta,z]\,G_1^{(2,2)}[\zeta,z]+$$
$$\frac{27}{4}\lambda^5\,G_1^{(1,2)}[\zeta,z]\,G_1^{(2,1)}[\zeta,z]\,G_1^{(2,2)}[\zeta,z]+\frac{5}{2}\lambda^5\,G_1^{(1,1)}[\zeta,z]\,G_1^{(2,2)}[\zeta,z]^2+$$
$$\frac{37}{24}\lambda^6\,G_1^{(2,2)}[\zeta,z]^3+\lambda^3\,G_1^{(1,1)}[\zeta,z]\,G_2^{(2,2)}[\zeta,z]+\frac{5}{4}\lambda^4\,G_1^{(2,2)}[\zeta,z]\,G_2^{(2,2)}[\zeta,z]+$$

$$\frac{1}{2}\lambda^2 G_3^{(2,2)}[\zeta, z] + \frac{1}{2}\lambda^3 G_1^{(1,0)}[\zeta, z] G_2^{(2,3)}[\zeta, z] + \lambda^4 G_1^{(2,1)}[\zeta, z] G_2^{(2,3)}[\zeta, z] +$$
$$\frac{1}{2}\lambda^3 G_1^{(0,2)}[\zeta, z] G_2^{(3,1)}[\zeta, z] + \frac{1}{2}\lambda^3 G_1^{(0,1)}[\zeta, z] G_2^{(3,2)}[\zeta, z] +$$
$$\lambda^4 G_1^{(1,2)}[\zeta, z] G_2^{(3,2)}[\zeta, z] + \frac{1}{2}\lambda^4 G_1^{(1,1)}[\zeta, z] G_2^{(3,3)}[\zeta, z] +$$
$$\frac{3}{4}\lambda^5 G_1^{(2,2)}[\zeta, z] G_2^{(3,3)}[\zeta, z] + \frac{1}{6}\lambda^3 G_3^{(3,3)}[\zeta, z] + \frac{1}{24}\lambda^4 G_3^{(4,4)}[\zeta, z]\Big) \epsilon^3 + O[\epsilon]^4$$

In[•]:= `(GG /. λ → 0) == F`

Out[•]= True

In[•]:= $\partial_\lambda GG$

Out[•]=
$$\left(G_1^{(1,1)}[\zeta, z] + \lambda\, G_1^{(2,2)}[\zeta, z]\right)\epsilon +$$
$$\left(G_1^{(0,1)}[\zeta, z]\, G_1^{(1,0)}[\zeta, z] + \lambda\, G_1^{(1,1)}[\zeta, z]^2 + G_2^{(1,1)}[\zeta, z] + 2\lambda\, G_1^{(1,0)}[\zeta, z]\, G_1^{(1,2)}[\zeta, z] + \right.$$
$$\lambda\, G_1^{(0,2)}[\zeta, z]\, G_1^{(2,0)}[\zeta, z] + 2\lambda\, G_1^{(0,1)}[\zeta, z]\, G_1^{(2,1)}[\zeta, z] + \frac{9}{2}\lambda^2\, G_1^{(1,2)}[\zeta, z]\, G_1^{(2,1)}[\zeta, z] +$$
$$\left. 3\lambda^2\, G_1^{(1,1)}[\zeta, z]\, G_1^{(2,2)}[\zeta, z] + \frac{5}{2}\lambda^3\, G_1^{(2,2)}[\zeta, z]^2 + \lambda\, G_2^{(2,2)}[\zeta, z] + \frac{1}{2}\lambda^2\, G_2^{(3,3)}[\zeta, z]\right)\epsilon^2 +$$
$$\left(G_2^{(0,1)}[\zeta, z]\, G_1^{(1,0)}[\zeta, z] + \lambda\, G_1^{(0,2)}[\zeta, z]\, G_1^{(1,0)}[\zeta, z]^2 + G_1^{(0,1)}[\zeta, z]\, G_2^{(1,0)}[\zeta, z] + \right.$$
$$2\lambda\, G_1^{(0,1)}[\zeta, z]\, G_1^{(1,0)}[\zeta, z]\, G_1^{(1,1)}[\zeta, z] + \lambda^2\, G_1^{(1,1)}[\zeta, z]^3 + 2\lambda\, G_1^{(1,1)}[\zeta, z]\, G_2^{(1,1)}[\zeta, z] +$$
$$G_3^{(1,1)}[\zeta, z] + 2\lambda\, G_2^{(1,0)}[\zeta, z]\, G_1^{(1,2)}[\zeta, z] + 6\lambda^2\, G_1^{(1,0)}[\zeta, z]\, G_1^{(1,1)}[\zeta, z]\, G_1^{(1,2)}[\zeta, z] +$$
$$2\lambda\, G_1^{(1,0)}[\zeta, z]\, G_2^{(1,2)}[\zeta, z] + \lambda\, G_1^{(0,1)}[\zeta, z]^2\, G_1^{(2,0)}[\zeta, z] + \lambda\, G_2^{(0,2)}[\zeta, z]\, G_1^{(2,0)}[\zeta, z] +$$
$$3\lambda^2\, G_1^{(0,2)}[\zeta, z]\, G_1^{(1,1)}[\zeta, z]\, G_1^{(2,0)}[\zeta, z] + \frac{9}{2}\lambda^2\, G_1^{(0,1)}[\zeta, z]\, G_1^{(1,2)}[\zeta, z]\, G_1^{(2,0)}[\zeta, z] +$$
$$6\lambda^3\, G_1^{(1,2)}[\zeta, z]^2\, G_1^{(2,0)}[\zeta, z] + \frac{3}{2}\lambda^2\, G_2^{(1,3)}[\zeta, z]\, G_1^{(2,0)}[\zeta, z] + \lambda\, G_1^{(0,2)}[\zeta, z]\, G_2^{(2,0)}[\zeta, z] +$$
$$2\lambda\, G_2^{(0,1)}[\zeta, z]\, G_1^{(2,1)}[\zeta, z] + \frac{9}{2}\lambda^2\, G_1^{(0,2)}[\zeta, z]\, G_1^{(1,0)}[\zeta, z]\, G_1^{(2,1)}[\zeta, z] +$$
$$6\lambda^2\, G_1^{(0,1)}[\zeta, z]\, G_1^{(1,1)}[\zeta, z]\, G_1^{(2,1)}[\zeta, z] + 18\lambda^3\, G_1^{(1,1)}[\zeta, z]\, G_1^{(1,2)}[\zeta, z]\, G_1^{(2,1)}[\zeta, z] +$$
$$\frac{9}{2}\lambda^2\, G_2^{(1,2)}[\zeta, z]\, G_1^{(2,1)}[\zeta, z] + 6\lambda^3\, G_1^{(0,2)}[\zeta, z]\, G_1^{(2,1)}[\zeta, z]^2 +$$
$$2\lambda\, G_1^{(0,1)}[\zeta, z]\, G_2^{(2,1)}[\zeta, z] + \frac{9}{2}\lambda^2\, G_1^{(1,2)}[\zeta, z]\, G_2^{(2,1)}[\zeta, z] +$$
$$3\lambda^2\, G_1^{(0,1)}[\zeta, z]\, G_1^{(1,0)}[\zeta, z]\, G_1^{(2,2)}[\zeta, z] + 6\lambda^3\, G_1^{(1,1)}[\zeta, z]^2\, G_1^{(2,2)}[\zeta, z] +$$
$$3\lambda^2\, G_2^{(1,1)}[\zeta, z]\, G_1^{(2,2)}[\zeta, z] + 10\lambda^3\, G_1^{(1,0)}[\zeta, z]\, G_1^{(1,2)}[\zeta, z]\, G_1^{(2,2)}[\zeta, z] +$$
$$5\lambda^3\, G_1^{(0,2)}[\zeta, z]\, G_1^{(2,0)}[\zeta, z]\, G_1^{(2,2)}[\zeta, z] + 10\lambda^3\, G_1^{(0,1)}[\zeta, z]\, G_1^{(2,1)}[\zeta, z]\, G_1^{(2,2)}[\zeta, z] +$$
$$\frac{135}{4}\lambda^4\, G_1^{(1,2)}[\zeta, z]\, G_1^{(2,1)}[\zeta, z]\, G_1^{(2,2)}[\zeta, z] + \frac{25}{2}\lambda^4\, G_1^{(1,1)}[\zeta, z]\, G_1^{(2,2)}[\zeta, z]^2 +$$
$$\frac{37}{4}\lambda^5\, G_1^{(2,2)}[\zeta, z]^3 + 3\lambda^2\, G_1^{(1,1)}[\zeta, z]\, G_2^{(2,2)}[\zeta, z] + 5\lambda^3\, G_1^{(2,2)}[\zeta, z]\, G_2^{(2,2)}[\zeta, z] +$$
$$\lambda\, G_3^{(2,2)}[\zeta, z] + \frac{3}{2}\lambda^2\, G_1^{(1,0)}[\zeta, z]\, G_2^{(2,3)}[\zeta, z] + 4\lambda^3\, G_1^{(2,1)}[\zeta, z]\, G_2^{(2,3)}[\zeta, z] +$$
$$\frac{3}{2}\lambda^2\, G_1^{(0,2)}[\zeta, z]\, G_2^{(3,1)}[\zeta, z] + \frac{3}{2}\lambda^2\, G_1^{(0,1)}[\zeta, z]\, G_2^{(3,2)}[\zeta, z] +$$
$$4\lambda^3\, G_1^{(1,2)}[\zeta, z]\, G_2^{(3,2)}[\zeta, z] + 2\lambda^3\, G_1^{(1,1)}[\zeta, z]\, G_2^{(3,3)}[\zeta, z] +$$
$$\left.\frac{15}{4}\lambda^4\, G_1^{(2,2)}[\zeta, z]\, G_2^{(3,3)}[\zeta, z] + \frac{1}{2}\lambda^2\, G_3^{(3,3)}[\zeta, z] + \frac{1}{6}\lambda^3\, G_3^{(4,4)}[\zeta, z]\right)\epsilon^3 + O[\epsilon]^4$$

In[•]:= `Simplify[(∂λGG) == (∂z∂ζGG) + (∂ζGG) (∂zGG)]`

Out[•]= True

In[•]:=
```
H0 = F;
Hk_ := Hk = (Hk-1;
  Expand[Hk-1 + Integrate[(∂z∂ζHk-1) + (∂ζHk-1) (∂zHk-1) - (∂λHk-1), {λ, 0, λ}]])
```

In[•]:= **H1**

Out[•]=
$$\left(G_1[\zeta, z] + \lambda\, G_1^{(1,1)}[\zeta, z]\right)\epsilon + \left(G_2[\zeta, z] + \lambda\, G_1^{(0,1)}[\zeta, z]\, G_1^{(1,0)}[\zeta, z] + \lambda\, G_2^{(1,1)}[\zeta, z]\right)\epsilon^2 + \left(G_3[\zeta, z] + \lambda\, G_2^{(0,1)}[\zeta, z]\, G_1^{(1,0)}[\zeta, z] + \lambda\, G_1^{(0,1)}[\zeta, z]\, G_2^{(1,0)}[\zeta, z] + \lambda\, G_3^{(1,1)}[\zeta, z]\right)\epsilon^3 + O[\epsilon]^4$$

In[•]:= **H2**

Out[•]=
$$\left(G_1[\zeta, z] + \lambda\, G_1^{(1,1)}[\zeta, z] + \frac{1}{2}\lambda^2\, G_1^{(2,2)}[\zeta, z]\right)\epsilon +$$
$$\left(G_2[\zeta, z] + \lambda\, G_1^{(0,1)}[\zeta, z]\, G_1^{(1,0)}[\zeta, z] + \frac{1}{2}\lambda^2\, G_1^{(1,1)}[\zeta, z]^2 + \lambda\, G_2^{(1,1)}[\zeta, z] + \lambda^2\, G_1^{(1,0)}[\zeta, z]\, G_1^{(1,2)}[\zeta, z] + \frac{1}{2}\lambda^2\, G_1^{(0,2)}[\zeta, z]\, G_1^{(2,0)}[\zeta, z] + \lambda^2\, G_1^{(0,1)}[\zeta, z]\, G_1^{(2,1)}[\zeta, z] + \frac{1}{3}\lambda^3\, G_1^{(1,2)}[\zeta, z]\, G_1^{(2,1)}[\zeta, z] + \frac{1}{2}\lambda^2\, G_2^{(2,2)}[\zeta, z]\right)\epsilon^2 +$$
$$\left(G_3[\zeta, z] + \lambda\, G_2^{(0,1)}[\zeta, z]\, G_1^{(1,0)}[\zeta, z] + \frac{1}{2}\lambda^2\, G_1^{(0,2)}[\zeta, z]\, G_1^{(1,0)}[\zeta, z]^2 + \lambda\, G_1^{(0,1)}[\zeta, z]\, G_2^{(1,0)}[\zeta, z] + \lambda^2\, G_1^{(0,1)}[\zeta, z]\, G_1^{(1,0)}[\zeta, z]\, G_1^{(1,1)}[\zeta, z] + \lambda^2\, G_1^{(1,1)}[\zeta, z]\, G_2^{(1,1)}[\zeta, z] + \lambda\, G_3^{(1,1)}[\zeta, z] + \lambda^2\, G_2^{(1,0)}[\zeta, z]\, G_1^{(1,2)}[\zeta, z] + \frac{1}{3}\lambda^3\, G_1^{(1,0)}[\zeta, z]\, G_1^{(1,1)}[\zeta, z]\, G_1^{(1,2)}[\zeta, z] + \lambda^2\, G_1^{(1,0)}[\zeta, z]\, G_2^{(1,2)}[\zeta, z] + \frac{1}{2}\lambda^2\, G_1^{(0,1)}[\zeta, z]^2\, G_1^{(2,0)}[\zeta, z] + \frac{1}{2}\lambda^2\, G_2^{(0,2)}[\zeta, z]\, G_1^{(2,0)}[\zeta, z] + \frac{1}{3}\lambda^3\, G_1^{(0,1)}[\zeta, z]\, G_1^{(1,2)}[\zeta, z]\, G_1^{(2,0)}[\zeta, z] + \frac{1}{2}\lambda^2\, G_1^{(0,2)}[\zeta, z]\, G_2^{(2,0)}[\zeta, z] + \lambda^2\, G_2^{(0,1)}[\zeta, z]\, G_1^{(2,1)}[\zeta, z] + \frac{1}{3}\lambda^3\, G_1^{(0,2)}[\zeta, z]\, G_1^{(1,0)}[\zeta, z]\, G_1^{(2,1)}[\zeta, z] + \frac{1}{3}\lambda^3\, G_1^{(0,1)}[\zeta, z]\, G_1^{(1,1)}[\zeta, z]\, G_1^{(2,1)}[\zeta, z] + \frac{1}{3}\lambda^3\, G_2^{(1,2)}[\zeta, z]\, G_1^{(2,1)}[\zeta, z] + \lambda^2\, G_1^{(0,1)}[\zeta, z]\, G_2^{(2,1)}[\zeta, z] + \frac{1}{3}\lambda^3\, G_1^{(1,2)}[\zeta, z]\, G_2^{(2,1)}[\zeta, z] + \frac{1}{2}\lambda^2\, G_3^{(2,2)}[\zeta, z]\right)\epsilon^3 + O[\epsilon]^4$$

In[•]:= **H3**

Out[•]=

$$\left(G_1[\zeta, z] + \lambda\, G_1^{(1,1)}[\zeta, z] + \frac{1}{2}\lambda^2\, G_1^{(2,2)}[\zeta, z]\right)\epsilon +$$
$$\left(G_2[\zeta, z] + \lambda\, G_1^{(0,1)}[\zeta, z]\, G_1^{(1,0)}[\zeta, z] + \frac{1}{2}\lambda^2\, G_1^{(1,1)}[\zeta, z]^2 + \right.$$
$$\lambda\, G_2^{(1,1)}[\zeta, z] + \lambda^2\, G_1^{(1,0)}[\zeta, z]\, G_1^{(1,2)}[\zeta, z] + \frac{1}{2}\lambda^2\, G_1^{(0,2)}[\zeta, z]\, G_1^{(2,0)}[\zeta, z] +$$
$$\lambda^2\, G_1^{(0,1)}[\zeta, z]\, G_1^{(2,1)}[\zeta, z] + \frac{3}{2}\lambda^3\, G_1^{(1,2)}[\zeta, z]\, G_1^{(2,1)}[\zeta, z] + \lambda^3\, G_1^{(1,1)}[\zeta, z]\, G_1^{(2,2)}[\zeta, z] +$$
$$\left.\frac{1}{12}\lambda^4\, G_1^{(2,2)}[\zeta, z]^2 + \frac{1}{2}\lambda^2\, G_2^{(2,2)}[\zeta, z] + \frac{1}{6}\lambda^3\, G_2^{(3,3)}[\zeta, z]\right)\epsilon^2 +$$
$$\left(G_3[\zeta, z] + \lambda\, G_2^{(0,1)}[\zeta, z]\, G_1^{(1,0)}[\zeta, z] + \frac{1}{2}\lambda^2\, G_1^{(0,2)}[\zeta, z]\, G_1^{(1,0)}[\zeta, z]^2 + \right.$$
$$\lambda\, G_1^{(0,1)}[\zeta, z]\, G_2^{(1,0)}[\zeta, z] + \lambda^2\, G_1^{(0,1)}[\zeta, z]\, G_1^{(1,0)}[\zeta, z]\, G_1^{(1,1)}[\zeta, z] + \frac{1}{3}\lambda^3\, G_1^{(1,1)}[\zeta, z]^3 +$$
$$\lambda^2\, G_1^{(1,1)}[\zeta, z]\, G_2^{(1,1)}[\zeta, z] + \lambda\, G_3^{(1,1)}[\zeta, z] + \lambda^2\, G_2^{(1,0)}[\zeta, z]\, G_1^{(1,2)}[\zeta, z] +$$
$$2\lambda^3\, G_1^{(1,0)}[\zeta, z]\, G_1^{(1,1)}[\zeta, z]\, G_1^{(1,2)}[\zeta, z] + \lambda^2\, G_1^{(1,0)}[\zeta, z]\, G_2^{(1,2)}[\zeta, z] +$$
$$\frac{1}{2}\lambda^2\, G_1^{(0,1)}[\zeta, z]^2\, G_1^{(2,0)}[\zeta, z] + \frac{1}{2}\lambda^2\, G_2^{(0,2)}[\zeta, z]\, G_1^{(2,0)}[\zeta, z] +$$
$$\lambda^3\, G_1^{(0,2)}[\zeta, z]\, G_1^{(1,1)}[\zeta, z]\, G_1^{(2,0)}[\zeta, z] + \frac{3}{2}\lambda^3\, G_1^{(0,1)}[\zeta, z]\, G_1^{(1,2)}[\zeta, z]\, G_1^{(2,0)}[\zeta, z] +$$
$$\frac{13}{24}\lambda^4\, G_1^{(1,2)}[\zeta, z]^2\, G_1^{(2,0)}[\zeta, z] + \frac{1}{2}\lambda^3\, G_2^{(1,3)}[\zeta, z]\, G_1^{(2,0)}[\zeta, z] +$$
$$\frac{1}{2}\lambda^2\, G_1^{(0,2)}[\zeta, z]\, G_2^{(2,0)}[\zeta, z] + \lambda^2\, G_2^{(0,1)}[\zeta, z]\, G_1^{(2,1)}[\zeta, z] +$$
$$\frac{3}{2}\lambda^3\, G_1^{(0,2)}[\zeta, z]\, G_1^{(1,0)}[\zeta, z]\, G_1^{(2,1)}[\zeta, z] + 2\lambda^3\, G_1^{(0,1)}[\zeta, z]\, G_1^{(1,1)}[\zeta, z]\, G_1^{(2,1)}[\zeta, z] +$$
$$\frac{3}{2}\lambda^4\, G_1^{(1,1)}[\zeta, z]\, G_1^{(1,2)}[\zeta, z]\, G_1^{(2,1)}[\zeta, z] + \frac{3}{2}\lambda^3\, G_2^{(1,2)}[\zeta, z]\, G_1^{(2,1)}[\zeta, z] +$$
$$\frac{13}{24}\lambda^4\, G_1^{(0,2)}[\zeta, z]\, G_1^{(2,1)}[\zeta, z]^2 + \lambda^2\, G_1^{(0,1)}[\zeta, z]\, G_2^{(2,1)}[\zeta, z] +$$
$$\frac{3}{2}\lambda^3\, G_1^{(1,2)}[\zeta, z]\, G_2^{(2,1)}[\zeta, z] + \lambda^3\, G_1^{(0,1)}[\zeta, z]\, G_1^{(1,0)}[\zeta, z]\, G_1^{(2,2)}[\zeta, z] +$$
$$\frac{1}{6}\lambda^4\, G_1^{(1,1)}[\zeta, z]^2\, G_1^{(2,2)}[\zeta, z] + \lambda^3\, G_2^{(1,1)}[\zeta, z]\, G_1^{(2,2)}[\zeta, z] +$$
$$\frac{7}{12}\lambda^4\, G_1^{(1,0)}[\zeta, z]\, G_1^{(1,2)}[\zeta, z]\, G_1^{(2,2)}[\zeta, z] + \frac{1}{6}\lambda^4\, G_1^{(0,2)}[\zeta, z]\, G_1^{(2,0)}[\zeta, z]\, G_1^{(2,2)}[\zeta, z] +$$
$$\frac{7}{12}\lambda^4\, G_1^{(0,1)}[\zeta, z]\, G_1^{(2,1)}[\zeta, z]\, G_1^{(2,2)}[\zeta, z] + \frac{2}{15}\lambda^5\, G_1^{(1,2)}[\zeta, z]\, G_1^{(2,1)}[\zeta, z]\, G_1^{(2,2)}[\zeta, z] +$$
$$\lambda^3\, G_1^{(1,1)}[\zeta, z]\, G_2^{(2,2)}[\zeta, z] + \frac{1}{6}\lambda^4\, G_1^{(2,2)}[\zeta, z]\, G_2^{(2,2)}[\zeta, z] + \frac{1}{2}\lambda^2\, G_3^{(2,2)}[\zeta, z] +$$
$$\frac{1}{2}\lambda^3\, G_1^{(1,0)}[\zeta, z]\, G_2^{(2,3)}[\zeta, z] + \frac{5}{24}\lambda^4\, G_1^{(2,1)}[\zeta, z]\, G_2^{(2,3)}[\zeta, z] +$$
$$\frac{1}{2}\lambda^3\, G_1^{(0,2)}[\zeta, z]\, G_2^{(3,1)}[\zeta, z] + \frac{1}{2}\lambda^3\, G_1^{(0,1)}[\zeta, z]\, G_2^{(3,2)}[\zeta, z] +$$
$$\left.\frac{5}{24}\lambda^4\, G_1^{(1,2)}[\zeta, z]\, G_2^{(3,2)}[\zeta, z] + \frac{1}{6}\lambda^3\, G_3^{(3,3)}[\zeta, z]\right)\epsilon^3 + O[\epsilon]^4$$

In[•]:= **H_4**

Out[•]=

$$\left(G_1[\zeta, z] + \lambda\, G_1^{(1,1)}[\zeta, z] + \frac{1}{2}\lambda^2\, G_1^{(2,2)}[\zeta, z]\right)\epsilon +$$
$$\left(G_2[\zeta, z] + \lambda\, G_1^{(0,1)}[\zeta, z]\, G_1^{(1,0)}[\zeta, z] + \frac{1}{2}\lambda^2\, G_1^{(1,1)}[\zeta, z]^2 +\right.$$
$$\lambda\, G_2^{(1,1)}[\zeta, z] + \lambda^2\, G_1^{(1,0)}[\zeta, z]\, G_1^{(1,2)}[\zeta, z] + \frac{1}{2}\lambda^2\, G_1^{(0,2)}[\zeta, z]\, G_1^{(2,0)}[\zeta, z] +$$
$$\lambda^2\, G_1^{(0,1)}[\zeta, z]\, G_1^{(2,1)}[\zeta, z] + \frac{3}{2}\lambda^3\, G_1^{(1,2)}[\zeta, z]\, G_1^{(2,1)}[\zeta, z] + \lambda^3\, G_1^{(1,1)}[\zeta, z]\, G_1^{(2,2)}[\zeta, z] +$$
$$\left.\frac{5}{8}\lambda^4\, G_1^{(2,2)}[\zeta, z]^2 + \frac{1}{2}\lambda^2\, G_2^{(2,2)}[\zeta, z] + \frac{1}{6}\lambda^3\, G_2^{(3,3)}[\zeta, z]\right)\epsilon^2 +$$
$$\left(G_3[\zeta, z] + \lambda\, G_2^{(0,1)}[\zeta, z]\, G_1^{(1,0)}[\zeta, z] + \frac{1}{2}\lambda^2\, G_1^{(0,2)}[\zeta, z]\, G_1^{(1,0)}[\zeta, z]^2 +\right.$$
$$\lambda\, G_1^{(0,1)}[\zeta, z]\, G_2^{(1,0)}[\zeta, z] + \lambda^2\, G_1^{(0,1)}[\zeta, z]\, G_1^{(1,0)}[\zeta, z]\, G_1^{(1,1)}[\zeta, z] +$$
$$\frac{1}{3}\lambda^3\, G_1^{(1,1)}[\zeta, z]^3 + \lambda^2\, G_1^{(1,1)}[\zeta, z]\, G_2^{(1,1)}[\zeta, z] + \lambda\, G_3^{(1,1)}[\zeta, z] +$$
$$\lambda^2\, G_2^{(1,0)}[\zeta, z]\, G_1^{(1,2)}[\zeta, z] + 2\lambda^3\, G_1^{(1,0)}[\zeta, z]\, G_1^{(1,1)}[\zeta, z]\, G_1^{(1,2)}[\zeta, z] +$$
$$\lambda^2\, G_1^{(1,0)}[\zeta, z]\, G_2^{(1,2)}[\zeta, z] + \frac{1}{2}\lambda^2\, G_1^{(0,1)}[\zeta, z]^2\, G_1^{(2,0)}[\zeta, z] +$$
$$\frac{1}{2}\lambda^2\, G_2^{(0,2)}[\zeta, z]\, G_1^{(2,0)}[\zeta, z] + \lambda^3\, G_1^{(0,2)}[\zeta, z]\, G_1^{(1,1)}[\zeta, z]\, G_1^{(2,0)}[\zeta, z] +$$
$$\frac{3}{2}\lambda^3\, G_1^{(0,1)}[\zeta, z]\, G_1^{(1,2)}[\zeta, z]\, G_1^{(2,0)}[\zeta, z] + \frac{3}{2}\lambda^4\, G_1^{(1,2)}[\zeta, z]^2\, G_1^{(2,0)}[\zeta, z] +$$
$$\frac{1}{2}\lambda^3\, G_2^{(1,3)}[\zeta, z]\, G_1^{(2,0)}[\zeta, z] + \frac{1}{2}\lambda^2\, G_1^{(0,2)}[\zeta, z]\, G_2^{(2,0)}[\zeta, z] +$$
$$\lambda^2\, G_2^{(0,1)}[\zeta, z]\, G_1^{(2,1)}[\zeta, z] + \frac{3}{2}\lambda^3\, G_1^{(0,2)}[\zeta, z]\, G_1^{(1,0)}[\zeta, z]\, G_1^{(2,1)}[\zeta, z] +$$
$$2\lambda^3\, G_1^{(0,1)}[\zeta, z]\, G_1^{(1,1)}[\zeta, z]\, G_1^{(2,1)}[\zeta, z] + \frac{9}{2}\lambda^4\, G_1^{(1,1)}[\zeta, z]\, G_1^{(1,2)}[\zeta, z]\, G_1^{(2,1)}[\zeta, z] +$$
$$\frac{3}{2}\lambda^3\, G_2^{(1,2)}[\zeta, z]\, G_1^{(2,1)}[\zeta, z] + \frac{3}{2}\lambda^4\, G_1^{(0,2)}[\zeta, z]\, G_1^{(2,1)}[\zeta, z]^2 +$$
$$\lambda^2\, G_1^{(0,1)}[\zeta, z]\, G_2^{(2,1)}[\zeta, z] + \frac{3}{2}\lambda^3\, G_1^{(1,2)}[\zeta, z]\, G_2^{(2,1)}[\zeta, z] +$$
$$\lambda^3\, G_1^{(0,1)}[\zeta, z]\, G_1^{(1,0)}[\zeta, z]\, G_1^{(2,2)}[\zeta, z] + \frac{3}{2}\lambda^4\, G_1^{(1,1)}[\zeta, z]^2\, G_1^{(2,2)}[\zeta, z] +$$
$$\lambda^3\, G_2^{(1,1)}[\zeta, z]\, G_1^{(2,2)}[\zeta, z] + \frac{5}{2}\lambda^4\, G_1^{(1,0)}[\zeta, z]\, G_1^{(1,2)}[\zeta, z]\, G_1^{(2,2)}[\zeta, z] +$$
$$\frac{5}{4}\lambda^4\, G_1^{(0,2)}[\zeta, z]\, G_1^{(2,0)}[\zeta, z]\, G_1^{(2,2)}[\zeta, z] + \frac{5}{2}\lambda^4\, G_1^{(0,1)}[\zeta, z]\, G_1^{(2,1)}[\zeta, z]\, G_1^{(2,2)}[\zeta, z] +$$
$$\frac{8}{3}\lambda^5\, G_1^{(1,2)}[\zeta, z]\, G_1^{(2,1)}[\zeta, z]\, G_1^{(2,2)}[\zeta, z] + \frac{3}{5}\lambda^5\, G_1^{(1,1)}[\zeta, z]\, G_1^{(2,2)}[\zeta, z]^2 +$$
$$\frac{1}{45}\lambda^6\, G_1^{(2,2)}[\zeta, z]^3 + \lambda^3\, G_1^{(1,1)}[\zeta, z]\, G_2^{(2,2)}[\zeta, z] + \frac{5}{4}\lambda^4\, G_1^{(2,2)}[\zeta, z]\, G_2^{(2,2)}[\zeta, z] +$$
$$\frac{1}{2}\lambda^2\, G_3^{(2,2)}[\zeta, z] + \frac{1}{2}\lambda^3\, G_1^{(1,0)}[\zeta, z]\, G_2^{(2,3)}[\zeta, z] + \lambda^4\, G_1^{(2,1)}[\zeta, z]\, G_2^{(2,3)}[\zeta, z] +$$
$$\frac{1}{2}\lambda^3\, G_1^{(0,2)}[\zeta, z]\, G_2^{(3,1)}[\zeta, z] + \frac{1}{2}\lambda^3\, G_1^{(0,1)}[\zeta, z]\, G_2^{(3,2)}[\zeta, z] +$$

$$\lambda^4\, G_1^{(1,2)}[\zeta, z]\, G_2^{(3,2)}[\zeta, z] + \frac{1}{2}\lambda^4\, G_1^{(1,1)}[\zeta, z]\, G_2^{(3,3)}[\zeta, z] + \frac{7}{60}\lambda^5\, G_1^{(2,2)}[\zeta, z]\, G_2^{(3,3)}[\zeta, z] + \frac{1}{6}\lambda^3\, G_3^{(3,3)}[\zeta, z] + \frac{1}{24}\lambda^4\, G_3^{(4,4)}[\zeta, z]\Big)\epsilon^3 + O[\epsilon]^4$$

In[•]:=
```
Simplify[H2$k == GG]
```

Out[•]= True

In[•]:=
```
T0 = F;
Tk_ := Tk = (Tk-1;
  Expand[Tk-1 + Integrate[(∂ζ Tk-1) (∂z Tk-1) - (∂λ Tk-1), {λ, 0, λ}]])
```

In[•]:=
```
T1
```

Out[•]= $G_1[\zeta, z]\,\epsilon + \left(G_2[\zeta, z] + \lambda\, G_1^{(0,1)}[\zeta, z]\, G_1^{(1,0)}[\zeta, z]\right)\epsilon^2 + \left(G_3[\zeta, z] + \lambda\, G_2^{(0,1)}[\zeta, z]\, G_1^{(1,0)}[\zeta, z] + \lambda\, G_1^{(0,1)}[\zeta, z]\, G_2^{(1,0)}[\zeta, z]\right)\epsilon^3 + O[\epsilon]^4$

In[•]:=
```
T2
```

Out[•]= $G_1[\zeta, z]\,\epsilon + \left(G_2[\zeta, z] + \lambda\, G_1^{(0,1)}[\zeta, z]\, G_1^{(1,0)}[\zeta, z]\right)\epsilon^2 + \Big(G_3[\zeta, z] + \lambda\, G_2^{(0,1)}[\zeta, z]\, G_1^{(1,0)}[\zeta, z] + \frac{1}{2}\lambda^2\, G_1^{(0,2)}[\zeta, z]\, G_1^{(1,0)}[\zeta, z]^2 + \lambda\, G_1^{(0,1)}[\zeta, z]\, G_2^{(1,0)}[\zeta, z] + \lambda^2\, G_1^{(0,1)}[\zeta, z]\, G_1^{(1,0)}[\zeta, z]\, G_1^{(1,1)}[\zeta, z] + \frac{1}{2}\lambda^2\, G_1^{(0,1)}[\zeta, z]^2\, G_1^{(2,0)}[\zeta, z]\Big)\epsilon^3 + O[\epsilon]^4$

In[•]:=
```
T2 - T1
```

Out[•]= $\left(\frac{1}{2}\lambda^2\, G_1^{(0,2)}[\zeta, z]\, G_1^{(1,0)}[\zeta, z]^2 + \lambda^2\, G_1^{(0,1)}[\zeta, z]\, G_1^{(1,0)}[\zeta, z]\, G_1^{(1,1)}[\zeta, z] + \frac{1}{2}\lambda^2\, G_1^{(0,1)}[\zeta, z]^2\, G_1^{(2,0)}[\zeta, z]\right)\epsilon^3 + O[\epsilon]^4$

In[•]:=
```
T3
```

Out[•]= $G_1[\zeta, z]\,\epsilon + \left(G_2[\zeta, z] + \lambda\, G_1^{(0,1)}[\zeta, z]\, G_1^{(1,0)}[\zeta, z]\right)\epsilon^2 + \Big(G_3[\zeta, z] + \lambda\, G_2^{(0,1)}[\zeta, z]\, G_1^{(1,0)}[\zeta, z] + \frac{1}{2}\lambda^2\, G_1^{(0,2)}[\zeta, z]\, G_1^{(1,0)}[\zeta, z]^2 + \lambda\, G_1^{(0,1)}[\zeta, z]\, G_2^{(1,0)}[\zeta, z] + \lambda^2\, G_1^{(0,1)}[\zeta, z]\, G_1^{(1,0)}[\zeta, z]\, G_1^{(1,1)}[\zeta, z] + \frac{1}{2}\lambda^2\, G_1^{(0,1)}[\zeta, z]^2\, G_1^{(2,0)}[\zeta, z]\Big)\epsilon^3 + O[\epsilon]^4$

In[•]:=
```
T4
```

Out[•]= $G_1[\zeta, z]\,\epsilon + \left(G_2[\zeta, z] + \lambda\, G_1^{(0,1)}[\zeta, z]\, G_1^{(1,0)}[\zeta, z]\right)\epsilon^2 + \Big(G_3[\zeta, z] + \lambda\, G_2^{(0,1)}[\zeta, z]\, G_1^{(1,0)}[\zeta, z] + \frac{1}{2}\lambda^2\, G_1^{(0,2)}[\zeta, z]\, G_1^{(1,0)}[\zeta, z]^2 + \lambda\, G_1^{(0,1)}[\zeta, z]\, G_2^{(1,0)}[\zeta, z] + \lambda^2\, G_1^{(0,1)}[\zeta, z]\, G_1^{(1,0)}[\zeta, z]\, G_1^{(1,1)}[\zeta, z] + \frac{1}{2}\lambda^2\, G_1^{(0,1)}[\zeta, z]^2\, G_1^{(2,0)}[\zeta, z]\Big)\epsilon^3 + O[\epsilon]^4$