```
In[•]:= ME = MatrixExp;

In[•]:= ρt = {{1, 0}, {0, 1}}; ρy = {{0, 0}, {-ϵ, 0}}; ρa = {{(1 + 1/ϵ)/2, 0}, {0, -(1 - 1/ϵ)/2}}; ρx = {{0, 1}, {0, 0}};
        Simplify@{ρa.ρx - ρx.ρa == ρx, ρa.ρy - ρy.ρa == -ρy, ρx.ρy - ρy.ρx == ρt - 2 ϵ ρa}
Out[•]= {True, True, True}

In[•]:= (prod = ME[τ ρt].ME[η ρy].ME[α ρa].ME[ξ ρx]) // MatrixForm
Out[•]//MatrixForm=
⎛ e^(α(1+ϵ)/(2ϵ)+τ)                    e^(α(1+ϵ)/(2ϵ)+τ) ξ                          ⎞
⎝ -e^(α(1+ϵ)/(2ϵ)+τ) ϵ η   e^(-α(-1+ϵ)/(2ϵ)+τ) - e^(α(1+ϵ)/(2ϵ)+τ) ϵ η ξ ⎠

In[•]:= {tr = Tr@prod, det = Det@prod} // Simplify
Out[•]= {e^(-α(-1+ϵ)/(2ϵ)+τ) (1 + e^α (1 - ϵ η ξ)), e^(α/ϵ+2 τ)}

In[•]:= {tr, det} /. ϵ → 0
```

Power: Infinite expression $\frac{1}{0}$ encountered.

Infinity: Indeterminate expression $e^{\text{ComplexInfinity}}$ encountered.

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General: Further output of Power::infy will be suppressed during this calculation.

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```
Out[•]= {Indeterminate, Indeterminate}

In[•]:= tr/Sqrt[det] // PowerExpand // FullSimplify
Out[•]= e^(-α/2) (1 + e^α (1 - ϵ η ξ))

In[•]:= tr/Sqrt[det] // PowerExpand // FullSimplify // Expand
Out[•]= e^(-α/2) + e^(α/2) - e^(α/2) ϵ η ξ

In[•]:= Tr[ME[η ρy].ME[λ (ρt - 2 ϵ ρa)].ME[ξ ρx]] // Simplify
Out[•]= e^(-ϵ λ) (1 + e^(2 ϵ λ) - ϵ η ξ)
```