Voronov on Algebras over Operads and BV-Algebras

November 5, 2018 7:32 PM

joint w/ Lucy Yang 8:30

Def A BV algebra is a graded commutative assoc. alg

multiplication $V \otimes V \to V$: $a \otimes b \to ab$

w/ a 2nd order differential operator

$\Delta : V \to V$ $|\Delta| = -1$ $\Delta^2 = 0$ $\Delta(1) = 0$

2nd order:

$$[[[\Delta, L_a], L_b], L_c] = 0$$

Δ: "The BV operator"

where $L_a x := a \cdot x$

Define $\{a,b\} = (-1)^{|a|}(\Delta(ab) - (\Delta a)b - (-1)^{|a|} a(\Delta b))$

a Lie bracket of degree -1

Δ is a derivation w.r.t. the bracket

Remark generalization: DG-BV-algebra

has another first order differential...

Also "commutative BV_∞-algebra"

Example 1. Theorem [Folklore] $\mathfrak{g}$ - graded Lie algebra,

then $S(\mathfrak{g}[-1])$ carries a canonical structure of a BV algebra, w/

$$\Delta(x_1 \ldots x_n) = \sum_{i<j} \pm x_1 \cdots [x_i, x_j] \ldots \hat{x}_j \ldots x_n$$

The "Chevalley-Eilenberg" differential.

2. (Terilla-Tradler-Wilson 2011) if A is graded Assoc.

2. (Terilla–Tradler–Wilson 2011) if A is graded Assoc. algebra then

$T(A[-1])$ carries a canonical BV structure.

w/ multiplication "the shuffle product"

& $\Delta(x_1 \dots x_n) = \sum_i x_1 \otimes \dots \otimes x_i x_{i+1} \otimes \dots \otimes x_n$

The Operad Prospective: The 3 graces:

The Lie operad, Koszul dual of COM; so $Lie = Com^!$ and $As = As^!$

$\rightsquigarrow$ generalize the previous stuff to algebras over quadratic operads.

<u>Thm</u> Let $\mathcal{O}$ be an operad which is
1. Connected. $\mathcal{O}(0) = \mathbb{Q}$ (or $\mathbb{K}$)
2. Of finite type $\dim \mathcal{O}(n)^p < \infty$
3. binary quadratic: $\mathcal{O} = \mathcal{F}(E)/(R)$ with $E \subset \mathcal{O}(2)$ $\quad R \subseteq \mathcal{F}(E)(3)$
4. Co-commutative Hopf (so $\mathcal{O}$ is an operad in the category of cocom. coalgs

under these assumptions on $\mathcal{O}$, let V be

<u>Examples</u> Ass, Comm, Pois Gerstenhaber

an $\mathcal{O}^!$-algebra. Then the cofree co-nilpotent $\mathcal{O}$-algebra $\mathcal{F}_{\mathcal{O}}(V[-1])$ has a canonical BV-algebra structure compatible w/ the $\mathcal{O}$-coalg structure. 9:14

Where

$$\mathcal{F}_{\mathcal{O}}^{c}(V[-1]) = \bigoplus_n \mathcal{O}(n)^* \otimes_{S_n} (V[-1])^{\otimes n}$$

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