Gabriel Drummond-Cole's talk

November 8, 2018 2:52 AM

Adjoint functors?

The operads $\mathcal{O}$ & $\mathcal{C}$ governing operads and co-operads?

"Every cyclic operad is an operad" $\Longleftrightarrow$

$\exists$ a forgetful $\mathcal{C} \to \mathcal{O}$

D-C: $\exists$ a left-adjoint (easy, like every universal construction)

$\exists$ a right adjoint (hard and surprising)

$$\begin{tikzcd} \mathcal{C} \arrow[r, "F"] & \mathcal{D} \arrow[l, bend right, "L"'] \arrow[l, bend left, "R"] \end{tikzcd}$$

$C \in \mathcal{C}$, $D \in \mathcal{D}$

$\mathcal{D}$: $\mathrm{Hom}(FC, D)$ $\quad$ $\mathrm{Hom}(D, FC)$

$\mathcal{C}$: $\mathrm{Hom}(C, RD)$ $\quad$ $\mathrm{Hom}(LD, C)$

E.g. 1. $\mathcal{C}$ = Groups $\quad$ $\mathcal{D}$ = sets $\quad$ S: forgetful.

$G \quad A$

$$\mathrm{Hom}_{sets}(A, SG) = \mathrm{Hom}_{grps}(LA, G)$$

LA is the free grp generated by A

2. $$\mathrm{AssAlg} \xrightarrow{F} \mathrm{LieAlg}$$

$$\mathrm{Hom}_L(\mathfrak{g}, FA) \cong \mathrm{Hom}(U(\mathfrak{g}), A)$$

So U is a left adjoint of F.

① : The colored operad whose algebras are operads.