Etingof@Colloquium: Representation Theory without Vector Spaces

November 14, 2018 4:10 PM

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G - group, k - alg closed field.

$\mathcal{C} = \mathrm{Rep}_k(G)$ category of f.d. reps of G.

(V, ρ_V) $\rho_V : G \to GL(V)$

$\mathcal{C}$ is: k-linear, abelian

- artinian objects have finite length
- Homs are f.d. over k.
- monoidal w/ associativity isomorphisms

$$\alpha : (X \otimes Y) \otimes Z \longrightarrow X \otimes (Y \otimes Z)$$

satisfying the pentagon etc.

- symmetric $c : X \otimes Y \longrightarrow Y \otimes X$

w/ $c^2 = Id$.

satisfies the hexagon axiom

- rigid : $X \longmapsto X^*$ satisfying duality axioms
- $\otimes$ is bilinear on morphisms.

$$(\Rightarrow (X \oplus Y) \otimes Z \cong X \otimes Z \oplus Y \otimes Z$$

- $\mathrm{End}(\mathbb{1}) = k$.

<u>Def</u> A category w/ such properties is called a "symmetric tensor category".

Generalize: $\mathcal{C} = \mathrm{Rep}\, G$ G affine group scheme/k

$H = \mathcal{O}(G)$ a commutative Hopf algebra

$\mathrm{Rep}\, G = \mathrm{Comod}\, H$ $\qquad V \to H \otimes V$

Forgetful functor $F : \mathrm{Rep}\, G \to \mathrm{Vec}_k$

F is a "symmetric tensor functor, exact".

called "Fiber functor".

$G = \mathrm{Aut}_\otimes(F)$ $\qquad \exists F : \mathcal{C} \to \mathrm{Vect}_k$

iff $\mathcal{C} \cong \mathrm{Rep}_G$

<u>Q</u> Is every symmetric tensor category of this form?

<u>Ans</u>: No; e.g., $\mathcal{C} = \mathrm{sVec}_k$ w/ sign-twisted c.

More generally Let G be an affine grp scheme in sVec_k (a supergroup)

$H = \mathcal{O}(G)$ a super-commutative Hopf algebra.

$G = \mathrm{spec}(H)$

Fix $z \in G(k)$ w/ $z^2 = 1$ $\quad \mathrm{Ad}(z) =$ parity on $\mathcal{O}(G)$

$\mathcal{C} = \mathrm{Rep}(G, z)$ reps of G on superspaces where z acts by parity.

$F : \mathcal{C} \to \mathrm{sVec}_k \qquad G = \mathrm{Aut}_\otimes(F)$

The existance of F is equiv. to $\mathcal{C}$ being
$Rep(G, z)$ 4:33