In[ ]:= **a = {{1, 0}, {0, 0}}; x = {{0, 1}, {0, 0}};**

In[ ]:= **a // MatrixForm**

Out[ ]//MatrixForm=

$$\begin{pmatrix} 1 & 0 \\ 0 & 0 \end{pmatrix}$$

In[ ]:= **x // MatrixForm**

Out[ ]//MatrixForm=

$$\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$$

In[ ]:= **a.x - x.a // MatrixForm**

Out[ ]//MatrixForm=

$$\begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix}$$

In[ ]:= **e[α_, ξ_] := MatrixExp[α a + ξ x];**
e[α, ξ] // MatrixForm

Out[ ]//MatrixForm=

$$\begin{pmatrix} e^{\alpha} & \frac{(-1+e^{\alpha})\,\xi}{\alpha} \\ 0 & 1 \end{pmatrix}$$

In[ ]:= **e[α_1, ξ_1].e[α_2, ξ_2] // Simplify // MatrixForm**

Out[ ]//MatrixForm=

$$\begin{pmatrix} e^{\alpha_1+\alpha_2} & \frac{(-1+e^{\alpha_1})\,\xi_1}{\alpha_1} + \frac{e^{\alpha_1}\,(-1+e^{\alpha_2})\,\xi_2}{\alpha_2} \\ 0 & 1 \end{pmatrix}$$

In[ ]:= **e[α_3, ξ_3] // MatrixForm**

Out[ ]//MatrixForm=

$$\begin{pmatrix} e^{\alpha_3} & \frac{(-1+e^{\alpha_3})\,\xi_3}{\alpha_3} \\ 0 & 1 \end{pmatrix}$$

In[ ]:= **Solve$\left[\frac{(-1 + e^{\alpha_1})\,\xi_1}{\alpha_1} + \frac{e^{\alpha_1}\,(-1 + e^{\alpha_2})\,\xi_2}{\alpha_2} == \frac{(-1 + e^{\alpha_3})\,\xi_3}{\alpha_3}$ /. $\alpha_3 \to \alpha_1 + \alpha_2$, $\xi_3\right]$**

Out[ ]= $\left\{\left\{\xi_3 \to \left((\alpha_1 + \alpha_2)\,\left(-\alpha_2\,\xi_1 + e^{\alpha_1}\,\alpha_2\,\xi_1 - e^{\alpha_1}\,\alpha_1\,\xi_2 + e^{\alpha_1+\alpha_2}\,\alpha_1\,\xi_2\right)\right) \big/ \left(\left(-1 + e^{\alpha_1+\alpha_2}\right)\,\alpha_1\,\alpha_2\right)\right\}\right\}$

In[ ]:= **Series$\left[\left((\alpha_1 + \alpha_2)\,\left(-\alpha_2\,\xi_1 + e^{\alpha_1}\,\alpha_2\,\xi_1 - e^{\alpha_1}\,\alpha_1\,\xi_2 + e^{\alpha_1+\alpha_2}\,\alpha_1\,\xi_2\right)\right) \big/ \left(\left(-1 + e^{\alpha_1+\alpha_2}\right)\,\alpha_1\,\alpha_2\right),$**
{α_1, 0, 2}, {α_2, 0, 2}$\right]$

Out[ ]= $\left(\,(\xi_1 + \xi_2) - \frac{\xi_1\,\alpha_2}{2} + \frac{1}{12}\,\xi_1\,\alpha_2^2 + O[\alpha_2]^3\right) + \left(\frac{\xi_2}{2} + \left(-\frac{\xi_1}{12} - \frac{\xi_2}{12}\right)\,\alpha_2 + \frac{1}{24}\,\xi_1\,\alpha_2^2 + O[\alpha_2]^3\right)\,\alpha_1 +$
$\left(\frac{\xi_2}{12} - \frac{\xi_2\,\alpha_2}{24} + \left(\frac{\xi_1}{180} + \frac{\xi_2}{180}\right)\,\alpha_2^2 + O[\alpha_2]^3\right)\,\alpha_1^2 + O[\alpha_1]^3$

In[ ]:= $\left((\alpha_1 + \alpha_2)\,\left(-\alpha_2\,\xi_1 + e^{\alpha_1}\,\alpha_2\,\xi_1 - e^{\alpha_1}\,\alpha_1\,\xi_2 + e^{\alpha_1+\alpha_2}\,\alpha_1\,\xi_2\right)\right) \big/ \left(\left(-1 + e^{\alpha_1+\alpha_2}\right)\,\alpha_1\,\alpha_2\right)$ **// Simplify**

Out[ ]= $\left((\alpha_1 + \alpha_2)\,\left(\left(-1 + e^{\alpha_1}\right)\,\alpha_2\,\xi_1 + e^{\alpha_1}\,\left(-1 + e^{\alpha_2}\right)\,\alpha_1\,\xi_2\right)\right) \big/ \left(\left(-1 + e^{\alpha_1+\alpha_2}\right)\,\alpha_1\,\alpha_2\right)$