Pym on Multiple zeta Values in Deformation Quantization

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https://bitbucket.org/bpym/starproducts With Peter Banks and Erik Panzer

X – a smooth manifold / $\mathbb{R}$ or $\mathbb{C}$

$\pi \in \Gamma(X, \Lambda^2 T_X)$ poisson

Def a deformation quantization of (X, π)

is

$$f \star g = fg + \hbar\{f,g\} + \hbar^2 B_2(f,g) \in \mathcal{O}(X)[[\hbar]]$$

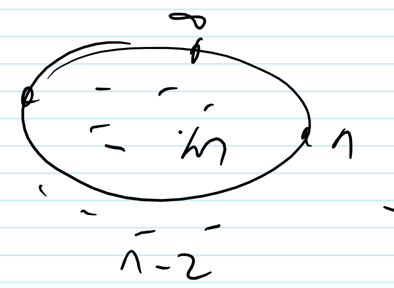
assoc. product.

Thm (Kontsevich '97) Every (X, π) has a "canonical" quantization

for $X = \mathbb{R}^n$ or $\mathbb{C}^n$,

$$f \star g = \sum \frac{\hbar^n}{n!} \sum_{\Gamma \in \mathrm{Graphs}(n)} c_\Gamma B_\Gamma(\pi^{\otimes n}, f, g)$$

where
$$c_\Gamma = \frac{1}{(2\pi i)^{\dim}} \int_{C_{n,m}} W_\Gamma$$

$$C_{n,m} = \{$$

$$\}$$

$$W_\Gamma \in A^\bullet(C_{n,m}) \subset \Omega^\bullet(C_{n,m})$$

generated by dlog(cross)

$\int(\text{ratios})$

Ex $\{x,y\} = \frac{1}{2}xy$ on $\mathbb{R}^2$

already here the quantization is not known explicitly.

$$x \star y = g(\hbar)xy \qquad y \star x = g(-\hbar)xy$$

$$g(\hbar) = 1 + \hbar/2 + \frac{\hbar^2}{24} - \frac{\hbar^3}{48} - \frac{\hbar^4}{1440} + \text{ stuff w/ } \zeta\text{-values.}$$

BTW, $x \star y = e^{\hbar} y \star x$

Def'n: MZV's

$$\zeta(n_1 \ldots n_d) = \sum_{0<k_1 \ldots <k_d} \frac{1}{k_1^{n_1} \cdots k_d^{n_d}}$$

$$w = \sum n_i \quad n_i \geq 1 \quad d_d \geq 2$$

$$Z_0 = \mathbb{Z}$$

$$Z_1 = \mathbb{Z}(i\pi)$$

$$Z_n = \mathbb{Z}\{\text{MZV's of wt } n\} + \mathbb{Z} \cdot i\pi\{\text{MZV at wt } n-1\}$$

Thm (Banks–Panzer–Pym, building on F. Brown)

Suppose $W \in A^{top}(C_{n,m})$ is such that

$I := \int W$ converges absolutely,

$I := \int_{C_{n,m}} W$ converges absolutely,

Then $I \in \begin{cases} (2\pi i)^{n-1} Z_{n-1} & m=0 \\ (2\pi i)^n Z_{n+m-2} & m>0 \end{cases}$

So total wt = dim of integration

$\#2\pi i$ factors = # of interior vertices.

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