Naef on The Goldman-Turaev Lie Bialgebra and the Kashiwara-Vergne Problem

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Setup: Σ surface w/ bdry $* \in \partial\Sigma$

$$\mathfrak{g} = K\pi_1 / [K\pi_1, K\pi_1] = K[S^1 \to \Sigma]$$
$$= |K\pi_1|$$

Def The Goldman bracket ... a Lie algebra.
Induced by a "double quasi-Poisson structure"

$$\{\cdot,\cdot\}_{MT} : K\pi_1 \otimes K\pi_1 \longrightarrow K\pi_1 \otimes K\pi_1$$

Massuyeau-Turaev.

Def Turaev cobracket δ
not well defined under $\{\varphi \sim \}$

Fix: Pick a "framing on Σ", use only loops w/ rotation number zero.

Prop $(\mathfrak{g}, [\,,\,], \delta)$ is an involutive Lie bialgebra $([\,,\,] \circ \delta = 0)$

$(K\pi_1, \{\,\}_{MT}, \mu)$ a "qBV" algebra.

All operations are cont. w.r.t filtration I^k,

$I = \ker(\epsilon : K\pi_1 \to K)$

Q Is $\hat{g} \overset{?}{\cong} gr\, g$ as Lie bialg?

Is $K\pi_1 \cong gr K\pi_1 \cong TH_1$ as qBV?

Thm (AKKN) For any $\Sigma_{g,n+1}$

1. $\hat{g} = gr\, g$ as Lie alg.

2. $(K\hat{\pi}, \{\ \}, \mu) \cong (TH_1, gr\{\ \} + r_{AM}, gr\,\mu + \underset{?}{h})$

r-matrix Alekseev-Meinrenken

(2 ⟹ 1 softly)

Def $SolKV^{i)} = \{\theta : K\hat{\pi}_1 \to TH_1$ iso of Hopf algebras s.t. induces $i\}$

$SolKV^{ii)}$ Same for Problem 2.

Thm (AKKN) Let $\theta \in SolKV^{i)}$ Then $\exists g \in TH_1$ group like s.t. $g\theta g^{-1} \in SolKV^{ii)}$.

Rem For $\Sigma = \Sigma_{n,1} =$ (oo)

SolKV is the "classical KV problem".

$K\pi_1 = K\langle X^{\pm 1}, Y^{\pm 1}\rangle \longrightarrow K\langle x, y\rangle$

via $X \to e^x \longrightarrow F(e^x)$

$\quad\;\; Y \to e^y \longrightarrow F(e^y)$

where $F \in \mathrm{SolKV}$.... Related to Duflo....

"All known solutions to KV come from Drinfel'd associators".

$$\mathrm{Assoc} \xrightarrow[\text{conj } 1\text{-}1]{} \mathrm{SolKV}$$