aS, with care

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$\mathcal{U}_{\gamma\epsilon;\hbar}$ conventions. "consolidate"

$q=\mathbb{e}^{\hbar\gamma\epsilon}$, $H=\langle a,x\rangle/([a,x]=\gamma x)$ with

$$A=\mathbb{e}^{-\hbar\epsilon a},\quad xA=qAx,\quad S_H(a,A,x)=(-a,A^{-1},-A^{-1}x),$$
$$\Delta_H(a,A,x)=(a_1+a_2,A_1A_2,x_1+A_1x_2)$$

$$S(e^{\zeta x})=e^{\zeta S(x)}=e^{-\zeta A^{-1}x}=$$

$$=\mathbb{O}_{Ax}\sum_{n=0}^{\infty}\frac{1}{n!}(-\zeta)^n A^{-n}x^n q^{-\binom{n}{2}}=\mathbb{O}_{Ax}\sum_{n=0}^{\infty}\frac{1}{n!}(-\zeta)^n A^{-n}x^n e^{-\hbar\gamma\epsilon\binom{n}{2}}$$

$$=\mathbb{O}_{Ax}\sum_{k=0}^{\infty}\frac{\epsilon^k}{k!}\left(\frac{-\hbar\gamma}{2}\right)^k\sum_{n=0}^{\infty}\frac{1}{n!}\binom{n}{2}^k(-\zeta)^n A^{-n}x^n$$

$$=\mathbb{O}_{Ax}\sum_{k=0}^{\infty}\frac{\epsilon^k}{k!}\left(\frac{-\hbar\gamma}{2}\right)^k(x^2\partial_x^2)^k e^{-\zeta A^{-1}x}$$