Noah

White: The center of the reflection equation algebra (and quantum group) for gl(n)

/$\mathbb{C}$

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W/ D. Jordan

1. Classical question: $GL_n \overset{\text{conj.}}{\curvearrowright} Mat_n$

So $GL_n \curvearrowright \mathcal{O}(Mat_n) = \mathbb{C}[e^i_j]$

Cayley-Hamilton: $E = (e^i_j)$

$$E^n - c_1^\circ E^{n-1} + \dots + (-1)^n c_n^\circ I = 0$$

$c_1^\circ = tr(E)$ $c_n^\circ = \det(E)$, and

$$c_k^\circ = \sum_{\substack{I \subset n \\ \#I = k}} \sum_{\sigma \in S_k} (-1)^{\ell(\sigma)} e^{i_1}_{i_{\sigma 1}} \dots e^{i_k}_{i_{\sigma k}}$$

Furthermore $\mathcal{O}(Mat_n)^{GL_n} \cong \mathbb{C}[c_1^\circ, \dots c_n^\circ]$

2. Main thm $q \in \mathbb{C}^\times$

$\mathcal{O}_q(Mat_n)$ = reflection eq'n algebra = generated by a^i_j subject to rel's dep. on q.

<u>Thm</u> (Jordan-W.)

$\mathcal{O}_q(Mat_n)$ has an identity, qC-H,

$$A^n - q^2 c_1 A^{n-1} + q^4 c_2 A^{n-2} \dots + (-q^2)^n c_n I = 0$$

and $Z(\mathcal{O}_q(Mat_n)) = \mathcal{O}_q(Mat_n)^{U_q(gl_n)}$

$= $ pol. algebra on c_k

$$c_k = \sum_{\substack{I\subset \underline{n} \\ |I|=k}} q^{-2wt(I)} \sum_{\sigma\in S_I} (-q)^{l(\sigma)} q^{e(\sigma)} a^{i_1}_{\sigma i_1} \cdots a^{i_k}_{\sigma i_k}$$

$wt(I) = \sum_{i\in I} i$ $\quad$ l is length in S_I as a subgroup of S_n

$e(\sigma) = \#\{i : \sigma(i) > i\}$ "exceedence"

$tr_q = c_1 \quad det_q = c_n$

Examples: $n=2$

$c_1 = q^{-2} a^1_1 + q^{-4} a^2_2$ $\quad$ known to Mudrov & Kulish

$c_2 = q^{-6}(a^1_1 a^2_2 - q^2 a^1_2 a^2_1)$

$n=3$: $\quad c_1 = q^{-2} a^1_1 + q^{-4} a^2_2 + q^{-6} a^3_3$

$c_2 = q^{-6}(\dots)$

$c_3 = \dots$

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3. relations Let R be the R-matrix for the vector rep of $U_q(GL_n)$

$$V = L_q(\omega_1)$$

$$R \in \mathrm{End}(V \otimes V)$$

Then $\mathcal{O}_q(\mathrm{Mat}_n)$ is the $\mathbb{C}$-alg generated by a^i_j & rels

$$R_{21} A_1 R_{12} A_2 = A_2 R_{21} A_1 R_{12}$$

where $A_1 = A \otimes id \quad A_2 = id \otimes A$

For example,

$m > n \qquad a^i_m a^i_n = q^{\delta_{in} - \delta_{im}}\ {}^i_n a^i_m$

$$+ \delta_{in}(q - q^{-1}) q^{-1} \sum_{p > i} a^i_p a^p_m$$

$-$ similar.

Context: Arose in factorizable scattering on the half-line. (Cherednik, ...)

& in study of braided Hopf algebras (Majid, ...)

& Provides quantization of Conjugacy classes