

Pensieve header: The yaxyax formulas for vanilla sl(2).

```
ME = MatrixExp; MF = MatrixForm;
Simp[sol_] := Flatten[sol] /. ConditionalExpression[ε_, _] => ε /.
  (var_ -> val_) => (var -> FullSimplify[PowerExpand[val]]);
```

$$\rho t = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}; \quad \rho y = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}; \quad \rho a = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}; \quad \rho x = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix};$$

```
eqn = Simplify[ME[η1 ρy].ME[α1 ρa].ME[ξ1 ρx].ME[η2 ρy].ME[α2 ρa].ME[ξ2 ρx] ==
  ME[η0 ρy].ME[α0 ρa].ME[ξ0 ρx]];
```

```
MF /@
eqn
```

$$\left(\begin{array}{c} e^{\alpha_1 + \alpha_2} (1 + \eta_2 \xi_1) \qquad e^{\alpha_1 - \alpha_2} \xi_1 + e^{\alpha_1 + \alpha_2} (1 + \eta_2 \xi_1) \xi_2 \\ e^{-\alpha_1 + \alpha_2} (\eta_2 + e^{2\alpha_1} \eta_1 (1 + \eta_2 \xi_1)) \quad e^{-\alpha_1 - \alpha_2} (1 + e^{2\alpha_1} \eta_1 \xi_1 + e^{2\alpha_2} \eta_2 \xi_2 + e^{2(\alpha_1 + \alpha_2)} \eta_1 (1 + \eta_2 \xi_1) \xi_2) \end{array} \right) ==$$

```
sol = Solve[Thread[Flatten /@ eqn], {η0, α0, ξ0}]
```

... Solve: Inconsistent or redundant transcendental equation. After reduction, the bad equation is $-\alpha_1 - \alpha_2 + \text{Log}[e^{-\alpha_0 + \alpha_1 + \alpha_2}] + \text{Log}[e^{\alpha_1 + \alpha_2} (1 + \eta_2 \xi_1)] == 0$.

... Solve: Inverse functions are being used by Solve, so some solutions may not be found; use Reduce for complete solution information.

$$\left\{ \left\{ \eta_0 \rightarrow \left(e^{-2\alpha_1} \left(1 + e^{2\alpha_1} \eta_1 \xi_1 - \frac{1}{1 + \eta_2 \xi_1} + e^{2\alpha_1 + 2\alpha_2} \eta_1 \xi_2 + e^{2\alpha_2} \eta_2 \xi_2 + e^{2\alpha_1 + 2\alpha_2} \eta_1 \eta_2 \xi_1 \xi_2 \right) \right) / \right. \right. \\ \left. \left(\xi_1 + e^{2\alpha_2} \xi_2 + e^{2\alpha_2} \eta_2 \xi_1 \xi_2 \right), \right. \\ \left. \alpha_0 \rightarrow \text{Log}[e^{\alpha_1 + \alpha_2} (1 + \eta_2 \xi_1)], \xi_0 \rightarrow \frac{e^{-2\alpha_2} (\xi_1 + e^{2\alpha_2} \xi_2 + e^{2\alpha_2} \eta_2 \xi_1 \xi_2)}{1 + \eta_2 \xi_1} \right\} \right\}$$

```
Simp[sol]
```

$$\left\{ \eta_0 \rightarrow \eta_1 + \frac{e^{-2\alpha_1} \eta_2}{1 + \eta_2 \xi_1}, \alpha_0 \rightarrow \alpha_1 + \alpha_2 + \text{Log}[1 + \eta_2 \xi_1], \xi_0 \rightarrow \frac{e^{-2\alpha_2} \xi_1}{1 + \eta_2 \xi_1} + \xi_2 \right\}$$

```
Simplify[eqn /. Simp[sol]]
```

True