Pensieve header: Reproducing commutation relations from a representation; the classical case. (Wrong; aborted).

Code borrowed from pensieve://2017-06/MultiYax.nb.

Representing yax

```
ρ1 = (0 0 1
      0 0 0
      0 0 0);
ρy = (0 0 0
      0 0 ϵ
      0 0 0); ρa = (0  0  0
                    0 -γ  0
                    0  0  0); ρx = (0 1 0
                                    0 0 0
                                    0 0 0); ρ0 = (0 0 0
                                                  0 0 0
                                                  0 0 0);
B[x_?MatrixQ, y_?MatrixQ] := x.y - y.x;
```

```
{B[ρa, ρx] == γ ρx, B[ρa, ρy] == -γ ρy,
 B[ρx, ρy] == ϵ ρ1, B[ρ1, ρy] == ρ0, B[ρ1, ρa] == ρ0, B[ρ1, ρx] == ρ0}
```

{True, True, True, True, True, True}

The 2-Stitch of Exponentials

```
P = IdentityMatrix[3];
Do[
  P = Expand[P.MatrixExp[η_i ρy].MatrixExp[α_i ρa].MatrixExp[ξ_i ρx]],
  {i, 1, 2}
 ];
P // MatrixForm
```

$$\begin{pmatrix} 1 & e^{-\gamma \alpha_2} \xi_1 + \xi_2 & \epsilon \eta_2 \xi_1 \\ 0 & e^{-\gamma \alpha_1 - \gamma \alpha_2} & \epsilon \eta_1 + e^{-\gamma \alpha_1} \epsilon \eta_2 \\ 0 & 0 & 1 \end{pmatrix}$$

```
Column[
 ϕ = First@Solve[Thread[Flatten /@
       (P == MatrixExp[η0 ρy].MatrixExp[α0 ρa].MatrixExp[ξ0 ρx].MatrixExp[τ0 ρ1])],
     {η0, α0, ξ0, τ0}] /. (v_ → ℰ_) :> (v → Expand@PowerExpand[ℰ])
]
```

Solve: Inverse functions are being used by Solve, so some solutions may not be found; use Reduce for complete solution information.

$\eta0 \to \eta_1 + e^{-\gamma \alpha_1} \eta_2$
$\alpha0 \to \alpha_1 + \alpha_2$
$\xi0 \to e^{-\gamma \alpha_2} \xi_1 + \xi_2$
$\tau0 \to \epsilon \eta_2 \xi_1$

```
Φ = Simplify[t0 τ0 + y0 η0 + a0 α0 + x0 ξ0 /. ϕ]
```

$a0 (\alpha_1 + \alpha_2) + y0 (\eta_1 + e^{-\gamma \alpha_1} \eta_2) + t0 \epsilon \eta_2 \xi_1 + x0 (e^{-\gamma \alpha_2} \xi_1 + \xi_2)$

```
Collect[Limit[Φ, ϵ → 0], {t0, y0, a0, xξ0}]
```

$a0\,(\alpha_1+\alpha_2) + y0\,(\eta_1 + e^{-\gamma\alpha_1}\eta_2) + x0\,(e^{-\gamma\alpha_2}\xi_1+\xi_2)$

```
Collect[Limit[Φ, ϵ → 0], {t0, y0, a0, xξ0}]
```

$a0\,(\alpha 1+\alpha 2) + y0\,(\eta 1 + e^{-\alpha 1\gamma}\eta 2) + e^{-\alpha 2\gamma}\,x0\,\xi 1 + x0\,\xi 2 + t0\,(-\eta 2\,\xi 1 + \tau 1 + \tau 2)$

Reproducing the Commutation Relations

```
Simplify@PowerExpand[(∂_{α1,ξ2} e^Φ - ∂_{ξ1,α2} e^Φ) /. {τ1 | η1 | α1 | ξ1 | τ2 | η2 | α2 | ξ2 → 0}]
```

$x0\,\gamma$

```
Simplify@PowerExpand[(∂_{α1,η2} e^Φ - ∂_{η1,α2} e^Φ) /. {τ1 | η1 | α1 | ξ1 | τ2 | η2 | α2 | ξ2 → 0}]
```

$-y0\,\gamma$

```
qbxy = Simplify@PowerExpand[(∂_{ξ1,η2} e^Φ - q ∂_{η1,ξ2} e^Φ) /. {τ1 | η1 | α1 | ξ1 | τ2 | η2 | α2 | ξ2 → 0}]
```

$-\frac{1}{\gamma\,\epsilon\,\hbar} e^{-\gamma\epsilon\hbar}\,(-1+e^{\gamma\epsilon\hbar})\,(t0 - 2\,a0\,\epsilon + e^{\gamma\epsilon\hbar}\,x0\,y0\,\gamma\,\epsilon\,\hbar)$

```
Limit[qbxy, ϵ → 0]
```

$-t0$