Pensieve header: The sl_2^ϵ dequantizator.

Code borrowed from Phi2CR-Classical.nb.

Monoblog on Oct 18, 2017:

$\mathcal{U}_{\hbar;\gamma\epsilon}$ conventions: $q=e^{\hbar\gamma\epsilon}$, $H=\langle a,x\rangle/([a,x]=\gamma x)$ with

$$A=e^{-\hbar\epsilon a},\quad xA=qAx,\quad S(a,A,x)=(-a,A^{-1},-A^{-1}x),$$
$$\Delta(a,A,x)=(a_1+a_2,A_1A_2,x_1+A_1x_2)$$

and dual $H^*=\langle b,y\rangle/([b,y]=-\epsilon y)$ with

$$B=e^{-\hbar\gamma b},\quad By=qyB,\quad S(b,B,y)=(-b,B^{-1},-yB^{-1}),$$
$$\Delta(b,B,y)=(b_1+b_2,B_1B_2,y_1B_2+y_2).$$

Pairing by $(a,x)^*=\hbar(b,y)$ making $\langle y^lb^i,a^jx^k\rangle=\delta_{ij}\delta_{kl}\hbar^{-(j+k)}j![k]_q!$ so $R=\sum\frac{\hbar^{j+k}y^kb^j\otimes a^jx^k}{j![k]_q!}$. Then $\mathcal{U}=H^{*cop}\otimes H$ with $(\phi f)(\psi g)=\langle\psi_1S^{-1}f_3\rangle\langle\psi_3,f_1\rangle(\phi\psi_2)(f_2g)$. With the central $t:=\epsilon a-\gamma b$, $T:=e^{\hbar t}=A^{-1}B$ get

$$[a,y]=-\gamma y,\quad [b,x]=\epsilon x,\quad xy-qyx=(1-TA^2)/\hbar.$$

At $\epsilon=0$, $\mathcal{U}_{\hbar;\gamma 0}=\langle b,y,a,x\rangle/([b,\cdot]=0,\ [a,x]=\gamma x,\ [a,y]=-\gamma y,\ [x,y]=(1-e^{-\hbar\gamma b})/\hbar)$ with $\Delta(b,y,a,x)=(b_1+b_2,y_1+e^{-\hbar\gamma b_1}y_2,a_1+a_2,x_1+x_2)$.

```
{rhs = Simplify[(1 - T A^2)/ħ /. {T → e^(ħ (ϵ a - b)), A → e^(-ħ ϵ a)} /. b → ϵ a - t], Limit[rhs, ħ → 0]}
```

$$\left\{-\frac{-1+e^{(t-2a\epsilon)\hbar}}{\hbar},\ -t+2a\epsilon\right\}$$

Representing $\mathfrak{g}^\epsilon$.

```
ME = MatrixExp;
Simp[sol_] :=
  Flatten[sol] /. C[_] → 0 /. (var_ → val_) :→ (var → Simplify[PowerExpand@val]);
```

```
ρt = (1 0
      0 1); ρy = (0 0
                  ϵ 0); ρa = ((1 + 1/ϵ)/2        0
                                 0        -(1 - 1/ϵ)/2); ρx = (0 1
                                                              0 0);
Simplify@{ρa.ρx - ρx.ρa == ρx, ρa.ρy - ρy.ρa == -ρy, ρx.ρy - ρy.ρx == 2 ϵ ρa - ρt}
```

```
{True, True, True}
```

The 2-Stitch of Exponentials

```
ϕ = {τ0 → -Log[1 + ϵ η2 ξ1]/ϵ + τ1 + τ2,
   η0 → η1 + e^(-α1) η2/(1 + ϵ η2 ξ1), α0 → 2 Log[1 + ϵ η2 ξ1] + α1 + α2, ξ0 → e^(-α2) ξ1/(1 + ϵ η2 ξ1) + ξ2};
Simplify[
 ME[τ1 ρt].ME[η1 ρy].ME[α1 ρa].ME[ξ1 ρx].ME[τ2 ρt].ME[η2 ρy].ME[α2 ρa].ME[ξ2 ρx] ==
  ME[τ0 ρt].ME[η0 ρy].ME[α0 ρa].ME[ξ0 ρx] /. ϕ]
```

```
True
```

```
Φ = Simplify[t₀ τ₀ + y₀ η₀ + a₀ α₀ + x₀ ξ₀ /. ϕ]
```

$$a_0\left(2\,\mathrm{Log}[1+\epsilon\,\eta_2\,\xi_1]+\alpha_1+\alpha_2\right)+y_0\left(\eta_1+\frac{e^{-\alpha_1}\,\eta_2}{1+\epsilon\,\eta_2\,\xi_1}\right)+x_0\left(\frac{e^{-\alpha_2}\,\xi_1}{1+\epsilon\,\eta_2\,\xi_1}+\xi_2\right)+t_0\left(-\frac{\mathrm{Log}[1+\epsilon\,\eta_2\,\xi_1]}{\epsilon}+\tau_1+\tau_2\right)$$

```
Collect[Limit[Φ, ϵ → 0], {t₀, y₀, a₀, x₀}]
```

$$a_0\,(\alpha_1+\alpha_2)+y_0\left(\eta_1+e^{-\alpha_1}\,\eta_2\right)+x_0\left(e^{-\alpha_2}\,\xi_1+\xi_2\right)+t_0\,(-\eta_2\,\xi_1+\tau_1+\tau_2)$$

Finding the Dequantizator

Should satisfy $x\,\hat{y}-q\,\hat{y}\,x=\frac{1-e^{\hbar(t-2\epsilon a)}}{\hbar}$ where $q=e^{\hbar\gamma\epsilon}$ (temporarily at $\gamma=1$).

The $x\,\hat{y}$ side

```
Simplify[∂_{ξ₁} e^Φ] /. {(τ | η | α | ξ)₁ → 0, (v : (τ | η | α | ξ))₂ :→ v, (v : (t | y | a | x))₀ :→ v}
```

$$e^{-\alpha+a\,\alpha+y\,\eta+x\,\xi+t\,\tau}\left(x+e^{\alpha}\,\eta\left(-t+2\,a\,\epsilon-y\,\epsilon\,\eta\right)\right)$$

Makes $x\,f[y,a-1]-(t-2\,a\,\epsilon+y\,\epsilon\,\partial_y)\,\partial_y f[y,a]$.

The $q\,\hat{y}\,x$ side

```
Simplify[∂_{ξ₂} e^Φ] /. {(v : (τ | η | α | ξ))₁ :→ v, (τ | η | α | ξ)₂ → 0, (v : (t | y | a | x))₀ :→ v}
```

$$e^{a\,\alpha+y\,\eta+x\,\xi+t\,\tau}\,x$$

Makes $x\,e^{\hbar\epsilon}\,f[y,a]$.

The overall equation

$x\left(f[y,a-1]-e^{\hbar\epsilon}\,f[y,a]\right)-(t-2\,a\,\epsilon+y\,\epsilon\,\partial_y)\,\partial_y f[y,a]=\frac{1-e^{\hbar(t-2\epsilon a)}}{\hbar}$.

At $\epsilon=0$ this is $x\,(f[y,a-1]-f[y,a])-t\,\partial_y f[y,a]=\frac{1-e^{\hbar t}}{\hbar}$, which is solved by $f[y,a]=y\,\frac{e^{\hbar t}-1}{\hbar t}$.

```
eqn[f_] := Simplify[x ((f /. a → a - 1) - e^(ħ ϵ) f) - (t - 2 ϵ a) ∂_y f + y ϵ ∂_{y,2} f - (1 - e^(ħ (t-2 ϵ a)))/ħ]
```

```
eqn[f[y, a]] /. ϵ → 0
```

$$\frac{1}{\hbar}\left(-1+e^{t\,\hbar}+x\,\hbar\,\left(f[y,-1+a]-f[y,a]\right)-t\,\hbar\,f^{(1,0)}[y,a]\right)$$

```
Simplify[eqn[y (e^(ħ t) - 1)/(ħ t)] /. ϵ → 0]
```

```
0
```

SeriesCoefficient$\left[\mathbf{eqn}\left[\mathbf{y}\,\frac{\mathbb{e}^{\hbar t}-1}{\hbar\,t}+\epsilon\,f_1[y,a]\right],\ \{\epsilon,0,1\}\right]$

$$-2\,a\,\mathbb{e}^{t\hbar}+\frac{x\,y}{t}-\frac{\mathbb{e}^{t\hbar}\,x\,y}{t}-\frac{2\,a}{t\,\hbar}+\frac{2\,a\,\mathbb{e}^{t\hbar}}{t\,\hbar}+x\,f_1[y,-1+a]-x\,f_1[y,a]-t\,f_1^{(1,0)}[y,a]$$

SeriesCoefficient$\left[\mathbf{eqn}\left[\mathbf{y}\,\frac{\mathbb{e}^{\hbar t}-1}{\hbar\,t}+\epsilon\left(\frac{y\,a}{t}-\frac{\mathbb{e}^{t\hbar}\,y\,a}{t}+f_1[y,a]\right)\right],\ \{\epsilon,0,1\}\right]$

$$-a-a\,\mathbb{e}^{t\hbar}-\frac{2\,a}{t\,\hbar}+\frac{2\,a\,\mathbb{e}^{t\hbar}}{t\,\hbar}+x\,f_1[y,-1+a]-x\,f_1[y,a]-t\,f_1^{(1,0)}[y,a]$$

Simplify[eqn[y g[x y, a]] //. x y → z]

$$\frac{1}{\hbar}\left(-1+\mathbb{e}^{(t-2a\epsilon)\hbar}+z\,\hbar\left(g[z,-1+a]-\mathbb{e}^{\epsilon\hbar}\,g[z,a]\right)-(t-2\,a\,\epsilon)\,\hbar\left(g[z,a]+z\,g^{(1,0)}[z,a]\right)+z\,\epsilon\,\hbar\left(2\,g^{(1,0)}[z,a]+z\,g^{(2,0)}[z,a]\right)\right)$$

$$\mathbf{eq}[g\_]:=\mathbf{Simplify}\left[\frac{1}{\hbar}\left(-1+\mathbb{e}^{(t-2a\epsilon)\hbar}+z\,\hbar\left((g\ /.\ a\to a-1)-\mathbb{e}^{\epsilon\hbar}\,g\right)-(t-2\,a\,\epsilon)\,\hbar\,(g+z\,\partial_z g)+z\,\epsilon\,\hbar\left(2\,\partial_z g+z\,\partial_{\{z,2\}}g\right)\right)\right]$$

eq[g[z, a]] == Simplify[eqn[y g[x y, a]] //. x y → z]

True

Simplify[eq[g[z, a]] /. ϵ → 0]

$$\frac{1}{\hbar}\left(-1+\mathbb{e}^{t\hbar}+z\,\hbar\,g[z,-1+a]-(t+z)\,\hbar\,g[z,a]-t\,z\,\hbar\,g^{(1,0)}[z,a]\right)$$

SeriesCoefficient[eq[g[z, a]], {ϵ, 0, 0}]

$$-\frac{1}{\hbar}+\frac{\mathbb{e}^{t\hbar}}{\hbar}+z\,g[z,-1+a]-t\,g[z,a]-z\,g[z,a]-t\,z\,g^{(1,0)}[z,a]$$

SeriesCoefficient$\left[\mathbf{eq}\left[\frac{\mathbb{e}^{\hbar t}-1}{\hbar\,t}+\epsilon\,g_1[z,a]\right],\ \{\epsilon,0,1\}\right]$

$$-2\,a\,\mathbb{e}^{t\hbar}+\frac{z}{t}-\frac{\mathbb{e}^{t\hbar}\,z}{t}-\frac{2\,a}{t\,\hbar}+\frac{2\,a\,\mathbb{e}^{t\hbar}}{t\,\hbar}+z\,g_1[z,-1+a]-t\,g_1[z,a]-z\,g_1[z,a]-t\,z\,g_1^{(1,0)}[z,a]$$

Simplify$\left[\mathbf{eqn}\left[x^{-1}\,g\left[t^{-1}\,x\,y,a\right]\right]\right]$ **//.** $t^{-1}\,x\,y\to z$

$$-\frac{1}{\hbar}+\frac{\mathbb{e}^{(t-2a\epsilon)\hbar}}{\hbar}+g[z,-1+a]-\mathbb{e}^{\epsilon\hbar}\,g[z,a]-g^{(1,0)}[z,a]+\frac{2\,a\,\epsilon\,g^{(1,0)}[z,a]}{t}+\frac{x\,y\,\epsilon\,g^{(2,0)}[z,a]}{t^2}$$

Simplify$\left[\mathbf{eqn}\left[x^{-1}\,g[x\,y,a]\right]\right]$ **//.** $x\,y\to z$

$$g[z,-1+a]+\frac{1}{\hbar}\left(-1+\mathbb{e}^{(t-2a\epsilon)\hbar}-\mathbb{e}^{\epsilon\hbar}\,\hbar\,g[z,a]-(t-2\,a\,\epsilon)\,\hbar\,g^{(1,0)}[z,a]+z\,\epsilon\,\hbar\,g^{(2,0)}[z,a]\right)$$

$$\mathbf{eq}[g\_]:=\mathbf{Simplify}\left[\frac{\mathbb{e}^{(t-2a\epsilon)\hbar}-1}{\hbar}+(g\ /.\ a\to a-1)-\mathbb{e}^{\epsilon\hbar}\,g-(t-2\,a\,\epsilon)\,\partial_z g+z\,\epsilon\,\partial_{\{z,2\}}g\right]$$

```
Simplify[eq[g[z, a]] == eqn[x^-1 g[x y, a]] //. x y → z]
```

True

Changing the dependent variable:

```
Collect[(z^(1-a+t/(2ϵ)))^-1 eq[z^((t-2aϵ)/(2ϵ)) g[z, a]], {g[z, a], g[z, -1 + a], g^(2,0)[z, a]}, Simplify]
```

$$\frac{\left(-1+e^{(t-2a\epsilon)\hbar}\right) z^{-1+a-\frac{t}{2\epsilon}}}{\hbar}+g[z,-1+a]-\frac{1}{4z^2\epsilon}\left(t^2+(2-4a)\,t\,\epsilon+4\epsilon\left(e^{\epsilon\hbar}z+(-1+a)\,a\,\epsilon\right)\right)g[z,a]+\epsilon\,g^{(2,0)}[z,a]$$

Changing the independent variable:

```
eq[g[z^((t+ϵ-2aϵ)/ϵ), a]]
```

$$-\frac{1}{\hbar}+\frac{e^{(t-2a\epsilon)\hbar}}{\hbar}-e^{\epsilon\hbar}g\left[z^{1-2a+\frac{t}{\epsilon}},a\right]+g\left[z^{3-2a+\frac{t}{\epsilon}},-1+a\right]+\frac{z^{1-4a+\frac{2t}{\epsilon}}(t+\epsilon-2a\epsilon)^2\,g^{(2,0)}\left[z^{1-2a+\frac{t}{\epsilon}},a\right]}{\epsilon}$$

Changing the dependent and the independent variable:

```
Collect[eq[z^s1 g[z^s2, a]], g^(1,0)[z^s2, a], Simplify]
```

$$\frac{-1+e^{(t-2a\epsilon)\hbar}}{\hbar}+z^{s_1}g\left[z^{s_2},-1+a\right]-z^{-1+s_1}g\left[z^{s_2},a\right]\left(e^{\epsilon\hbar}z+(t+\epsilon-2a\epsilon)\,s_1-\epsilon\,s_1^2\right)+z^{-1+s_1+s_2}s_2\left(-t-\epsilon+2a\epsilon+2\epsilon s_1+\epsilon s_2\right)g^{(1,0)}\left[z^{s_2},a\right]+z^{-1+s_1+2s_2}\epsilon\,s_2^2\,g^{(2,0)}\left[z^{s_2},a\right]$$

Expanding in ϵ:

```
Simplify[eq[g[z, a]] /. ϵ → 0]
```

$$g[z,-1+a]-\frac{1-e^{t\hbar}+\hbar\,g[z,a]+t\hbar\,g^{(1,0)}[z,a]}{\hbar}$$

```
SeriesCoefficient[eq[g[z, a]], {ϵ, 0, 0}]
```

$$-\frac{1}{\hbar}+\frac{e^{t\hbar}}{\hbar}+g[z,-1+a]-g[z,a]-t\,g^{(1,0)}[z,a]$$

```
Series[-1/ℏ + e^(t ℏ)/ℏ, {ℏ, 0, 3}]
```

$$t+\frac{t^2\hbar}{2}+\frac{t^3\hbar^2}{6}+\frac{t^4\hbar^3}{24}+O[\hbar]^4$$

```
SeriesCoefficient[eq[(e^(ℏ t) - 1)/(ℏ t) z + ϵ g1[z, a]], {ϵ, 0, 1}]
```

$$-2a\,e^{t\hbar}+\frac{z}{t}-\frac{e^{t\hbar}z}{t}-\frac{2a}{t\hbar}+\frac{2a\,e^{t\hbar}}{t\hbar}+g_1[z,-1+a]-g_1[z,a]-t\,g_1{}^{(1,0)}[z,a]$$

$$\mathtt{Series}\left[-2\,a\,e^{t\hbar}+\frac{z}{t}-\frac{e^{t\hbar}z}{t}-\frac{2\,a}{t\,\hbar}+\frac{2\,a\,e^{t\hbar}}{t\,\hbar},\ \{\hbar,0,2\}\right]$$

$$(-a\,t-z)\,\hbar+\left(-\frac{2\,a\,t^2}{3}-\frac{t\,z}{2}\right)\hbar^2+O[\hbar]^3$$

$$\mathtt{SeriesCoefficient}\Big[\mathtt{eq}\Big[\frac{e^{\hbar t}-1}{\hbar\,t}\,z+\epsilon\left(\frac{z}{t}\left(-2\,a\,e^{t\hbar}-\frac{2\,a}{t\,\hbar}+\frac{2\,a\,e^{t\hbar}}{t\,\hbar}\right)+\frac{z}{2\,t}\left(\frac{z}{t}+\frac{e^{t\hbar}z}{t}+\frac{2\,z}{t^2\hbar}-\frac{2\,e^{t\hbar}z}{t^2\hbar}\right)+g_1[z,a]\right)\Big],\ \{\epsilon,0,1\}\Big]$$

$$g_1[z,-1+a]-g_1[z,a]-t\,g_1^{(1,0)}[z,a]$$

$$\mathtt{Series}\left[\frac{z}{t}\left(-2\,a\,e^{t\hbar}-\frac{2\,a}{t\,\hbar}+\frac{2\,a\,e^{t\hbar}}{t\,\hbar}\right)+\frac{z}{2\,t}\left(\frac{z}{t}+\frac{e^{t\hbar}z}{t}+\frac{2\,z}{t^2\hbar}-\frac{2\,e^{t\hbar}z}{t^2\hbar}\right),\ \{t,0,1\}\right]$$

$$\left(-a\,z\,\hbar+\frac{z^2\hbar^2}{12}\right)+\frac{1}{24}\left(-16\,a\,z\,\hbar^2+z^2\hbar^3\right)t+O[t]^2$$

$$\mathtt{FullSimplify}\left[\frac{z}{t}\left(-2\,a\,e^{t\hbar}-\frac{2\,a}{t\,\hbar}+\frac{2\,a\,e^{t\hbar}}{t\,\hbar}\right)+\frac{z}{2\,t}\left(\frac{z}{t}+\frac{e^{t\hbar}z}{t}+\frac{2\,z}{t^2\hbar}-\frac{2\,e^{t\hbar}z}{t^2\hbar}\right)\right]$$

$$\frac{1}{2\,t^3\hbar}z\left(z\left(2+t\,\hbar+e^{t\hbar}(-2+t\,\hbar)\right)-4\,a\,t\left(1+e^{t\hbar}(-1+t\,\hbar)\right)\right)$$

$$\mathtt{Expand@SeriesCoefficient}\Big[\mathtt{eq}\Big[\frac{e^{\hbar t}-1}{\hbar\,t}\,z+\epsilon\left(\frac{z}{t}\left(-2\,a\,e^{t\hbar}-\frac{2\,a}{t\,\hbar}+\frac{2\,a\,e^{t\hbar}}{t\,\hbar}\right)+\frac{z}{2\,t}\left(\frac{z}{t}+\frac{e^{t\hbar}z}{t}+\frac{2\,z}{t^2\hbar}-\frac{2\,e^{t\hbar}z}{t^2\hbar}\right)+\epsilon\,g_2[z,a]\right)\Big],\ \{\epsilon,0,2\}\Big]$$

$$-\frac{4\,a^2e^{t\hbar}}{t}+\frac{z}{t^2}+\frac{4\,a\,z}{t^2}+\frac{e^{t\hbar}z}{t^2}-\frac{z^2}{t^3}+\frac{e^{t\hbar}z^2}{t^3}-\frac{4\,a^2}{t^2\hbar}+\frac{4\,a^2e^{t\hbar}}{t^2\hbar}+\frac{2\,z}{t^3\hbar}+\frac{4\,a\,z}{t^3\hbar}-\frac{2\,e^{t\hbar}z}{t^3\hbar}-\frac{4\,a\,e^{t\hbar}z}{t^3\hbar}+$$
$$2\,a^2e^{t\hbar}\hbar+\frac{z\,\hbar}{2\,t}-\frac{e^{t\hbar}z\,\hbar}{2\,t}+\frac{2\,a\,e^{t\hbar}z\,\hbar}{t}-\frac{z^2\hbar}{2\,t^2}-\frac{e^{t\hbar}z^2\hbar}{2\,t^2}+g_2[z,-1+a]-g_2[z,a]-t\,g_2^{(1,0)}[z,a]$$

$$\mathtt{Expand@SeriesCoefficient}\Big[\mathtt{eq}\Big[\frac{e^{\hbar t}-1}{\hbar\,t}\,z+\epsilon\Big(\frac{z}{t}\left(-2\,a\,e^{t\hbar}-\frac{2\,a}{t\,\hbar}+\frac{2\,a\,e^{t\hbar}}{t\,\hbar}\right)+\frac{z}{2\,t}\left(\frac{z}{t}+\frac{e^{t\hbar}z}{t}+\frac{2\,z}{t^2\hbar}-\frac{2\,e^{t\hbar}z}{t^2\hbar}\right)+$$
$$\epsilon\Big(\frac{z\,a^2}{t}\left(-\frac{4\,e^{t\hbar}}{t}-\frac{4}{t^2\hbar}+\frac{4\,e^{t\hbar}}{t^2\hbar}+2\,e^{t\hbar}\hbar\right)+a\,\frac{z^2}{2\,t}\left(\frac{4}{t^2}+\frac{8\,e^{t\hbar}}{t^2}+\frac{12}{t^3\hbar}-\frac{12\,e^{t\hbar}}{t^3\hbar}-\frac{2\,e^{t\hbar}\hbar}{t}\right)+$$
$$\frac{1}{12\,t^5\hbar}z^2\big(-12\,t+12\,e^{t\hbar}t-24\,z+24\,e^{t\hbar}z+6\,t^2\hbar-18\,e^{t\hbar}t^2\hbar-12\,t\,z\,\hbar-12\,e^{t\hbar}t\,z\,\hbar+$$
$$3\,t^3\hbar^2+9\,e^{t\hbar}t^3\hbar^2-2\,t^2z\,\hbar^2+2\,e^{t\hbar}t^2z\,\hbar^2\big)+g_2[z,a]\Big)\Big)\Big],\ \{\epsilon,0,2\}\Big]$$

$$g_2[z,-1+a]-g_2[z,a]-t\,g_2^{(1,0)}[z,a]$$

```
Expand@SeriesCoefficient[
  eq[(e^(ħ t) - 1)/(ħ t) z + ϵ (z/t (-2 a e^(t ħ) - (2 a)/(t ħ) + (2 a e^(t ħ))/(t ħ)) + z/(2 t) (z/t + (e^(t ħ) z)/t + (2 z)/(t^2 ħ) - (2 e^(t ħ) z)/(t^2 ħ)) +
      ϵ ((z a^2)/t (-(4 e^(t ħ))/t - 4/(t^2 ħ) + (4 e^(t ħ))/(t^2 ħ) + 2 e^(t ħ) ħ) + a z^2/(2 t) (4/t^2 + (8 e^(t ħ))/t^2 + 12/(t^3 ħ) - (12 e^(t ħ))/(t^3 ħ) - (2 e^(t ħ) ħ)/t) +
        1/(12 t^5 ħ) z^2 (-12 t + 12 e^(t ħ) t - 24 z + 24 e^(t ħ) z + 6 t^2 ħ - 18 e^(t ħ) t^2 ħ - 12 t z ħ - 12 e^(t ħ) t z ħ +
          3 t^3 ħ^2 + 9 e^(t ħ) t^3 ħ^2 - 2 t^2 z ħ^2 + 2 e^(t ħ) t^2 z ħ^2) + ϵ g3[z, a]))], {ϵ, 0, 3}]
```

$$-\frac{8 a^3 e^{t\hbar}}{t^2}+\frac{z}{t^3}+\frac{6 a z}{t^3}+\frac{12 a^2 z}{t^3}-\frac{3 e^{t\hbar} z}{t^3}+\frac{2 a e^{t\hbar} z}{t^3}+\frac{12 a^2 e^{t\hbar} z}{t^3}-\frac{5 z^2}{t^4}-\frac{12 a z^2}{t^4}-\frac{7 e^{t\hbar} z^2}{t^4}+\frac{2 z^3}{t^5}-\frac{2 e^{t\hbar} z^3}{t^5}-\frac{8 a^3}{t^3\hbar}+\frac{8 a^3 e^{t\hbar}}{t^3\hbar}-\frac{2 z}{t^4\hbar}+\frac{8 a z}{t^4\hbar}+\frac{24 a^2 z}{t^4\hbar}+\frac{2 e^{t\hbar} z}{t^4\hbar}-\frac{8 a e^{t\hbar} z}{t^4\hbar}-\frac{24 a^2 e^{t\hbar} z}{t^4\hbar}-\frac{12 z^2}{t^5\hbar}-\frac{12 a z^2}{t^5\hbar}+\frac{12 e^{t\hbar} z^2}{t^5\hbar}+\frac{12 a e^{t\hbar} z^2}{t^5\hbar}+\frac{4 a^3 e^{t\hbar}\hbar}{t}+\frac{z\hbar}{2t^2}+\frac{2 a z\hbar}{t^2}+\frac{3 e^{t\hbar} z\hbar}{2t^2}-\frac{2 z^2\hbar}{t^3}-\frac{3 a z^2\hbar}{t^3}+\frac{3 e^{t\hbar} z^2\hbar}{t^3}-\frac{3 a e^{t\hbar} z^2\hbar}{t^3}+\frac{z^3\hbar}{t^4}+\frac{e^{t\hbar} z^3\hbar}{t^4}-\frac{4}{3}a^3 e^{t\hbar}\hbar^2+\frac{z\hbar^2}{6t}-\frac{e^{t\hbar} z\hbar^2}{6t}+\frac{a e^{t\hbar} z\hbar^2}{t}-\frac{2a^2 e^{t\hbar} z\hbar^2}{t}-\frac{z^2\hbar^2}{2t^2}-\frac{e^{t\hbar} z^2\hbar^2}{t^2}+\frac{a e^{t\hbar} z^2\hbar^2}{t^2}+\frac{z^3\hbar^2}{6t^3}-\frac{e^{t\hbar} z^3\hbar^2}{6t^3}+g_3[z,-1+a]-g_3[z,a]-t\,g_3^{(1,0)}[z,a]$$

```
Series[(e^(ħ t) - 1)/(ħ t) z + ϵ (z/t (-2 a e^(t ħ) - (2 a)/(t ħ) + (2 a e^(t ħ))/(t ħ)) + z/(2 t) (z/t + (e^(t ħ) z)/t + (2 z)/(t^2 ħ) - (2 e^(t ħ) z)/(t^2 ħ)) +
    ϵ ((z a^2)/t (-(4 e^(t ħ))/t - 4/(t^2 ħ) + (4 e^(t ħ))/(t^2 ħ) + 2 e^(t ħ) ħ) + a z^2/(2 t) (4/t^2 + (8 e^(t ħ))/t^2 + 12/(t^3 ħ) - (12 e^(t ħ))/(t^3 ħ) - (2 e^(t ħ) ħ)/t) +
      1/(12 t^5 ħ) z^2 (-12 t + 12 e^(t ħ) t - 24 z + 24 e^(t ħ) z + 6 t^2 ħ - 18 e^(t ħ) t^2 ħ - 12 t z ħ - 12 e^(t ħ) t z ħ +
        3 t^3 ħ^2 + 9 e^(t ħ) t^3 ħ^2 - 2 t^2 z ħ^2 + 2 e^(t ħ) t^2 z ħ^2) + ϵ g3[z, a])), {t, 0, 2}]
```

$$\frac{z^2\epsilon^2\hbar^2}{6t}+\frac{1}{360}\left(360 z-360 a z\epsilon\hbar+30 z^2\epsilon\hbar^2+240 a^2 z\epsilon^2\hbar^2+60 z^2\epsilon^2\hbar^3-30 a z^2\epsilon^2\hbar^3+z^3\epsilon^2\hbar^4+360\epsilon^3 g_3[z,a]\right)+\frac{1}{720}\left(360 z\hbar-480 a z\epsilon\hbar^2+30 z^2\epsilon\hbar^3+360 a^2 z\epsilon^2\hbar^3+51 z^2\epsilon^2\hbar^4-36 a z^2\epsilon^2\hbar^4+z^3\epsilon^2\hbar^5\right)t+\frac{1}{10080}\left(1680 z\hbar^2-2520 a z\epsilon\hbar^3+126 z^2\epsilon\hbar^4+2016 a^2 z\epsilon^2\hbar^4+203 z^2\epsilon^2\hbar^5-168 a z^2\epsilon^2\hbar^5+4 z^3\epsilon^2\hbar^6\right)t^2+O[t]^3$$

```
eqng = FullSimplify[ħ (x ((y g[x y, a - 1]) - e^(ħ ϵ) (y g[x y, a])) -
     (t - 2 ϵ a) ∂y (y g[x y, a]) + y ϵ ∂{y,2} (y g[x y, a]) - (1 - e^(ħ (t-2 ϵ a)))/ħ)] //. x y → z
```

$$-1+e^{(t-2a\epsilon)\hbar}+z\hbar\, g[z,-1+a]-\left(t+e^{\epsilon\hbar}z-2a\epsilon\right)\hbar\, g[z,a]+z\hbar\left(\left(-t+2\left(1+a\right)\epsilon\right)g^{(1,0)}[z,a]+z\epsilon\, g^{(2,0)}[z,a]\right)$$

```
eqng = FullSimplify@PowerExpand[
  Simplify[
   ℏ (x ((y g[Log[x y], a - 1]) - e^(ℏ ϵ) (y g[Log[x y], a])) -
       (t - 2 ϵ a) ∂_y (y g[Log[x y], a]) + y ϵ ∂_{y,2} (y g[Log[x y], a])) - (1 - e^(ℏ (t-2 ϵ a)))
   ] //. x y → e^z
  ]
```

$$-1 + e^{(t-2a\epsilon)\hbar} + \hbar\left(e^z g[z, -1+a] - \left(e^{z+\epsilon\hbar} + t - 2a\epsilon\right) g[z, a] + \left(-t + \epsilon + 2a\epsilon\right) g^{(1,0)}[z, a] + \epsilon\, g^{(2,0)}[z, a]\right)$$

```
FullSimplify[eqng /. ϵ → 0]
```

$$-1 + e^{t\hbar} + e^z \hbar\, g[z, -1+a] - \hbar\left(\left(e^z + t\right) g[z, a] + t\, g^{(1,0)}[z, a]\right)$$