Pushforwards on Sep 14

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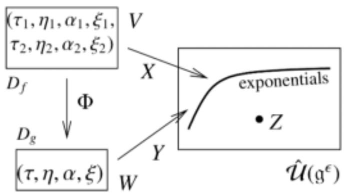
Cheat Sheet Pushforwards

http://drorbn.net/AcademicPensieve/2017-09/
modified September 13, 2017.



Definition. PS :=(Power Series).

For a vector space V, let $\mathcal{D}_0(V)$ denote the space of distributions on V whose support is $\{0\}$. Via the Laplace transform $\mathcal{D}_0(V)$ can be identified with $\mathcal{S}(V)$; we have $\mathcal{L}_V\colon \mathcal{D}_0(V)\to\mathcal{S}(V)$.

Challenge. With $\Phi\colon V\to W$ a PS map near 0 (so $\Phi\in \mathrm{mor}_{PS}(V\to W) := W\otimes\mathcal{S}^+(V^*)$) and with $D_f\in\mathcal{D}_0(V)$, understand $\Phi_*D_f\in\mathcal{D}_0(W)$.

Challenge. With $\Phi=(\phi_j(\alpha_i))$ and $Z=\zeta(\partial_{\alpha_i})$, set $\Phi_*Z := \mathbb{e}^{\sum\partial_{\beta_j}\phi_j(\partial_{a_i})}\zeta(a_i)\big|_{a_i=0}$. With $(a_i,y_i,x_i,t_i) := (\partial_{\alpha_i},\partial_{\eta_i},\partial_{\xi_i},\partial_{\tau_i})$, compute/implement Φ_*Z, with

$$Z=\omega\exp\left(\sum\lambda_{ij}t_ia_j+\sum q_{ij}y_ix_j+\epsilon P_0\right),$$

$\lambda_{ij}\in\mathbb{Z}$, $\omega, q_{ij}\in R:=\mathbb{Q}(T_i=\mathbb{e}^{t_i})$, $P_0\in R[a_i,y_i,x_i]$, and

$$\begin{aligned}\Phi^*(\bar\alpha_i) &= \sum\psi^1_{ij}\alpha_j+\epsilon P_1,\\ \Phi^*(\bar\eta_i) &= \sum\psi^2_{ij}\eta_j+\epsilon P_2,\\ \Phi^*(\bar\xi_i) &= \sum\psi^3_{ij}\xi_j+\epsilon P_3,\\ \Phi^*(\bar\tau_i) &= \sum\psi^4_{ij}\tau_j+\sum\gamma_{ij}\eta_i\xi_j+\epsilon P_4,\end{aligned}$$

$\psi^{1,4}_{ij}\in\mathbb{Z}$, $\psi^{2,3}\in R$, $P_{1,4}\in\mathbb{Q}[x_i,y_i]$, $P_{2,3}\in R[x_i,y_i]$, $\gamma_{ij}\in R$.

Example. 2017-07/Multi-beta-yax.nb: In $\mathcal{U}_{\gamma^{-1};\gamma\beta}$ where $q=\mathbb{e}^\beta$, $\prod_{i=1}^2\mathbb{e}^{\eta_iy}\mathbb{e}^{\alpha_ia}\mathbb{e}^{\xi_ix}=\mathbb{e}^{\eta y}\mathbb{e}^{\alpha a}\mathbb{e}^{\xi x}\mathbb{e}^{\tau t}$, with

$$\begin{aligned}\eta &= \eta_1+\eta_2e^{-\gamma\alpha_1}-\beta\gamma\eta_2^2\xi_1e^{-\gamma\alpha_1}+\ldots=\eta_1+\delta\eta_2e^{\beta-\alpha_1\gamma}\\ \alpha &= \alpha_1+\alpha_2+2\beta\eta_2\xi_1+\ldots=\alpha_1+\alpha_2-2\left(\beta+\log\delta\right)/\gamma\\ \xi &= \xi_1e^{-\gamma\alpha_2}+\xi_2-\beta\gamma\eta_2\xi_1^2e^{-\gamma\alpha_2}+\ldots=\delta\xi_1e^{\beta-\alpha_2\gamma}+\xi_2\\ \tau &= -\eta_2\xi_1+\beta\eta_2\xi_1\left(\gamma\eta_2\xi_1+1\right)/2+\ldots=\left(\beta+\log\delta\right)/(\beta\gamma)\end{aligned}$$

and $\delta:=\left(\left(e^\beta-1\right)\gamma\eta_2\xi_1+e^\beta\right)^{-1}=1-(1+\gamma\eta_1\xi_1)\beta+\ldots$.

Include Lemma 3 from QuantizedLogos.nb.

t is central or class-0

x,y,z is/are ordinary or class-1

a is Cartan or class-∞

$$V=V_0\oplus V_1\oplus V_\infty \qquad W=W_0\oplus W_1\oplus W_\infty$$

$$\Phi: V\longrightarrow W \quad \text{is} \quad \begin{pmatrix}\Phi_0\\ \Phi_1\\ \Phi_\infty\end{pmatrix}$$

where 0. Φ_1 & Φ_∞ are indep of V_0

Φ_0 is affine linear in V_0

F depends arbitrarily on V_0

∞.