Pushforwards on Sep 13

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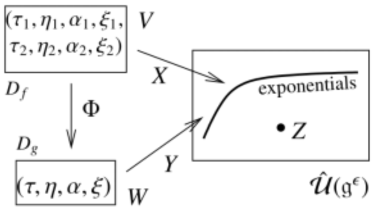
Cheat Sheet Pushforwards

http://drorbn.net/AcademicPensieve/2017-09/
modified September 13, 2017.

$f \in S(V)$



$g \in S(W)$

Challenge. With $\Phi = (\phi_j(\alpha_i))$ and $Z = \zeta(\partial_{\alpha_i})$, set $\Phi_* Z := \left. e^{\sum \partial_{\beta_j} \phi_j(\partial_{a_i})} \zeta(a_i) \right|_{a_i = 0}$. With $(a_i, y_i, x_i, t_i) := (\partial_{\alpha_i}, \partial_{\eta_i}, \partial_{\xi_i}, \partial_{\tau_i})$, compute/implement $\Phi_* Z$, with

$$Z = \omega \exp\left(\sum \lambda_{ij} t_i a_j + \sum q_{ij} y_i x_j + \epsilon P_0 \right),$$

$\lambda_{ij} \in \mathbb{Z}$, $\omega, q_{ij} \in R := \mathbb{Q}(T_i = e^{t_i})$, $P_0 \in R[a_i, y_i, x_i]$, and

$$\begin{aligned}
\Phi^*(\bar{\alpha}_i) &= \sum \psi^1_{ij} \alpha_j + \epsilon P_1, \\
\Phi^*(\bar{\eta}_i) &= \sum \psi^2_{ij} \eta_j + \epsilon P_2, \\
\Phi^*(\bar{\xi}_i) &= \sum \psi^3_{ij} \xi_j + \epsilon P_3, \\
\Phi^*(\bar{\tau}_i) &= \sum \psi^4_{ij} \tau_j + \sum \gamma_{ij} \eta_i \xi_j + \epsilon P_4,
\end{aligned}$$

$\psi^{1,4}_{ij} \in \mathbb{Z}$, $\psi^{2,3} \in R$, $P_{1,4} \in \mathbb{Q}[x_i, y_i]$, $P_{2,3} \in R[x_i, y_i]$, $\gamma_{ij} \in R$.

Example. 2017-07/Multi-beta-yax.nb: In $\mathcal{U}_{\gamma^{-1};\gamma\beta}$ where $q = e^\beta$, $\prod_{i=1}^2 e^{\eta_i y} e^{\alpha_i a} e^{\xi_i x} = e^{\eta y} e^{\alpha a} e^{\xi x} e^{\tau t}$, with

$$\begin{aligned}
\eta &= \eta_1 + \eta_2 e^{-\gamma\alpha_1} - \beta\gamma\eta_2^2\xi_1 e^{-\gamma\alpha_1} + \ldots = \eta_1 + \delta\eta_2 e^{\beta - \alpha_1\gamma} \\
\alpha &= \alpha_1 + \alpha_2 + 2\beta\eta_2\xi_1 + \ldots = \alpha_1 + \alpha_2 - 2(\beta + \log\delta)/\gamma \\
\xi &= \xi_1 e^{-\gamma\alpha_2} + \xi_2 - \beta\gamma\eta_2\xi_1^2 e^{-\gamma\alpha_2} + \ldots = \delta\xi_1 e^{\beta - \alpha_2\gamma} + \xi_2 \\
\tau &= -\eta_2\xi_1 + \beta\eta_2\xi_1(\gamma\eta_2\xi_1 + 1)/2 + \ldots = (\beta + \log\delta)/(\beta\gamma)
\end{aligned}$$

and $\delta := \left(\left(e^\beta - 1\right)\gamma\eta_2\xi_1 + e^\beta \right)^{-1} = 1 - (1 + \gamma\eta_1\xi_1)\beta + \ldots$.

$V \xrightarrow{\Phi} W$ $\qquad$ $\Phi \in \mathrm{End}(W^*, S(V^*))$

$S(V) \longrightarrow$ $\qquad$ $= W \otimes S(V^*)$

$\mathrm{Mor}(V, W) = W \otimes S^+(V^*)$ $\qquad$ S^+: no degree 0 parts.

$f \in S(V) \longleftrightarrow D_f \in \mathcal{D}_0(V)$ "distributions supported at 0"

$\xleftarrow{\mathcal{L}_V}$