Andras Szenes on Integrating over Hilbert schemes

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$\mathrm{Fun}(\mathbb{A}^d) = \mathbb{C}[x_1, \dots x_d]$

$n \longrightarrow \mathrm{Hilb}^n = \{I \subset R : \mathrm{codim} = n\}$

$\supset \{S \subset \mathbb{A}^d : |S| = n\}$

$\cup$

$\mathrm{Hilb}_0^n = \{I \subset R_0 : \mathrm{supp}\, I = 0\}$ $\quad \mathrm{Supp}\, I = \{p : \forall f \in I,\ f(p) = 0\}$

At $d=1$ $\mathrm{Hilb}^n = \{I_f : f = x^n + a_{n-1}x^{n-1} + \dots + a_0\} \cong \mathbb{C}^n$.

At $d=2$ Hilb^n is smooth, $\dim = 2n$

$\dim \mathrm{Hilb}_0^n = n-1$ & it is $\underbrace{\ }_{\text{often}}$ singular.