Graphical Pushforwards

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(170805) With $\Phi = (\phi_j(\alpha_i))$ and $Z = \zeta(\partial_{\alpha_i})$, set $\Phi_* Z := e^{\sum \partial_{\beta_j}\phi_j(\partial_{\alpha_i})}\zeta(a_i)\Big|_{a_i=0}$. **Challenge.** With $(a_i, y_i, x_i, t_i) := (\partial_{\alpha_i}, \partial_{\eta_i}, \partial_{\xi_i}, \partial_{\tau_i})$, compute/implement $\Phi_* Z$, with

$$Z = \omega \exp\left(\sum \lambda_{ij} t_i a_j + \sum q_{ij} y_i x_j + \epsilon P_0\right),$$

$\lambda_{ij} \in \mathbb{Z}$, $\omega, q_{ij} \in R := \mathbb{Q}(T_i = e^{t_i})$, $P_0 \in R[a_i, y_i, x_i]$, and

$$\begin{aligned}
\Phi^*(\bar{\alpha}_i) &= \sum \psi^1_{ij}\alpha_j + \epsilon P_1,\\
\Phi^*(\bar{\eta}_i) &= \sum \psi^2_{ij}\eta_j + \epsilon P_2,\\
\Phi^*(\bar{\xi}_i) &= \sum \psi^3_{ij}\xi_j + \epsilon P_3,\\
\Phi^*(\bar{\tau}_i) &= \sum \psi^4_{ij}\tau_j + \sum \gamma_{ij}\eta_i\xi_j + \epsilon P_4,
\end{aligned}$$

$\psi^{1,4}_{ij} \in \mathbb{Z}$, $\psi^{2,3} \in R$, $P_{1,4} \in \mathbb{Q}[x_i, y_i]$, $P_{2,3} \in R[x_i, y_i]$, $\gamma_{ij} \in R$.

The Z side: The $\emptyset$ side: