Convergence issues, I

I need a Convergence theorem!

August 10, 2017 12:38 AM

Dror Bar-Natan: Talks: Sydney-1708: **Poly-Poly Extras**

ωεβ:=http://drorbn.net/Sydney-1708/
Slides w/ no URL should be banned!

Warning. Conventions on this page change randomly from line to line.

The Algebra. $\mathcal{U}_{\hbar;\alpha\beta}$ conventions: $q=\mathbb{e}^{\hbar\alpha\beta}$, $H=\langle a,x\rangle/([a,x]=\alpha x)$ with

$$A=\mathbb{e}^{-\hbar\beta a},\quad xA=qAx,\quad S(a,A,x)=(-a,A^{-1},-A^{-1}x),$$
$$\Delta(a,A,x)=(a_1+a_2,A_1A_2,x_1+A_1x_2)$$

and dual $H^*=\langle b,y\rangle/([b,y]=-\beta y)$ with

$$B=\mathbb{e}^{-\hbar\alpha b},\quad By=qyB,\quad S(b,B,y)=(-b,B^{-1},-yB^{-1}),$$
$$\Delta(b,B,y)=(b_1+b_2,B_1B_2,y_1B_2+y_2).$$

Pairing by $(a,x)^*=\hbar(b,y)$ making $\langle y^lb^j,a^jx^k\rangle=\delta_{ij}\delta_{kl}i![k]_q!$. Then $\mathcal{U}=H^{*cop}\otimes H$ with $(\phi f)(\psi g)=\langle\psi_1S^{-1}f_3\rangle\langle\psi_3,f_1\rangle(\phi\psi_2)(f_2g)$. With the central $t:=\beta a-\alpha b$, $T:=e^{\hbar t}=A^{-1}B$ get

$$[a,y]=-\alpha y,\quad xy-qyx=(1-TA^2)/\hbar.$$

Benkart-Witherspoon, 2017-06/BW.nb: At $\alpha\beta\hbar=\sigma-\rho$, represented by $y\to\begin{pmatrix}0&0\\-\mathbb{e}^{\rho}&0\end{pmatrix}$, $a\to\frac{\alpha}{\rho-\sigma}\begin{pmatrix}\rho&0\\0&\sigma\end{pmatrix}$, $A\to\begin{pmatrix}\mathbb{e}^{\rho}&0\\0&\mathbb{e}^{\sigma}\end{pmatrix}$, $x\to\frac{\mathbb{e}^{\rho}-\mathbb{e}^{\sigma}}{\hbar\mathbb{e}^{\rho+\sigma}}\begin{pmatrix}0&1\\0&0\end{pmatrix}$, $t\to\frac{\rho+\sigma}{\hbar}\begin{pmatrix}1&0\\0&1\end{pmatrix}$, $T\to\frac{1}{\mathbb{e}^{\rho+\sigma}}\begin{pmatrix}1&0\\0&1\end{pmatrix}$, $b\to\frac{\beta}{\sigma-\rho}\begin{pmatrix}\sigma&0\\0&\rho\end{pmatrix}$, $B\to\begin{pmatrix}\mathbb{e}^{-\sigma}&0\\0&\mathbb{e}^{-\rho}\end{pmatrix}$.

The R-Matrix. With $[n]_q:=(q^n-1)/(q-1)$, $[n]_q!:=[1]q\cdot\ldots\cdot[n]_q$ and $\mathbb{e}_q^x:=\sum_{n\geq 0}\frac{x^n}{[n]_q!}$, we have the mysterious Quesne formula of arXiv:math-ph/0305003: $\mathbb{e}_q^x=\mathbb{e}^x\exp\left(\sum_{k\geq 2}\frac{(1-q)^kx^k}{k(1-q^k)}\right)$. Then $R_{ij}:=\mathbb{O}\left(yb\otimes ax:\mathbb{e}^{\hbar ba}\mathbb{e}_q^{\hbar yx}\right)$.

ħ-convergent?
tybβ-convergent?

***xa* Swaps.**

***ay* Swaps.**

***xy* Swaps.**

The Drinfel'd Element.

Putting Everything Together.

Non ħ homogenuous.

Non tybβ homogeneous at ħ=1

Q: is it enough to consider $\hbar=\alpha^{-1}$?

what was I thinking when I made that choice?

All relations are ħ/ybβ/xaα-balanced!

q & R are likewise.