Anna Lachowska, The center of the small quantum group in type A

August 30, 2017 6:01 AM

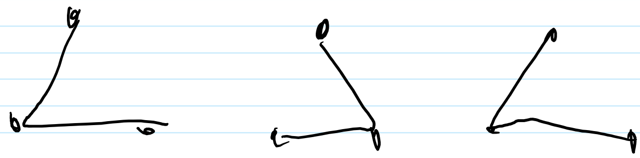
Diagonal Co-Invariants by Haiman

$$DC_n = \mathbb{C}[x_1 \dots x_n, y_1 \dots y_n] / \mathbb{C}[x_1 \dots x_n, y_1 \dots y_n]^{S_n}_+$$

where for $\sigma \in S_n$, $\sigma x_i = x_{\sigma i}$, $\sigma y_i = y_{\sigma i}$

$$\Rightarrow \dim DC_n = (n+1)^{n-1}$$

$$DC_2 = \langle \quad \rangle / \begin{matrix} \cancel{x_1+x_2} & x_1x_2 & x_1y_1+x_2y_2 \\ \cancel{y_1+y_2} & y_1y_2 & x_1y_2+x_2y_1 \end{matrix} \Big\} 6$$

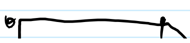


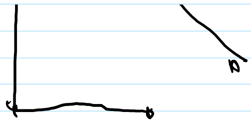
DC_3

$$\frac{4\cdot 3\cdot 2}{2} = 12$$

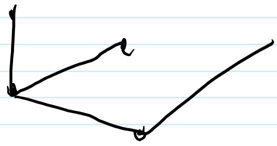
4

DC_4





60



$5 \cdot 4 \cdot 3 = 60$

5.

A COMBINATORIAL FORMULA FOR THE CHARACTER OF THE DIAGONAL COINVARIANTS

J. HAGLUND, M. HAIMAN, N. LOEHR, J. B. REMMEL, AND A. ULYANOV

ABSTRACT. Let R_n be the ring of coinvariants for the diagonal action of the symmetric group S_n. It is known that the character of R_n as a doubly-graded S_n module can be expressed using the Frobenius characteristic map as ∇e_n, where e_n is the n-th elementary symmetric function, and ∇ is an operator from the theory of Macdonald polynomials.

We conjecture a combinatorial formula for ∇e_n and prove that it has many desirable properties which support our conjecture. In particular, we prove that our formula is a symmetric function (which is not obvious) and that it is Schur positive. These results make use of the theory of ribbon tableau generating functions of Lascoux, Leclerc and Thibon. We also show that a variety of earlier conjectures and theorems on ∇e_n are special cases of our conjecture.

Finally, we extend our conjectures on ∇e_n and several of the results supporting them to higher powers $\nabla^m e_n$.

1. INTRODUCTION

1.1. Let R_n be the ring of coinvariants for the diagonal action of the symmetric group S_n on $\mathbb{C}^n \oplus \mathbb{C}^n$. In other words,

$$R_n = \mathbb{C}[\mathbf{x}, \mathbf{y}]/I, \tag{1}$$

where $\mathbb{C}[\mathbf{x}, \mathbf{y}] = \mathbb{C}[x_1, y_1, \ldots, x_n, y_n]$ is the ring of polynomial functions on $\mathbb{C}^n \oplus \mathbb{C}^n$, the symmetric group acts "diagonally" (*i.e.*, permuting the x and y variables simultaneously), and the ideal $I = ((\mathbf{x}, \mathbf{y}) \cap \mathbb{C}[\mathbf{x}, \mathbf{y}]^{S_n})$ is generated by all S_n-invariant polynomials without constant term. The S_n action respects the double grading

$$R_n = \bigoplus_{r,s} (R_n)_{r,s} \tag{2}$$

given by the x and y degrees.