# The bch co-product

July 14, 2017 9:47 AM

(on $g^*$ as an Abelian alg.)

In $[a,x]=\gamma x$

$bch \in S(g^*)\otimes S(g^*)\otimes g$

$bch^* \in g^*\otimes S(g)\otimes S(g)$

$\Delta(q)$

$= Hom(g, S(g^*)\otimes S(g^*))$

$q\in g^*$ $p\in g$

$= Hom(g, U(g^*)\otimes U(g^*))$

co products need to satisfy $\Delta([q_1,q_2])=[\Delta(q_1),\Delta(q_2)]$

For bch both sides are 0,

$$e^{p_1}e^{p_2}=e^{bch(p_1,p_2)}$$

In bch2D.nb:

```
In[28]:= 𝔼[M_] /; MatrixQ[M] := MatrixExp[M];
         a = (γ 0
              0 0); x = (0 1
                         0 0);
         a.x - x.a == γ x
Out[30]= True

In[32]:= 𝔼[α a + ξ x] // Simplify // MatrixForm
Out[32]//MatrixForm=
```

$$\begin{pmatrix} e^{\alpha\gamma} & \frac{(-1+e^{\alpha\gamma})\xi}{\alpha\gamma} \\ 0 & 1 \end{pmatrix}$$

In[33]:= $\mathbb{E}[\alpha_1 a+\xi_1 x].\mathbb{E}[\alpha_2 a+\xi_2 x] == \mathbb{E}\left[(\alpha_1+\alpha_2)\,a+\frac{(\alpha_1+\alpha_2)\left((-1+e^{\gamma\alpha_1})\,\alpha_2\,\xi_1+e^{\gamma\alpha_1}(-1+e^{\gamma\alpha_2})\,\alpha_1\,\xi_2\right)}{(-1+e^{\gamma(\alpha_1+\alpha_2)})\,\alpha_1\,\alpha_2}\,x\right]$ // Simplify

Out[33]= True

$$\frac{(\alpha_1+\alpha_2)\left((-1+e^{\gamma\alpha_1})\,\alpha_2\,\xi_1+e^{\gamma\alpha_1}(-1+e^{\gamma\alpha_2})\,\alpha_1\,\xi_2\right)}{(-1+e^{\gamma(\alpha_1+\alpha_2)})\,\alpha_1\,\alpha_2}$$

After rescaling $\xi$, this is $\xi_1+e^{\gamma\alpha_1}\xi_2$