

Determining the Pairing

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claim A Hopf pairing $\langle, \rangle: A \otimes B \rightarrow \mathbb{F}$ between two Hopf algebras A & B satisfying (*) is determined by its values on multiplicative generators of A & B .

Proof

$\begin{matrix} A & B \\ \langle \text{one}, \text{one} \rangle \end{matrix}$
 \Downarrow determines
 $\langle \text{one}, \text{many} \rangle$
 \Downarrow determines
 $\langle \text{many}, \text{many} \rangle$

(*) we need to know that in A
 $\Delta(\text{gen}) = \sum g_{\text{out}} \otimes g_{\text{in}}$

(*) in B is not needed here.

Also, if $a \in A$ & $b \in B$ are primitive, then

$$\langle e^a, e^b \rangle = e^{\langle a, b \rangle}$$

So for the purposes of the above, group-like elements count as "generators".