

Pensieve header: Working in the double of the 2D pencil, as determined in "Doubling.nb"; continued pensieve://2017-08/.

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SetDirectory["C:\\drorbn\\AcademicPensieve\\2017-06"];
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The "degree carrier" is \hbar , and all "coupling constants" are proportional to it.

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(*$TD=∞; ħ /: ħ^d.; /; d>$TD :=0;*)
$TβD = 2; β /: β^d.; /; d > $TβD := 0;
```

The Doubled Algebra $\mathcal{U}_{\hbar\alpha\beta}$

Change relative to "Doubling.nb": We use $t = \beta a - \alpha b$ and $T = e^{\hbar t} = A^{-1} B$ instead of b and B .

Implementing $\mathcal{U}_{\hbar\alpha\beta}$

With $q = e^{\hbar\alpha\beta}$, $A = e^{-\hbar\beta a}$, and $[f, g]_q := fg - qgf$, our algebra is $\mathcal{U}_{\hbar\alpha\beta} = \langle t, y, a, x \rangle / \mathcal{R}$, where $\mathcal{R} = ([t, *] = 0, [a, y] = -\alpha y, [x, y]_q = \hbar^{-1}(1 - TA^2), [x, a] = -\alpha x)$.

$$\text{Series}\left[\hbar^{-1} \left(1 - e^{\hbar t - 2\hbar\beta a}\right), \{\beta\beta, 0, 5\}\right]$$

$$-\frac{-1 + e^{t\hbar}}{\hbar} + 2a e^{t\hbar} \beta\beta - \frac{1}{2} (2^2 a^2 e^{t\hbar} \hbar) \beta\beta^2 +$$

$$\frac{1}{6} 2^3 a^3 e^{t\hbar} \hbar^2 \beta\beta^3 - \frac{1}{24} (2^4 a^4 e^{t\hbar} \hbar^3) \beta\beta^4 + \frac{1}{120} 2^5 a^5 e^{t\hbar} \hbar^4 \beta\beta^5 + O[\beta\beta]^6$$

```
q := Sum[(ħ α β)^k / k!, {k, 0, Min[$TβD]}];
AlgebraAtom = y | a | x;
PBWRule = {y → 1, a → 2, x → 3};
B[U@a, U@y] = -α U@y; B[U@x, U@a] = -α U@x;
(*B[U@x, U@y] = (q-1) UU[y, x] + UU@Sum[-(t-2 a β)^(k+1) ħ^k / (k+1)!, {k, 0, $TD}]; *)
B[U@x, U@y] := (q - 1) UU[y, x] + UU[ħ^-1 (1 - Sum[T (-2 a β ħ)^k / k!, {k, 0, $TβD}])];
```

```
x_ ≤ y_ := OrderedQ[{x, y} /. PBWRule]; x_ < y_ := ! OrderedQ[{y, x} /. PBWRule];
Simp[ε_] := Collect[ε, _U, Expand];
```

```
U_i[ε_] := ε /. {t → t_i, T → T_i, u_U → Replace[u, x_ → x_i, 1]};
B[U[(x_)_i], U[(y_)_i]] := B[U[x_i], U[y_i]] = U_i[B[U@x, U@y]];
B[U[(x_)_i], U[(y_)_j]] /; i != j := 0;
B[x_, x_] = 0;
B[U[y_], U[x_]] = Simp[-B[U[x], U[y]]];
B[x_, y_] := x**y - y**x;
```

```

Unprotect[NonCommutativeMultiply];
NonCommutativeMultiply[x_] := x;
0 ** _ = _ ** 0 = 0;
x_ ** U[] := x; U[] ** x_ := x;
(a_ * x_U) ** (b_ * y_U) := If[ab === 0, 0, Simp[ab (x**y)]];
(a_ * x_U) ** y_ := Simp[a (x**y)]; x_ ** (a_ * y_U) := Simp[a (x**y)];
(x_Plus) ** y_ := (#**y) & /@ x; x_ ** (y_Plus) := (x**#) & /@ y;

```

```

U[xx____, x_] ** U[y_, yy____] :=
  If[x ≤ y, U[xx, x, y, yy], U@xx ** (U@y ** U@x + B[U@x, U@y]) ** U@yy];

```

```

UU[c_. * (L : AlgebraAtom)^n_, r____] /; FreeQ[c, AlgebraAtom] :=
  Expand[c UU[Sequence @@ Table[L, {n}], r]];
UU[c_. * L : AlgebraAtom, r____] := Expand[c U[L] ** UU[r]];
UU[c_, r____] /; FreeQ[c, AlgebraAtom] := Expand[c UU[r]];
UU[] = U[];
UU[L_Plus, r____] := UU[#, r] & /@ L;
UU[L_, r____] := UU[Expand[L], r];

```

```

O[poly_, specs____] := Module[{vs, us, z},
  vs = Join @@ (First /@ {specs});
  us = Join @@ ({specs} /. (L_ -> s_) -> (L /. x_i_ -> x_s));
  Simp@Total[CoefficientRules[Normal@Series[poly, {ħ, 0, $TD}], vs] /.
    (p_ -> c_) -> c UU @@ (us^p)
]

```

$\$T\beta D = 5$; $B[U@x, U@y]$ // Simp

$$\left(\frac{1}{\hbar} - \frac{T}{\hbar}\right) U[] + 2 T \beta U[a] - 2 T \beta^2 \hbar U[a, a] + \left(\alpha \beta \hbar + \frac{1}{2} \alpha^2 \beta^2 \hbar^2 + \frac{1}{6} \alpha^3 \beta^3 \hbar^3 + \frac{1}{24} \alpha^4 \beta^4 \hbar^4 + \frac{1}{120} \alpha^5 \beta^5 \hbar^5\right) U[y, x] + \frac{4}{3} T \beta^3 \hbar^2 U[a, a, a] - \frac{2}{3} T \beta^4 \hbar^3 U[a, a, a, a] + \frac{4}{15} T \beta^5 \hbar^4 U[a, a, a, a, a]$$

$\$T\beta D = 2$; $z1 = U[y, y, a, a, x, x]$; $z2 = U[y, a, x]$; $z3 = U[y, y, a, x]$;

$z1 ** (z2 ** z3) - (z1 ** z2) ** z3$ // Simp

0

$adx[\mathcal{E}__] := B[U@x, \mathcal{E}_]$;

$\$T\beta D = 2$; Simp[NestList[adx, U@y, $\$T\beta D + 2$]]

$$\{U[y], \left(\frac{1}{\hbar} - \frac{T}{\hbar}\right) U[] + 2 T \beta U[a] - 2 T \beta^2 \hbar U[a, a] + \left(\alpha \beta \hbar + \frac{1}{2} \alpha^2 \beta^2 \hbar^2\right) U[y, x], \left(\alpha \beta - 3 T \alpha \beta + \frac{1}{2} \alpha^2 \beta^2 \hbar - \frac{5}{2} T \alpha^2 \beta^2 \hbar\right) U[x] + 6 T \alpha \beta^2 \hbar U[a, x] + \alpha^2 \beta^2 \hbar^2 U[y, x, x], \left(\alpha^2 \beta^2 \hbar - 7 T \alpha^2 \beta^2 \hbar\right) U[x, x], 0\}$$

```

G = Simp[NestList[adx, U@y, $TβD + 1].Table[μk/k!, {k, 0, $TβD + 1}]]
( $\frac{\mu}{\hbar} - \frac{T\mu}{\hbar}$ ) U[] + 2 T β μ U[a] + ( $\frac{1}{2} \alpha \beta \mu^2 - \frac{3}{2} T \alpha \beta \mu^2 + \frac{1}{4} \alpha^2 \beta^2 \mu^2 \hbar - \frac{5}{4} T \alpha^2 \beta^2 \mu^2 \hbar$ ) U[x] +
U[y] - 2 T β2 μ ħ U[a, a] + 3 T α β2 μ2 ħ U[a, x] + ( $\frac{1}{6} \alpha^2 \beta^2 \mu^3 \hbar - \frac{7}{6} T \alpha^2 \beta^2 \mu^3 \hbar$ ) U[x, x] +
( $\alpha \beta \mu \hbar + \frac{1}{2} \alpha^2 \beta^2 \mu \hbar^2$ ) U[y, x] +  $\frac{1}{2} \alpha^2 \beta^2 \mu^2 \hbar^2$  U[y, x, x]

$TβD = 0;
adx[ε_] := B[U@x, ε];
G = Simp[NestList[adx, U@y, $TβD + 1].Table[μk/k!, {k, 0, $TβD + 1}]];
F0 = e $-\frac{(-\mu+T\mu)\nu}{\hbar}$ ;
F = Sum[F0 f1,i,j,k[ν] β1 UU[yi, aj, xk],
  {1, 0, $TβD}, {i, 0, 1}, {j, 0, 1}, {k, 0, Min[1, 2 1 - i - j]}];
ownows = Cases[F, f___[ν], ∞];
bas = Union@@Table[β1 Cases[Coefficient[F, β, 1], _U, ∞], {1, 0, $TβD}];
eqns = Flatten[{(Coefficient[F - U[], #] /. ν → 0) == 0,
  Expand[F0-1 Coefficient[Simp[F ** G - U[y] ** F - ∂νF], #] == 0} & /@ bas];
{sol} = DSolve[eqns, ownows, ν];
FF = Simp[F0-1 F /. sol];
LFF = Simp@Sum[
  SeriesCoefficient[Log[1 + x], {x, 0, k}] * NonCommutativeMultiply@@Table[FF - U[], {k}],
  {k, 0, $TβD}]
0

$TβD = 1;
adx[ε_] := B[U@x, ε];
G = Simp[NestList[adx, U@y, $TβD + 1].Table[μk/k!, {k, 0, $TβD + 1}]];
F0 = e $-\frac{(-\mu+T\mu)\nu}{\hbar}$ ;
F = Sum[F0 f1,i,j,k[ν] β1 UU[yi, aj, xk],
  {1, 0, $TβD}, {i, 0, 1}, {j, 0, 1}, {k, 0, Min[1, 2 1 - i - j]}];
ownows = Cases[F, f___[ν], ∞];
bas = Union@@Table[β1 Cases[Coefficient[F, β, 1], _U, ∞], {1, 0, $TβD}];
eqns = Flatten[{(Coefficient[F - U[], #] /. ν → 0) == 0,
  Expand[F0-1 Coefficient[Simp[F ** G - U[y] ** F - ∂νF], #] == 0} & /@ bas];
{sol} = DSolve[eqns, ownows, ν];
FF = Simp[F0-1 F /. sol];
LFF = Simp@Sum[
  SeriesCoefficient[Log[1 + x], {x, 0, k}] * NonCommutativeMultiply@@Table[FF - U[], {k}],
  {k, 0, $TβD}]
( $\frac{\alpha \beta \mu^2 \nu^2}{4 \hbar} - \frac{T \alpha \beta \mu^2 \nu^2}{\hbar} + \frac{3 T^2 \alpha \beta \mu^2 \nu^2}{4 \hbar}$ ) U[] + 2 T β μ ν U[a] +
( $\frac{1}{2} \alpha \beta \mu^2 \nu - \frac{3}{2} T \alpha \beta \mu^2 \nu$ ) U[x] + ( $\frac{1}{2} \alpha \beta \mu \nu^2 - \frac{3}{2} T \alpha \beta \mu \nu^2$ ) U[y] + α β μ ν ħ U[y, x]

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```

$TβD = 2;
adx[ε_] := B[U@x, ε];
G = Simp[NestList[adx, U@y, $TβD + 1].Table[μk/k!, {k, 0, $TβD + 1}]]];
F0 = e- $\frac{(-μ+Tμ)v}{\hbar}$ ;
F = Sum[F0 f1,i,j,k[v] β1 UU[yi, aj, xk],
  {1, 0, $TβD}, {i, 0, 1}, {j, 0, 1}, {k, 0, Min[1, 2 1 - i - j]}];
unowns = Cases[F, f___[v], ∞];
bas = Union@@Table[β1 Cases[Coefficient[F, β, 1], _U, ∞], {1, 0, $TβD}];
eqns = Flatten[{(Coefficient[F - U[], #] /. v → 0) == 0,
  Expand[F0-1 Coefficient[Simp[F ** G - U[y] ** F - ∂vF], #] == 0} & /@ bas];
{sol} = DSolve[eqns, unowns, v];
FF = Simp[F0-1 F /. sol];
LFF = Simp@Sum[
  SeriesCoefficient[Log[1 + x], {x, 0, k}] * NonCommutativeMultiply@@Table[FF - U[], {k}],
  {k, 0, $TβD}]

```

$$\left(\frac{1}{8} \alpha^2 \beta^2 \mu^2 v^2 - \frac{3}{4} T \alpha^2 \beta^2 \mu^2 v^2 + \frac{5}{8} T^2 \alpha^2 \beta^2 \mu^2 v^2 + \frac{\alpha \beta \mu^2 v^2}{4 \hbar} - \frac{T \alpha \beta \mu^2 v^2}{\hbar} + \right.$$

$$\left. \frac{3 T^2 \alpha \beta \mu^2 v^2}{4 \hbar} + \frac{\alpha^2 \beta^2 \mu^3 v^3}{72 \hbar} - \frac{5 T \alpha^2 \beta^2 \mu^3 v^3}{24 \hbar} + \frac{5 T^2 \alpha^2 \beta^2 \mu^3 v^3}{24 \hbar} - \frac{T^3 \alpha^2 \beta^2 \mu^3 v^3}{72 \hbar} \right) U[] +$$

$$(2 T \beta \mu v + 2 T \alpha \beta^2 \mu^2 v^2 - 3 T^2 \alpha \beta^2 \mu^2 v^2) U[a] + \left(\frac{1}{2} \alpha \beta \mu^2 v - \frac{3}{2} T \alpha \beta \mu^2 v + \frac{1}{6} \alpha^2 \beta^2 \mu^3 v^2 - \right.$$

$$\left. \frac{4}{3} T \alpha^2 \beta^2 \mu^3 v^2 + \frac{7}{6} T^2 \alpha^2 \beta^2 \mu^3 v^2 + \frac{1}{4} \alpha^2 \beta^2 \mu^2 v \hbar - \frac{5}{4} T \alpha^2 \beta^2 \mu^2 v \hbar \right) U[x] +$$

$$\left(\frac{1}{2} \alpha \beta \mu v^2 - \frac{3}{2} T \alpha \beta \mu v^2 + \frac{1}{6} \alpha^2 \beta^2 \mu^2 v^3 - \frac{4}{3} T \alpha^2 \beta^2 \mu^2 v^3 + \frac{7}{6} T^2 \alpha^2 \beta^2 \mu^2 v^3 + \right.$$

$$\left. \frac{1}{4} \alpha^2 \beta^2 \mu v^2 \hbar - \frac{5}{4} T \alpha^2 \beta^2 \mu v^2 \hbar \right) U[y] - 2 T \beta^2 \mu v \hbar U[a, a] +$$

$$3 T \alpha \beta^2 \mu^2 v \hbar U[a, x] + \left(\frac{1}{6} \alpha^2 \beta^2 \mu^3 v \hbar - \frac{7}{6} T \alpha^2 \beta^2 \mu^3 v \hbar \right) U[x, x] +$$

$$3 T \alpha \beta^2 \mu v^2 \hbar U[y, a] +$$

$$\left(\alpha \beta \mu v \hbar + \frac{3}{4} \alpha^2 \beta^2 \mu^2 v^2 \hbar - \frac{11}{4} T \alpha^2 \beta^2 \mu^2 v^2 \hbar + \frac{1}{2} \alpha^2 \beta^2 \mu v \hbar^2 \right) U[y, x] +$$

$$\left(\frac{1}{6} \alpha^2 \beta^2 \mu v^3 \hbar - \frac{7}{6} T \alpha^2 \beta^2 \mu v^3 \hbar \right) U[y, y] +$$

$$\frac{1}{2} \alpha^2 \beta^2 \mu^2 v \hbar^2 U[y, x, x] + \frac{1}{2} \alpha^2 \beta^2 \mu v^2 \hbar^2 U[y, y, x]$$

```

$TβD = 3;
adx[ε_] := B[U@x, ε];
G = Simp[NestList[adx, U@y, $TβD + 1].Table[μk/k!, {k, 0, $TβD + 1}]]];
F0 = e- $\frac{(-\mu+T\mu)\nu}{\hbar}$ ;
F = Sum[F0 f1,i,j,k[ν] β1 UU[yi, aj, xk],
  {1, 0, $TβD}, {i, 0, 1}, {j, 0, 1}, {k, 0, Min[1, 2 1 - i - j]}];
unowns = Cases[F, f___[ν], ∞];
bas = Union@@Table[β1 Cases[Coefficient[F, β, 1], _U, ∞], {1, 0, $TβD}];
eqns = Flatten[{{Coefficient[F - U[], #] /. ν → 0} == 0,
  Expand[F0-1 Coefficient[Simp[F ** G - U[y] ** F - ∂νF], #] == 0} & /@ bas];
{sol} = DSolve[eqns, unowns, ν];
SortBy[sol, #[[2]] &]

```

$$\{f_{1,0,1,1}[\nu] \rightarrow 0, f_{1,1,1,0}[\nu] \rightarrow 0, f_{2,0,1,2}[\nu] \rightarrow 0, f_{2,0,2,1}[\nu] \rightarrow 0, f_{2,0,2,2}[\nu] \rightarrow 0, f_{2,1,1,2}[\nu] \rightarrow 0,$$

$$f_{2,1,2,0}[\nu] \rightarrow 0, f_{2,1,2,1}[\nu] \rightarrow 0, f_{2,2,1,0}[\nu] \rightarrow 0, f_{2,2,1,1}[\nu] \rightarrow 0, f_{2,2,2,0}[\nu] \rightarrow 0,$$

$$f_{3,0,1,3}[\nu] \rightarrow 0, f_{3,0,2,2}[\nu] \rightarrow 0, f_{3,0,2,3}[\nu] \rightarrow 0, f_{3,0,3,1}[\nu] \rightarrow 0, f_{3,0,3,2}[\nu] \rightarrow 0,$$

$$f_{3,0,3,3}[\nu] \rightarrow 0, f_{3,1,1,3}[\nu] \rightarrow 0, f_{3,1,2,2}[\nu] \rightarrow 0, f_{3,1,2,3}[\nu] \rightarrow 0, f_{3,1,3,0}[\nu] \rightarrow 0,$$

$$f_{3,1,3,1}[\nu] \rightarrow 0, f_{3,1,3,2}[\nu] \rightarrow 0, f_{3,2,1,3}[\nu] \rightarrow 0, f_{3,2,2,0}[\nu] \rightarrow 0, f_{3,2,2,1}[\nu] \rightarrow 0,$$

$$f_{3,2,2,2}[\nu] \rightarrow 0, f_{3,2,3,0}[\nu] \rightarrow 0, f_{3,2,3,1}[\nu] \rightarrow 0, f_{3,3,1,0}[\nu] \rightarrow 0, f_{3,3,1,1}[\nu] \rightarrow 0,$$

$$f_{3,3,1,2}[\nu] \rightarrow 0, f_{3,3,2,0}[\nu] \rightarrow 0, f_{3,3,2,1}[\nu] \rightarrow 0, f_{3,3,3,0}[\nu] \rightarrow 0, f_{0,0,0,0}[\nu] \rightarrow 1,$$

$$f_{1,0,1,0}[\nu] \rightarrow 2 T \mu \nu, f_{1,0,0,1}[\nu] \rightarrow -\frac{1}{2} (-\alpha \mu^2 + 3 T \alpha \mu^2) \nu, f_{1,1,0,0}[\nu] \rightarrow -\frac{1}{2} (-\alpha \mu + 3 T \alpha \mu) \nu^2,$$

$$f_{1,0,0,0}[\nu] \rightarrow \frac{(-1 + T) (-\alpha \mu^2 + 3 T \alpha \mu^2) \nu^2}{4 \hbar}, f_{1,1,0,1}[\nu] \rightarrow \alpha \mu \nu \hbar, f_{2,1,1,1}[\nu] \rightarrow 2 T \alpha \mu^2 \nu^2 \hbar,$$

$$f_{2,2,0,2}[\nu] \rightarrow \frac{1}{2} \alpha^2 \mu^2 \nu^2 \hbar^2, f_{3,2,1,2}[\nu] \rightarrow T \alpha^2 \mu^3 \nu^3 \hbar^2, f_{3,3,0,3}[\nu] \rightarrow \frac{1}{6} \alpha^3 \mu^3 \nu^3 \hbar^3,$$

$$f_{2,0,2,0}[\nu] \rightarrow 2 (T^2 \mu^2 \nu^2 - T \mu \nu \hbar), f_{2,0,1,1}[\nu] \rightarrow T \alpha \mu^3 \nu^2 - 3 T^2 \alpha \mu^3 \nu^2 + 3 T \alpha \mu^2 \nu \hbar,$$

$$f_{2,0,0,2}[\nu] \rightarrow \frac{1}{24} (3 \alpha^2 \mu^4 \nu^2 - 18 T \alpha^2 \mu^4 \nu^2 + 27 T^2 \alpha^2 \mu^4 \nu^2 + 4 \alpha^2 \mu^3 \nu \hbar - 28 T \alpha^2 \mu^3 \nu \hbar),$$

$$f_{2,1,1,0}[\nu] \rightarrow T \alpha \mu^2 \nu^3 - 3 T^2 \alpha \mu^2 \nu^3 + 3 T \alpha \mu \nu^2 \hbar,$$

$$f_{2,0,1,0}[\nu] \rightarrow \frac{1}{2 \hbar} (T \alpha \mu^3 \nu^3 - 4 T^2 \alpha \mu^3 \nu^3 + 3 T^3 \alpha \mu^3 \nu^3 + 4 T \alpha \mu^2 \nu^2 \hbar - 6 T^2 \alpha \mu^2 \nu^2 \hbar),$$

$$f_{2,2,0,0}[\nu] \rightarrow \frac{1}{24} (3 \alpha^2 \mu^2 \nu^4 - 18 T \alpha^2 \mu^2 \nu^4 + 27 T^2 \alpha^2 \mu^2 \nu^4 + 4 \alpha^2 \mu \nu^3 \hbar - 28 T \alpha^2 \mu \nu^3 \hbar),$$

$$f_{3,0,3,0}[\nu] \rightarrow \frac{4}{3} (T^3 \mu^3 \nu^3 - 3 T^2 \mu^2 \nu^2 \hbar + T \mu \nu \hbar^2),$$

$$f_{2,1,0,1}[\nu] \rightarrow \frac{1}{4} (2 \alpha^2 \mu^3 \nu^3 - 10 T \alpha^2 \mu^3 \nu^3 + 12 T^2 \alpha^2 \mu^3 \nu^3 + 5 \alpha^2 \mu^2 \nu^2 \hbar - 21 T \alpha^2 \mu^2 \nu^2 \hbar + 2 \alpha^2 \mu \nu \hbar^2),$$

$$f_{3,0,2,1}[\nu] \rightarrow T^2 \alpha \mu^4 \nu^3 - 3 T^3 \alpha \mu^4 \nu^3 - T \alpha \mu^3 \nu^2 \hbar + 9 T^2 \alpha \mu^3 \nu^2 \hbar - 3 T \alpha \mu^2 \nu \hbar^2,$$

$$f_{2,1,0,2}[\nu] \rightarrow \frac{1}{2} (\alpha^2 \mu^3 \nu^2 \hbar - 3 T \alpha^2 \mu^3 \nu^2 \hbar + \alpha^2 \mu^2 \nu \hbar^2),$$

$$f_{2,0,0,1}[\nu] \rightarrow \frac{1}{24 \hbar} (3 \alpha^2 \mu^4 \nu^3 - 21 T \alpha^2 \mu^4 \nu^3 + 45 T^2 \alpha^2 \mu^4 \nu^3 - 27 T^3 \alpha^2 \mu^4 \nu^3 + 10 \alpha^2 \mu^3 \nu^2 \hbar -$$

$$68 T \alpha^2 \mu^3 \nu^2 \hbar + 82 T^2 \alpha^2 \mu^3 \nu^2 \hbar + 6 \alpha^2 \mu^2 \nu \hbar^2 - 30 T \alpha^2 \mu^2 \nu \hbar^2), f_{3,0,1,2}[\nu] \rightarrow$$

$$\frac{1}{12} (3 T \alpha^2 \mu^5 \nu^3 - 18 T^2 \alpha^2 \mu^5 \nu^3 + 27 T^3 \alpha^2 \mu^5 \nu^3 + 22 T \alpha^2 \mu^4 \nu^2 \hbar - 82 T^2 \alpha^2 \mu^4 \nu^2 \hbar + 28 T \alpha^2 \mu^3 \nu \hbar^2),$$

$$f_{3,0,0,3}[\nu] \rightarrow \frac{1}{48} (\alpha^3 \mu^6 \nu^3 - 9 T \alpha^3 \mu^6 \nu^3 + 27 T^2 \alpha^3 \mu^6 \nu^3 - 27 T^3 \alpha^3 \mu^6 \nu^3 +$$

$$\begin{aligned}
 & 4 \alpha^3 \mu^5 \nu^2 \hbar - 40 T \alpha^3 \mu^5 \nu^2 \hbar + 84 T^2 \alpha^3 \mu^5 \nu^2 \hbar + 2 \alpha^3 \mu^4 \nu \hbar^2 - 30 T \alpha^3 \mu^4 \nu \hbar^2), \\
 f_{3,1,2,0}[\nu] & \rightarrow T^2 \alpha \mu^3 \nu^4 - 3 T^3 \alpha \mu^3 \nu^4 - T \alpha \mu^2 \nu^3 \hbar + 9 T^2 \alpha \mu^2 \nu^3 \hbar - 3 T \alpha \mu \nu^2 \hbar^2, \\
 f_{2,2,0,1}[\nu] & \rightarrow \frac{1}{2} (\alpha^2 \mu^2 \nu^3 \hbar - 3 T \alpha^2 \mu^2 \nu^3 \hbar + \alpha^2 \mu \nu^2 \hbar^2), \\
 f_{2,1,0,0}[\nu] & \rightarrow \frac{1}{24 \hbar} (3 \alpha^2 \mu^3 \nu^4 - 21 T \alpha^2 \mu^3 \nu^4 + 45 T^2 \alpha^2 \mu^3 \nu^4 - 27 T^3 \alpha^2 \mu^3 \nu^4 + \\
 & 10 \alpha^2 \mu^2 \nu^3 \hbar - 68 T \alpha^2 \mu^2 \nu^3 \hbar + 82 T^2 \alpha^2 \mu^2 \nu^3 \hbar + 6 \alpha^2 \mu \nu^2 \hbar^2 - 30 T \alpha^2 \mu \nu^2 \hbar^2), \\
 f_{3,1,2,1}[\nu] & \rightarrow 2 (T^2 \alpha \mu^3 \nu^3 \hbar - T \alpha \mu^2 \nu^2 \hbar^2), \quad f_{3,0,2,0}[\nu] \rightarrow \frac{1}{2 \hbar} (T^2 \alpha \mu^4 \nu^4 - 4 T^3 \alpha \mu^4 \nu^4 + 3 T^4 \alpha \mu^4 \nu^4 - \\
 & T \alpha \mu^3 \nu^3 \hbar + 12 T^2 \alpha \mu^3 \nu^3 \hbar - 15 T^3 \alpha \mu^3 \nu^3 \hbar - 4 T \alpha \mu^2 \nu^2 \hbar^2 + 12 T^2 \alpha \mu^2 \nu^2 \hbar^2), \quad f_{3,1,1,1}[\nu] \rightarrow \\
 & \frac{1}{2} (2 T \alpha^2 \mu^4 \nu^4 - 10 T^2 \alpha^2 \mu^4 \nu^4 + 12 T^3 \alpha^2 \mu^4 \nu^4 + 15 T \alpha^2 \mu^3 \nu^3 \hbar - 45 T^2 \alpha^2 \mu^3 \nu^3 \hbar + 23 T \alpha^2 \mu^2 \nu^2 \hbar^2), \\
 f_{2,0,0,0}[\nu] & \rightarrow \frac{1}{288 \hbar^2} (9 \alpha^2 \mu^4 \nu^4 - 72 T \alpha^2 \mu^4 \nu^4 + 198 T^2 \alpha^2 \mu^4 \nu^4 - 216 T^3 \alpha^2 \mu^4 \nu^4 + \\
 & 81 T^4 \alpha^2 \mu^4 \nu^4 + 40 \alpha^2 \mu^3 \nu^3 \hbar - 312 T \alpha^2 \mu^3 \nu^3 \hbar + 600 T^2 \alpha^2 \mu^3 \nu^3 \hbar - \\
 & 328 T^3 \alpha^2 \mu^3 \nu^3 \hbar + 36 \alpha^2 \mu^2 \nu^2 \hbar^2 - 216 T \alpha^2 \mu^2 \nu^2 \hbar^2 + 180 T^2 \alpha^2 \mu^2 \nu^2 \hbar^2), \\
 f_{3,1,1,2}[\nu] & \rightarrow T \alpha^2 \mu^4 \nu^3 \hbar - 3 T^2 \alpha^2 \mu^4 \nu^3 \hbar + 4 T \alpha^2 \mu^3 \nu^2 \hbar^2, \quad f_{3,2,1,0}[\nu] \rightarrow \\
 & \frac{1}{12} (3 T \alpha^2 \mu^3 \nu^5 - 18 T^2 \alpha^2 \mu^3 \nu^5 + 27 T^3 \alpha^2 \mu^3 \nu^5 + 22 T \alpha^2 \mu^2 \nu^4 \hbar - 82 T^2 \alpha^2 \mu^2 \nu^4 \hbar + 28 T \alpha^2 \mu \nu^3 \hbar^2), \\
 f_{3,2,1,1}[\nu] & \rightarrow T \alpha^2 \mu^3 \nu^4 \hbar - 3 T^2 \alpha^2 \mu^3 \nu^4 \hbar + 4 T \alpha^2 \mu^2 \nu^3 \hbar^2, \\
 f_{3,3,0,0}[\nu] & \rightarrow \frac{1}{48} (\alpha^3 \mu^3 \nu^6 - 9 T \alpha^3 \mu^3 \nu^6 + 27 T^2 \alpha^3 \mu^3 \nu^6 - 27 T^3 \alpha^3 \mu^3 \nu^6 + \\
 & 4 \alpha^3 \mu^2 \nu^5 \hbar - 40 T \alpha^3 \mu^2 \nu^5 \hbar + 84 T^2 \alpha^3 \mu^2 \nu^5 \hbar + 2 \alpha^3 \mu \nu^4 \hbar^2 - 30 T \alpha^3 \mu \nu^4 \hbar^2), \\
 f_{3,0,1,1}[\nu] & \rightarrow \frac{1}{12 \hbar} (3 T \alpha^2 \mu^5 \nu^4 - 21 T^2 \alpha^2 \mu^5 \nu^4 + 45 T^3 \alpha^2 \mu^5 \nu^4 - 27 T^4 \alpha^2 \mu^5 \nu^4 + 31 T \alpha^2 \mu^4 \nu^3 \hbar - \\
 & 158 T^2 \alpha^2 \mu^4 \nu^3 \hbar + 163 T^3 \alpha^2 \mu^4 \nu^3 \hbar + 74 T \alpha^2 \mu^3 \nu^2 \hbar^2 - 194 T^2 \alpha^2 \mu^3 \nu^2 \hbar^2 + 30 T \alpha^2 \mu^2 \nu \hbar^3), \\
 f_{3,1,0,2}[\nu] & \rightarrow \frac{1}{48} (9 \alpha^3 \mu^5 \nu^4 - 69 T \alpha^3 \mu^5 \nu^4 + 171 T^2 \alpha^3 \mu^5 \nu^4 - 135 T^3 \alpha^3 \mu^5 \nu^4 + 60 \alpha^3 \mu^4 \nu^3 \hbar - \\
 & 416 T \alpha^3 \mu^4 \nu^3 \hbar + 644 T^2 \alpha^3 \mu^4 \nu^3 \hbar + 92 \alpha^3 \mu^3 \nu^2 \hbar^2 - 508 T \alpha^3 \mu^3 \nu^2 \hbar^2 + 24 \alpha^3 \mu^2 \nu \hbar^3), \quad f_{3,1,0,3}[\nu] \rightarrow \\
 & \frac{1}{24} (3 \alpha^3 \mu^5 \nu^3 \hbar - 18 T \alpha^3 \mu^5 \nu^3 \hbar + 27 T^2 \alpha^3 \mu^5 \nu^3 \hbar + 10 \alpha^3 \mu^4 \nu^2 \hbar^2 - 46 T \alpha^3 \mu^4 \nu^2 \hbar^2 + 4 \alpha^3 \mu^3 \nu \hbar^3), \\
 f_{3,0,0,2}[\nu] & \rightarrow \frac{1}{96 \hbar} (3 \alpha^3 \mu^6 \nu^4 - 30 T \alpha^3 \mu^6 \nu^4 + 108 T^2 \alpha^3 \mu^6 \nu^4 - 162 T^3 \alpha^3 \mu^6 \nu^4 + \\
 & 81 T^4 \alpha^3 \mu^6 \nu^4 + 24 \alpha^3 \mu^5 \nu^3 \hbar - 240 T \alpha^3 \mu^5 \nu^3 \hbar + 696 T^2 \alpha^3 \mu^5 \nu^3 \hbar - 576 T^3 \alpha^3 \mu^5 \nu^3 \hbar + \\
 & 46 \alpha^3 \mu^4 \nu^2 \hbar^2 - 472 T \alpha^3 \mu^4 \nu^2 \hbar^2 + 858 T^2 \alpha^3 \mu^4 \nu^2 \hbar^2 + 16 \alpha^3 \mu^3 \nu \hbar^3 - 192 T \alpha^3 \mu^3 \nu \hbar^3), \\
 f_{3,1,1,0}[\nu] & \rightarrow \frac{1}{12 \hbar} (3 T \alpha^2 \mu^4 \nu^5 - 21 T^2 \alpha^2 \mu^4 \nu^5 + 45 T^3 \alpha^2 \mu^4 \nu^5 - 27 T^4 \alpha^2 \mu^4 \nu^5 + 31 T \alpha^2 \mu^3 \nu^4 \hbar - \\
 & 158 T^2 \alpha^2 \mu^3 \nu^4 \hbar + 163 T^3 \alpha^2 \mu^3 \nu^4 \hbar + 74 T \alpha^2 \mu^2 \nu^3 \hbar^2 - 194 T^2 \alpha^2 \mu^2 \nu^3 \hbar^2 + 30 T \alpha^2 \mu \nu^2 \hbar^3), \\
 f_{3,2,0,1}[\nu] & \rightarrow \frac{1}{48} (9 \alpha^3 \mu^4 \nu^5 - 69 T \alpha^3 \mu^4 \nu^5 + 171 T^2 \alpha^3 \mu^4 \nu^5 - 135 T^3 \alpha^3 \mu^4 \nu^5 + 60 \alpha^3 \mu^3 \nu^4 \hbar - \\
 & 416 T \alpha^3 \mu^3 \nu^4 \hbar + 644 T^2 \alpha^3 \mu^3 \nu^4 \hbar + 92 \alpha^3 \mu^2 \nu^3 \hbar^2 - 508 T \alpha^3 \mu^2 \nu^3 \hbar^2 + 24 \alpha^3 \mu \nu^2 \hbar^3), \\
 f_{3,0,1,0}[\nu] & \rightarrow \frac{1}{144 \hbar^2} (9 T \alpha^2 \mu^5 \nu^5 - 72 T^2 \alpha^2 \mu^5 \nu^5 + 198 T^3 \alpha^2 \mu^5 \nu^5 - 216 T^4 \alpha^2 \mu^5 \nu^5 + 81 T^5 \alpha^2 \mu^5 \nu^5 + \\
 & 112 T \alpha^2 \mu^4 \nu^4 \hbar - 708 T^2 \alpha^2 \mu^4 \nu^4 \hbar + 1248 T^3 \alpha^2 \mu^4 \nu^4 \hbar - 652 T^4 \alpha^2 \mu^4 \nu^4 \hbar + 348 T \alpha^2 \mu^3 \nu^3 \hbar^2 - \\
 & 1416 T^2 \alpha^2 \mu^3 \nu^3 \hbar^2 + 1164 T^3 \alpha^2 \mu^3 \nu^3 \hbar^2 + 216 T \alpha^2 \mu^2 \nu^2 \hbar^3 - 360 T^2 \alpha^2 \mu^2 \nu^2 \hbar^3), \quad f_{3,2,0,2}[\nu] \rightarrow \\
 & \frac{1}{8} (3 \alpha^3 \mu^4 \nu^4 \hbar - 16 T \alpha^3 \mu^4 \nu^4 \hbar + 21 T^2 \alpha^3 \mu^4 \nu^4 \hbar + 14 \alpha^3 \mu^3 \nu^3 \hbar^2 - 54 T \alpha^3 \mu^3 \nu^3 \hbar^2 + 12 \alpha^3 \mu^2 \nu^2 \hbar^3),
 \end{aligned}$$

$$\begin{aligned}
 f_{3,2,0,3}[\nu] &\rightarrow \frac{1}{4} \left(\alpha^3 \mu^4 \nu^3 \hbar^2 - 3 T \alpha^3 \mu^4 \nu^3 \hbar^2 + 2 \alpha^3 \mu^3 \nu^2 \hbar^3 \right), f_{3,3,0,1}[\nu] \rightarrow \\
 &\frac{1}{24} \left(3 \alpha^3 \mu^3 \nu^5 \hbar - 18 T \alpha^3 \mu^3 \nu^5 \hbar + 27 T^2 \alpha^3 \mu^3 \nu^5 \hbar + 10 \alpha^3 \mu^2 \nu^4 \hbar^2 - 46 T \alpha^3 \mu^2 \nu^4 \hbar^2 + 4 \alpha^3 \mu \nu^3 \hbar^3 \right), \\
 f_{3,2,0,0}[\nu] &\rightarrow \frac{1}{96 \hbar} \left(3 \alpha^3 \mu^4 \nu^6 - 30 T \alpha^3 \mu^4 \nu^6 + 108 T^2 \alpha^3 \mu^4 \nu^6 - 162 T^3 \alpha^3 \mu^4 \nu^6 + \right. \\
 &81 T^4 \alpha^3 \mu^4 \nu^6 + 24 \alpha^3 \mu^3 \nu^5 \hbar - 240 T \alpha^3 \mu^3 \nu^5 \hbar + 696 T^2 \alpha^3 \mu^3 \nu^5 \hbar - 576 T^3 \alpha^3 \mu^3 \nu^5 \hbar + \\
 &46 \alpha^3 \mu^2 \nu^4 \hbar^2 - 472 T \alpha^3 \mu^2 \nu^4 \hbar^2 + 858 T^2 \alpha^3 \mu^2 \nu^4 \hbar^2 + 16 \alpha^3 \mu \nu^3 \hbar^3 - 192 T \alpha^3 \mu \nu^3 \hbar^3 \left. \right), \\
 f_{3,3,0,2}[\nu] &\rightarrow \frac{1}{4} \left(\alpha^3 \mu^3 \nu^4 \hbar^2 - 3 T \alpha^3 \mu^3 \nu^4 \hbar^2 + 2 \alpha^3 \mu^2 \nu^3 \hbar^3 \right), \\
 f_{3,1,0,1}[\nu] &\rightarrow \frac{1}{288 \hbar} \left(27 \alpha^3 \mu^5 \nu^5 - 252 T \alpha^3 \mu^5 \nu^5 + 846 T^2 \alpha^3 \mu^5 \nu^5 - 1188 T^3 \alpha^3 \mu^5 \nu^5 + 567 T^4 \alpha^3 \mu^5 \nu^5 + \right. \\
 &250 \alpha^3 \mu^4 \nu^4 \hbar - 2226 T \alpha^3 \mu^4 \nu^4 \hbar + 5814 T^2 \alpha^3 \mu^4 \nu^4 \hbar - 4414 T^3 \alpha^3 \mu^4 \nu^4 \hbar + 624 \alpha^3 \mu^3 \nu^3 \hbar^2 - \\
 &5112 T \alpha^3 \mu^3 \nu^3 \hbar^2 + 7944 T^2 \alpha^3 \mu^3 \nu^3 \hbar^2 + 432 \alpha^3 \mu^2 \nu^2 \hbar^3 - 2880 T \alpha^3 \mu^2 \nu^2 \hbar^3 + 48 \alpha^3 \mu \nu \hbar^4 \left. \right), \\
 f_{3,0,0,1}[\nu] &\rightarrow \frac{1}{576 \hbar^2} \left(9 \alpha^3 \mu^6 \nu^5 - 99 T \alpha^3 \mu^6 \nu^5 + 414 T^2 \alpha^3 \mu^6 \nu^5 - 810 T^3 \alpha^3 \mu^6 \nu^5 + 729 T^4 \alpha^3 \mu^6 \nu^5 - \right. \\
 &243 T^5 \alpha^3 \mu^6 \nu^5 + 100 \alpha^3 \mu^5 \nu^4 \hbar - 1080 T \alpha^3 \mu^5 \nu^4 \hbar + 3840 T^2 \alpha^3 \mu^5 \nu^4 \hbar - 5320 T^3 \alpha^3 \mu^5 \nu^4 \hbar + \\
 &2460 T^4 \alpha^3 \mu^5 \nu^4 \hbar + 312 \alpha^3 \mu^4 \nu^3 \hbar^2 - 3288 T \alpha^3 \mu^4 \nu^3 \hbar^2 + 8808 T^2 \alpha^3 \mu^4 \nu^3 \hbar^2 - 6408 T^3 \alpha^3 \mu^4 \nu^3 \hbar^2 + \\
 &288 \alpha^3 \mu^3 \nu^2 \hbar^3 - 2880 T \alpha^3 \mu^3 \nu^2 \hbar^3 + 4320 T^2 \alpha^3 \mu^3 \nu^2 \hbar^3 + 48 \alpha^3 \mu^2 \nu \hbar^4 - 432 T \alpha^3 \mu^2 \nu \hbar^4 \left. \right), \\
 f_{3,1,0,0}[\nu] &\rightarrow \frac{1}{576 \hbar^2} \left(9 \alpha^3 \mu^5 \nu^6 - 99 T \alpha^3 \mu^5 \nu^6 + 414 T^2 \alpha^3 \mu^5 \nu^6 - 810 T^3 \alpha^3 \mu^5 \nu^6 + 729 T^4 \alpha^3 \mu^5 \nu^6 - \right. \\
 &243 T^5 \alpha^3 \mu^5 \nu^6 + 100 \alpha^3 \mu^4 \nu^5 \hbar - 1080 T \alpha^3 \mu^4 \nu^5 \hbar + 3840 T^2 \alpha^3 \mu^4 \nu^5 \hbar - 5320 T^3 \alpha^3 \mu^4 \nu^5 \hbar + \\
 &2460 T^4 \alpha^3 \mu^4 \nu^5 \hbar + 312 \alpha^3 \mu^3 \nu^4 \hbar^2 - 3288 T \alpha^3 \mu^3 \nu^4 \hbar^2 + 8808 T^2 \alpha^3 \mu^3 \nu^4 \hbar^2 - 6408 T^3 \alpha^3 \mu^3 \nu^4 \hbar^2 + \\
 &288 \alpha^3 \mu^2 \nu^3 \hbar^3 - 2880 T \alpha^3 \mu^2 \nu^3 \hbar^3 + 4320 T^2 \alpha^3 \mu^2 \nu^3 \hbar^3 + 48 \alpha^3 \mu \nu^2 \hbar^4 - 432 T \alpha^3 \mu \nu^2 \hbar^4 \left. \right), \\
 f_{3,0,0,0}[\nu] &\rightarrow \frac{1}{1152 \hbar^3} \left(3 \alpha^3 \mu^6 \nu^6 - 36 T \alpha^3 \mu^6 \nu^6 + 171 T^2 \alpha^3 \mu^6 \nu^6 - 408 T^3 \alpha^3 \mu^6 \nu^6 + 513 T^4 \alpha^3 \mu^6 \nu^6 - \right. \\
 &324 T^5 \alpha^3 \mu^6 \nu^6 + 81 T^6 \alpha^3 \mu^6 \nu^6 + 40 \alpha^3 \mu^5 \nu^5 \hbar - 472 T \alpha^3 \mu^5 \nu^5 \hbar + 1968 T^2 \alpha^3 \mu^5 \nu^5 \hbar - \\
 &3664 T^3 \alpha^3 \mu^5 \nu^5 \hbar + 3112 T^4 \alpha^3 \mu^5 \nu^5 \hbar - 984 T^5 \alpha^3 \mu^5 \nu^5 \hbar + 156 \alpha^3 \mu^4 \nu^4 \hbar^2 - 1800 T \alpha^3 \mu^4 \nu^4 \hbar^2 + \\
 &6048 T^2 \alpha^3 \mu^4 \nu^4 \hbar^2 - 7608 T^3 \alpha^3 \mu^4 \nu^4 \hbar^2 + 3204 T^4 \alpha^3 \mu^4 \nu^4 \hbar^2 + 192 \alpha^3 \mu^3 \nu^3 \hbar^3 - 2112 T \alpha^3 \mu^3 \nu^3 \hbar^3 + \\
 &4800 T^2 \alpha^3 \mu^3 \nu^3 \hbar^3 - 2880 T^3 \alpha^3 \mu^3 \nu^3 \hbar^3 + 48 \alpha^3 \mu^2 \nu^2 \hbar^4 - 480 T \alpha^3 \mu^2 \nu^2 \hbar^4 + 432 T^2 \alpha^3 \mu^2 \nu^2 \hbar^4 \left. \right) \}
 \end{aligned}$$

FF = Simp[F0⁻¹ F /. sol];

{FF /. ν → 0, Simp[F0 FF ** G - F0 U[y] ** FF - ∂_ν (F0 FF)]}

{U[], 0}

FF

$$\begin{aligned}
 &\left(1 + \frac{1}{8} \alpha^2 \beta^2 \mu^2 \nu^2 - \frac{3}{4} T \alpha^2 \beta^2 \mu^2 \nu^2 + \frac{5}{8} T^2 \alpha^2 \beta^2 \mu^2 \nu^2 + \frac{1}{6} \alpha^3 \beta^3 \mu^3 \nu^3 - \frac{11}{6} T \alpha^3 \beta^3 \mu^3 \nu^3 + \frac{25}{6} T^2 \alpha^3 \beta^3 \mu^3 \nu^3 - \right. \\
 &\frac{5}{2} T^3 \alpha^3 \beta^3 \mu^3 \nu^3 + \frac{\alpha^3 \beta^3 \mu^6 \nu^6}{384 \hbar^3} - \frac{T \alpha^3 \beta^3 \mu^6 \nu^6}{32 \hbar^3} + \frac{19 T^2 \alpha^3 \beta^3 \mu^6 \nu^6}{128 \hbar^3} - \frac{17 T^3 \alpha^3 \beta^3 \mu^6 \nu^6}{48 \hbar^3} + \\
 &\frac{57 T^4 \alpha^3 \beta^3 \mu^6 \nu^6}{128 \hbar^3} - \frac{9 T^5 \alpha^3 \beta^3 \mu^6 \nu^6}{32 \hbar^3} + \frac{9 T^6 \alpha^3 \beta^3 \mu^6 \nu^6}{32 \hbar^2} + \frac{\alpha^2 \beta^2 \mu^4 \nu^4}{32 \hbar^2} - \frac{T \alpha^2 \beta^2 \mu^4 \nu^4}{4 \hbar^2} + \\
 &\frac{11 T^2 \alpha^2 \beta^2 \mu^4 \nu^4}{16 \hbar^2} - \frac{3 T^3 \alpha^2 \beta^2 \mu^4 \nu^4}{4 \hbar^2} + \frac{9 T^4 \alpha^2 \beta^2 \mu^4 \nu^4}{32 \hbar^2} + \frac{5 \alpha^3 \beta^3 \mu^5 \nu^5}{144 \hbar^2} - \frac{59 T \alpha^3 \beta^3 \mu^5 \nu^5}{144 \hbar^2} + \\
 &\frac{41 T^2 \alpha^3 \beta^3 \mu^5 \nu^5}{24 \hbar^2} - \frac{229 T^3 \alpha^3 \beta^3 \mu^5 \nu^5}{72 \hbar^2} + \frac{389 T^4 \alpha^3 \beta^3 \mu^5 \nu^5}{144 \hbar^2} - \frac{41 T^5 \alpha^3 \beta^3 \mu^5 \nu^5}{48 \hbar^2} + \frac{\alpha \beta \mu^2 \nu^2}{4 \hbar} -
 \end{aligned}$$

$$\begin{aligned}
 & \frac{T \alpha \beta \mu^2 \nu^2}{\hbar} + \frac{3 T^2 \alpha \beta \mu^2 \nu^2}{4 \hbar} + \frac{5 \alpha^2 \beta^2 \mu^3 \nu^3}{36 \hbar} - \frac{13 T \alpha^2 \beta^2 \mu^3 \nu^3}{12 \hbar} + \frac{25 T^2 \alpha^2 \beta^2 \mu^3 \nu^3}{12 \hbar} - \\
 & \frac{41 T^3 \alpha^2 \beta^2 \mu^3 \nu^3}{36 \hbar} + \frac{13 \alpha^3 \beta^3 \mu^4 \nu^4}{96 \hbar} - \frac{25 T \alpha^3 \beta^3 \mu^4 \nu^4}{16 \hbar} + \frac{21 T^2 \alpha^3 \beta^3 \mu^4 \nu^4}{4 \hbar} - \frac{317 T^3 \alpha^3 \beta^3 \mu^4 \nu^4}{48 \hbar} + \\
 & \left. \frac{89 T^4 \alpha^3 \beta^3 \mu^4 \nu^4}{32 \hbar} + \frac{1}{24} \alpha^3 \beta^3 \mu^2 \nu^2 \hbar - \frac{5}{12} T \alpha^3 \beta^3 \mu^2 \nu^2 \hbar + \frac{3}{8} T^2 \alpha^3 \beta^3 \mu^2 \nu^2 \hbar \right) U[] + \\
 & \left(2 T \beta \mu \nu + 2 T \alpha \beta^2 \mu^2 \nu^2 - 3 T^2 \alpha \beta^2 \mu^2 \nu^2 + \frac{29}{12} T \alpha^2 \beta^3 \mu^3 \nu^3 - \frac{59}{6} T^2 \alpha^2 \beta^3 \mu^3 \nu^3 + \frac{97}{12} T^3 \alpha^2 \beta^3 \mu^3 \nu^3 + \right. \\
 & \frac{T \alpha^2 \beta^3 \mu^5 \nu^5}{16 \hbar^2} - \frac{T^2 \alpha^2 \beta^3 \mu^5 \nu^5}{2 \hbar^2} + \frac{11 T^3 \alpha^2 \beta^3 \mu^5 \nu^5}{8 \hbar^2} - \frac{3 T^4 \alpha^2 \beta^3 \mu^5 \nu^5}{2 \hbar^2} + \frac{9 T^5 \alpha^2 \beta^3 \mu^5 \nu^5}{16 \hbar^2} + \\
 & \frac{T \alpha \beta^2 \mu^3 \nu^3}{2 \hbar} - \frac{2 T^2 \alpha \beta^2 \mu^3 \nu^3}{\hbar} + \frac{3 T^3 \alpha \beta^2 \mu^3 \nu^3}{2 \hbar} + \frac{7 T \alpha^2 \beta^3 \mu^4 \nu^4}{9 \hbar} - \frac{59 T^2 \alpha^2 \beta^3 \mu^4 \nu^4}{12 \hbar} + \\
 & \left. \frac{26 T^3 \alpha^2 \beta^3 \mu^4 \nu^4}{3 \hbar} - \frac{163 T^4 \alpha^2 \beta^3 \mu^4 \nu^4}{36 \hbar} + \frac{3}{2} T \alpha^2 \beta^3 \mu^2 \nu^2 \hbar - \frac{5}{2} T^2 \alpha^2 \beta^3 \mu^2 \nu^2 \hbar \right) U[a] + \\
 & \left(\frac{1}{2} \alpha \beta \mu^2 \nu - \frac{3}{2} T \alpha \beta \mu^2 \nu + \frac{5}{12} \alpha^2 \beta^2 \mu^3 \nu^2 - \frac{17}{6} T \alpha^2 \beta^2 \mu^3 \nu^2 + \frac{41}{12} T^2 \alpha^2 \beta^2 \mu^3 \nu^2 + \frac{13}{24} \alpha^3 \beta^3 \mu^4 \nu^3 - \right. \\
 & \frac{137}{24} T \alpha^3 \beta^3 \mu^4 \nu^3 + \frac{367}{24} T^2 \alpha^3 \beta^3 \mu^4 \nu^3 - \frac{89}{8} T^3 \alpha^3 \beta^3 \mu^4 \nu^3 + \frac{\alpha^3 \beta^3 \mu^6 \nu^5}{64 \hbar^2} - \frac{11 T \alpha^3 \beta^3 \mu^6 \nu^5}{64 \hbar^2} + \\
 & \frac{23 T^2 \alpha^3 \beta^3 \mu^6 \nu^5}{32 \hbar^2} - \frac{45 T^3 \alpha^3 \beta^3 \mu^6 \nu^5}{32 \hbar^2} + \frac{81 T^4 \alpha^3 \beta^3 \mu^6 \nu^5}{64 \hbar^2} - \frac{27 T^5 \alpha^3 \beta^3 \mu^6 \nu^5}{64 \hbar^2} + \frac{\alpha^2 \beta^2 \mu^4 \nu^3}{8 \hbar} - \\
 & \frac{7 T \alpha^2 \beta^2 \mu^4 \nu^3}{8 \hbar} + \frac{15 T^2 \alpha^2 \beta^2 \mu^4 \nu^3}{8 \hbar} - \frac{9 T^3 \alpha^2 \beta^2 \mu^4 \nu^3}{8 \hbar} + \frac{25 \alpha^3 \beta^3 \mu^5 \nu^4}{144 \hbar} - \frac{15 T \alpha^3 \beta^3 \mu^5 \nu^4}{8 \hbar} + \\
 & \frac{20 T^2 \alpha^3 \beta^3 \mu^5 \nu^4}{3 \hbar} - \frac{665 T^3 \alpha^3 \beta^3 \mu^5 \nu^4}{72 \hbar} + \frac{205 T^4 \alpha^3 \beta^3 \mu^5 \nu^4}{48 \hbar} + \frac{1}{4} \alpha^2 \beta^2 \mu^2 \nu \hbar - \frac{5}{4} T \alpha^2 \beta^2 \mu^2 \nu \hbar + \\
 & \left. \frac{1}{2} \alpha^3 \beta^3 \mu^3 \nu^2 \hbar - 5 T \alpha^3 \beta^3 \mu^3 \nu^2 \hbar + \frac{15}{2} T^2 \alpha^3 \beta^3 \mu^3 \nu^2 \hbar + \frac{1}{12} \alpha^3 \beta^3 \mu^2 \nu \hbar^2 - \frac{3}{4} T \alpha^3 \beta^3 \mu^2 \nu \hbar^2 \right) U[x] + \\
 & \left(\frac{1}{2} \alpha \beta \mu \nu^2 - \frac{3}{2} T \alpha \beta \mu \nu^2 + \frac{5}{12} \alpha^2 \beta^2 \mu^2 \nu^3 - \frac{17}{6} T \alpha^2 \beta^2 \mu^2 \nu^3 + \frac{41}{12} T^2 \alpha^2 \beta^2 \mu^2 \nu^3 + \frac{13}{24} \alpha^3 \beta^3 \mu^3 \nu^4 - \right. \\
 & \frac{137}{24} T \alpha^3 \beta^3 \mu^3 \nu^4 + \frac{367}{24} T^2 \alpha^3 \beta^3 \mu^3 \nu^4 - \frac{89}{8} T^3 \alpha^3 \beta^3 \mu^3 \nu^4 + \frac{\alpha^3 \beta^3 \mu^5 \nu^6}{64 \hbar^2} - \frac{11 T \alpha^3 \beta^3 \mu^5 \nu^6}{64 \hbar^2} + \\
 & \frac{23 T^2 \alpha^3 \beta^3 \mu^5 \nu^6}{32 \hbar^2} - \frac{45 T^3 \alpha^3 \beta^3 \mu^5 \nu^6}{32 \hbar^2} + \frac{81 T^4 \alpha^3 \beta^3 \mu^5 \nu^6}{64 \hbar^2} - \frac{27 T^5 \alpha^3 \beta^3 \mu^5 \nu^6}{64 \hbar^2} + \frac{\alpha^2 \beta^2 \mu^3 \nu^4}{8 \hbar} - \\
 & \frac{7 T \alpha^2 \beta^2 \mu^3 \nu^4}{8 \hbar} + \frac{15 T^2 \alpha^2 \beta^2 \mu^3 \nu^4}{8 \hbar} - \frac{9 T^3 \alpha^2 \beta^2 \mu^3 \nu^4}{8 \hbar} + \frac{25 \alpha^3 \beta^3 \mu^4 \nu^5}{144 \hbar} - \frac{15 T \alpha^3 \beta^3 \mu^4 \nu^5}{8 \hbar} + \\
 & \frac{20 T^2 \alpha^3 \beta^3 \mu^4 \nu^5}{3 \hbar} - \frac{665 T^3 \alpha^3 \beta^3 \mu^4 \nu^5}{72 \hbar} + \frac{205 T^4 \alpha^3 \beta^3 \mu^4 \nu^5}{48 \hbar} + \frac{1}{4} \alpha^2 \beta^2 \mu \nu^2 \hbar - \frac{5}{4} T \alpha^2 \beta^2 \mu \nu^2 \hbar + \\
 & \left. \frac{1}{2} \alpha^3 \beta^3 \mu^2 \nu^3 \hbar - 5 T \alpha^3 \beta^3 \mu^2 \nu^3 \hbar + \frac{15}{2} T^2 \alpha^3 \beta^3 \mu^2 \nu^3 \hbar + \frac{1}{12} \alpha^3 \beta^3 \mu \nu^2 \hbar^2 - \frac{3}{4} T \alpha^3 \beta^3 \mu \nu^2 \hbar^2 \right) U[y] + \\
 & \left(2 T^2 \beta^2 \mu^2 \nu^2 - \frac{1}{2} T \alpha \beta^3 \mu^3 \nu^3 + 6 T^2 \alpha \beta^3 \mu^3 \nu^3 - \frac{15}{2} T^3 \alpha \beta^3 \mu^3 \nu^3 + \frac{T^2 \alpha \beta^3 \mu^4 \nu^4}{2 \hbar} - \right. \\
 & \left. \frac{2 T^3 \alpha \beta^3 \mu^4 \nu^4}{\hbar} + \frac{3 T^4 \alpha \beta^3 \mu^4 \nu^4}{2 \hbar} - 2 T \beta^2 \mu \nu \hbar - 2 T \alpha \beta^3 \mu^2 \nu^2 \hbar + 6 T^2 \alpha \beta^3 \mu^2 \nu^2 \hbar \right) U[a, a] + \\
 & \left(T \alpha \beta^2 \mu^3 \nu^2 - 3 T^2 \alpha \beta^2 \mu^3 \nu^2 + \frac{31}{12} T \alpha^2 \beta^3 \mu^4 \nu^3 - \frac{79}{6} T^2 \alpha^2 \beta^3 \mu^4 \nu^3 + \frac{163}{12} T^3 \alpha^2 \beta^3 \mu^4 \nu^3 + \right.
 \end{aligned}$$

$$\begin{aligned}
 & \left(\frac{T \alpha^2 \beta^3 \mu^5 \nu^4}{4 \hbar} - \frac{7 T^2 \alpha^2 \beta^3 \mu^5 \nu^4}{4 \hbar} + \frac{15 T^3 \alpha^2 \beta^3 \mu^5 \nu^4}{4 \hbar} - \frac{9 T^4 \alpha^2 \beta^3 \mu^5 \nu^4}{4 \hbar} + \right. \\
 & \left. 3 T \alpha \beta^2 \mu^2 \nu \hbar + \frac{37}{6} T \alpha^2 \beta^3 \mu^3 \nu^2 \hbar - \frac{97}{6} T^2 \alpha^2 \beta^3 \mu^3 \nu^2 \hbar + \frac{5}{2} T \alpha^2 \beta^3 \mu^2 \nu \hbar^2 \right) U[\mathbf{a}, \mathbf{x}] + \\
 & \left(\frac{1}{8} \alpha^2 \beta^2 \mu^4 \nu^2 - \frac{3}{4} T \alpha^2 \beta^2 \mu^4 \nu^2 + \frac{9}{8} T^2 \alpha^2 \beta^2 \mu^4 \nu^2 + \frac{1}{4} \alpha^3 \beta^3 \mu^5 \nu^3 - \frac{5}{2} T \alpha^3 \beta^3 \mu^5 \nu^3 + \right. \\
 & \frac{29}{4} T^2 \alpha^3 \beta^3 \mu^5 \nu^3 - 6 T^3 \alpha^3 \beta^3 \mu^5 \nu^3 + \frac{\alpha^3 \beta^3 \mu^6 \nu^4}{32 \hbar} - \frac{5 T \alpha^3 \beta^3 \mu^6 \nu^4}{16 \hbar} + \frac{9 T^2 \alpha^3 \beta^3 \mu^6 \nu^4}{8 \hbar} - \\
 & \frac{27 T^3 \alpha^3 \beta^3 \mu^6 \nu^4}{16 \hbar} + \frac{27 T^4 \alpha^3 \beta^3 \mu^6 \nu^4}{32 \hbar} + \frac{1}{6} \alpha^2 \beta^2 \mu^3 \nu \hbar - \frac{7}{6} T \alpha^2 \beta^2 \mu^3 \nu \hbar + \frac{23}{48} \alpha^3 \beta^3 \mu^4 \nu^2 \hbar - \\
 & \left. \frac{59}{12} T \alpha^3 \beta^3 \mu^4 \nu^2 \hbar + \frac{143}{16} T^2 \alpha^3 \beta^3 \mu^4 \nu^2 \hbar + \frac{1}{6} \alpha^3 \beta^3 \mu^3 \nu \hbar^2 - 2 T \alpha^3 \beta^3 \mu^3 \nu \hbar^2 \right) U[\mathbf{x}, \mathbf{x}] + \\
 & \left(T \alpha \beta^2 \mu^2 \nu^3 - 3 T^2 \alpha \beta^2 \mu^2 \nu^3 + \frac{31}{12} T \alpha^2 \beta^3 \mu^3 \nu^4 - \frac{79}{6} T^2 \alpha^2 \beta^3 \mu^3 \nu^4 + \frac{163}{12} T^3 \alpha^2 \beta^3 \mu^3 \nu^4 + \right. \\
 & \frac{T \alpha^2 \beta^3 \mu^4 \nu^5}{4 \hbar} - \frac{7 T^2 \alpha^2 \beta^3 \mu^4 \nu^5}{4 \hbar} + \frac{15 T^3 \alpha^2 \beta^3 \mu^4 \nu^5}{4 \hbar} - \frac{9 T^4 \alpha^2 \beta^3 \mu^4 \nu^5}{4 \hbar} + \\
 & \left. 3 T \alpha \beta^2 \mu \nu^2 \hbar + \frac{37}{6} T \alpha^2 \beta^3 \mu^2 \nu^3 \hbar - \frac{97}{6} T^2 \alpha^2 \beta^3 \mu^2 \nu^3 \hbar + \frac{5}{2} T \alpha^2 \beta^3 \mu \nu^2 \hbar^2 \right) U[\mathbf{y}, \mathbf{a}] + \\
 & \left(\frac{1}{2} \alpha^2 \beta^2 \mu^3 \nu^3 - \frac{5}{2} T \alpha^2 \beta^2 \mu^3 \nu^3 + 3 T^2 \alpha^2 \beta^2 \mu^3 \nu^3 + \frac{125}{144} \alpha^3 \beta^3 \mu^4 \nu^4 - \frac{371}{48} T \alpha^3 \beta^3 \mu^4 \nu^4 + \right. \\
 & \frac{323}{16} T^2 \alpha^3 \beta^3 \mu^4 \nu^4 - \frac{2207}{144} T^3 \alpha^3 \beta^3 \mu^4 \nu^4 + \frac{3 \alpha^3 \beta^3 \mu^5 \nu^5}{32 \hbar} - \frac{7 T \alpha^3 \beta^3 \mu^5 \nu^5}{8 \hbar} + \\
 & \frac{47 T^2 \alpha^3 \beta^3 \mu^5 \nu^5}{16 \hbar} - \frac{33 T^3 \alpha^3 \beta^3 \mu^5 \nu^5}{8 \hbar} + \frac{63 T^4 \alpha^3 \beta^3 \mu^5 \nu^5}{32 \hbar} + \alpha \beta \mu \nu \hbar + \frac{5}{4} \alpha^2 \beta^2 \mu^2 \nu^2 \hbar - \\
 & \frac{21}{4} T \alpha^2 \beta^2 \mu^2 \nu^2 \hbar + \frac{13}{6} \alpha^3 \beta^3 \mu^3 \nu^3 \hbar - \frac{71}{4} T \alpha^3 \beta^3 \mu^3 \nu^3 \hbar + \frac{331}{12} T^2 \alpha^3 \beta^3 \mu^3 \nu^3 \hbar + \\
 & \left. \frac{1}{2} \alpha^2 \beta^2 \mu \nu \hbar^2 + \frac{3}{2} \alpha^3 \beta^3 \mu^2 \nu^2 \hbar^2 - 10 T \alpha^3 \beta^3 \mu^2 \nu^2 \hbar^2 + \frac{1}{6} \alpha^3 \beta^3 \mu \nu \hbar^3 \right) U[\mathbf{y}, \mathbf{x}] + \\
 & \left(\frac{1}{8} \alpha^2 \beta^2 \mu^2 \nu^4 - \frac{3}{4} T \alpha^2 \beta^2 \mu^2 \nu^4 + \frac{9}{8} T^2 \alpha^2 \beta^2 \mu^2 \nu^4 + \frac{1}{4} \alpha^3 \beta^3 \mu^3 \nu^5 - \frac{5}{2} T \alpha^3 \beta^3 \mu^3 \nu^5 + \right. \\
 & \frac{29}{4} T^2 \alpha^3 \beta^3 \mu^3 \nu^5 - 6 T^3 \alpha^3 \beta^3 \mu^3 \nu^5 + \frac{\alpha^3 \beta^3 \mu^4 \nu^6}{32 \hbar} - \frac{5 T \alpha^3 \beta^3 \mu^4 \nu^6}{16 \hbar} + \frac{9 T^2 \alpha^3 \beta^3 \mu^4 \nu^6}{8 \hbar} - \\
 & \frac{27 T^3 \alpha^3 \beta^3 \mu^4 \nu^6}{16 \hbar} + \frac{27 T^4 \alpha^3 \beta^3 \mu^4 \nu^6}{32 \hbar} + \frac{1}{6} \alpha^2 \beta^2 \mu \nu^3 \hbar - \frac{7}{6} T \alpha^2 \beta^2 \mu \nu^3 \hbar + \frac{23}{48} \alpha^3 \beta^3 \mu^2 \nu^4 \hbar - \\
 & \left. \frac{59}{12} T \alpha^3 \beta^3 \mu^2 \nu^4 \hbar + \frac{143}{16} T^2 \alpha^3 \beta^3 \mu^2 \nu^4 \hbar + \frac{1}{6} \alpha^3 \beta^3 \mu \nu^3 \hbar^2 - 2 T \alpha^3 \beta^3 \mu \nu^3 \hbar^2 \right) U[\mathbf{y}, \mathbf{y}] + \\
 & \left(\frac{4}{3} T^3 \beta^3 \mu^3 \nu^3 - 4 T^2 \beta^3 \mu^2 \nu^2 \hbar + \frac{4}{3} T \beta^3 \mu \nu \hbar^2 \right) U[\mathbf{a}, \mathbf{a}, \mathbf{a}] + \\
 & \left(T^2 \alpha \beta^3 \mu^4 \nu^3 - 3 T^3 \alpha \beta^3 \mu^4 \nu^3 - T \alpha \beta^3 \mu^3 \nu^2 \hbar + 9 T^2 \alpha \beta^3 \mu^3 \nu^2 \hbar - 3 T \alpha \beta^3 \mu^2 \nu \hbar^2 \right) U[\mathbf{a}, \mathbf{a}, \mathbf{x}] + \\
 & \left(\frac{1}{4} T \alpha^2 \beta^3 \mu^5 \nu^3 - \frac{3}{2} T^2 \alpha^2 \beta^3 \mu^5 \nu^3 + \frac{9}{4} T^3 \alpha^2 \beta^3 \mu^5 \nu^3 + \right. \\
 & \left. \frac{11}{6} T \alpha^2 \beta^3 \mu^4 \nu^2 \hbar - \frac{41}{6} T^2 \alpha^2 \beta^3 \mu^4 \nu^2 \hbar + \frac{7}{3} T \alpha^2 \beta^3 \mu^3 \nu \hbar^2 \right) U[\mathbf{a}, \mathbf{x}, \mathbf{x}] + \\
 & \left(\frac{1}{48} \alpha^3 \beta^3 \mu^6 \nu^3 - \frac{3}{16} T \alpha^3 \beta^3 \mu^6 \nu^3 + \frac{9}{16} T^2 \alpha^3 \beta^3 \mu^6 \nu^3 - \frac{9}{16} T^3 \alpha^3 \beta^3 \mu^6 \nu^3 + \frac{1}{12} \alpha^3 \beta^3 \mu^5 \nu^2 \hbar - \right.
 \end{aligned}$$

$$\begin{aligned}
 & \left(\frac{5}{6} T \alpha^3 \beta^3 \mu^5 \nu^2 \hbar + \frac{7}{4} T^2 \alpha^3 \beta^3 \mu^5 \nu^2 \hbar + \frac{1}{24} \alpha^3 \beta^3 \mu^4 \nu \hbar^2 - \frac{5}{8} T \alpha^3 \beta^3 \mu^4 \nu \hbar^2 \right) U[x, x, x] + \\
 & \left(T^2 \alpha \beta^3 \mu^3 \nu^4 - 3 T^3 \alpha \beta^3 \mu^3 \nu^4 - T \alpha \beta^3 \mu^2 \nu^3 \hbar + 9 T^2 \alpha \beta^3 \mu^2 \nu^3 \hbar - 3 T \alpha \beta^3 \mu \nu^2 \hbar^2 \right) U[y, a, a] + \\
 & \left(T \alpha^2 \beta^3 \mu^4 \nu^4 - 5 T^2 \alpha^2 \beta^3 \mu^4 \nu^4 + 6 T^3 \alpha^2 \beta^3 \mu^4 \nu^4 + 2 T \alpha \beta^2 \mu^2 \nu^2 \hbar + \right. \\
 & \quad \left. \frac{15}{2} T \alpha^2 \beta^3 \mu^3 \nu^3 \hbar - \frac{45}{2} T^2 \alpha^2 \beta^3 \mu^3 \nu^3 \hbar + \frac{23}{2} T \alpha^2 \beta^3 \mu^2 \nu^2 \hbar^2 \right) U[y, a, x] + \\
 & \left(\frac{3}{16} \alpha^3 \beta^3 \mu^5 \nu^4 - \frac{23}{16} T \alpha^3 \beta^3 \mu^5 \nu^4 + \frac{57}{16} T^2 \alpha^3 \beta^3 \mu^5 \nu^4 - \frac{45}{16} T^3 \alpha^3 \beta^3 \mu^5 \nu^4 + \frac{1}{2} \alpha^2 \beta^2 \mu^3 \nu^2 \hbar - \right. \\
 & \quad \frac{3}{2} T \alpha^2 \beta^2 \mu^3 \nu^2 \hbar + \frac{5}{4} \alpha^3 \beta^3 \mu^4 \nu^3 \hbar - \frac{26}{3} T \alpha^3 \beta^3 \mu^4 \nu^3 \hbar + \frac{161}{12} T^2 \alpha^3 \beta^3 \mu^4 \nu^3 \hbar + \\
 & \quad \left. \frac{1}{2} \alpha^2 \beta^2 \mu^2 \nu \hbar^2 + \frac{23}{12} \alpha^3 \beta^3 \mu^3 \nu^2 \hbar^2 - \frac{127}{12} T \alpha^3 \beta^3 \mu^3 \nu^2 \hbar^2 + \frac{1}{2} \alpha^3 \beta^3 \mu^2 \nu \hbar^3 \right) U[y, x, x] + \\
 & \left(\frac{1}{4} T \alpha^2 \beta^3 \mu^3 \nu^5 - \frac{3}{2} T^2 \alpha^2 \beta^3 \mu^3 \nu^5 + \frac{9}{4} T^3 \alpha^2 \beta^3 \mu^3 \nu^5 + \frac{11}{6} T \alpha^2 \beta^3 \mu^2 \nu^4 \hbar - \right. \\
 & \quad \left. \frac{41}{6} T^2 \alpha^2 \beta^3 \mu^2 \nu^4 \hbar + \frac{7}{3} T \alpha^2 \beta^3 \mu \nu^3 \hbar^2 \right) U[y, y, a] + \\
 & \left(\frac{3}{16} \alpha^3 \beta^3 \mu^4 \nu^5 - \frac{23}{16} T \alpha^3 \beta^3 \mu^4 \nu^5 + \frac{57}{16} T^2 \alpha^3 \beta^3 \mu^4 \nu^5 - \frac{45}{16} T^3 \alpha^3 \beta^3 \mu^4 \nu^5 + \frac{1}{2} \alpha^2 \beta^2 \mu^2 \nu^3 \hbar - \right. \\
 & \quad \frac{3}{2} T \alpha^2 \beta^2 \mu^2 \nu^3 \hbar + \frac{5}{4} \alpha^3 \beta^3 \mu^3 \nu^4 \hbar - \frac{26}{3} T \alpha^3 \beta^3 \mu^3 \nu^4 \hbar + \frac{161}{12} T^2 \alpha^3 \beta^3 \mu^3 \nu^4 \hbar + \\
 & \quad \left. \frac{1}{2} \alpha^2 \beta^2 \mu \nu^2 \hbar^2 + \frac{23}{12} \alpha^3 \beta^3 \mu^2 \nu^3 \hbar^2 - \frac{127}{12} T \alpha^3 \beta^3 \mu^2 \nu^3 \hbar^2 + \frac{1}{2} \alpha^3 \beta^3 \mu \nu^2 \hbar^3 \right) U[y, y, x] + \\
 & \left(\frac{1}{48} \alpha^3 \beta^3 \mu^3 \nu^6 - \frac{3}{16} T \alpha^3 \beta^3 \mu^3 \nu^6 + \frac{9}{16} T^2 \alpha^3 \beta^3 \mu^3 \nu^6 - \frac{9}{16} T^3 \alpha^3 \beta^3 \mu^3 \nu^6 + \frac{1}{12} \alpha^3 \beta^3 \mu^2 \nu^5 \hbar - \right. \\
 & \quad \left. \frac{5}{6} T \alpha^3 \beta^3 \mu^2 \nu^5 \hbar + \frac{7}{4} T^2 \alpha^3 \beta^3 \mu^2 \nu^5 \hbar + \frac{1}{24} \alpha^3 \beta^3 \mu \nu^4 \hbar^2 - \frac{5}{8} T \alpha^3 \beta^3 \mu \nu^4 \hbar^2 \right) U[y, y, y] + \\
 & \left(2 T^2 \alpha \beta^3 \mu^3 \nu^3 \hbar - 2 T \alpha \beta^3 \mu^2 \nu^2 \hbar^2 \right) U[y, a, a, x] + \\
 & \left(T \alpha^2 \beta^3 \mu^4 \nu^3 \hbar - 3 T^2 \alpha^2 \beta^3 \mu^4 \nu^3 \hbar + 4 T \alpha^2 \beta^3 \mu^3 \nu^2 \hbar^2 \right) U[y, a, x, x] + \\
 & \left(\frac{1}{8} \alpha^3 \beta^3 \mu^5 \nu^3 \hbar - \frac{3}{4} T \alpha^3 \beta^3 \mu^5 \nu^3 \hbar + \frac{9}{8} T^2 \alpha^3 \beta^3 \mu^5 \nu^3 \hbar + \right. \\
 & \quad \left. \frac{5}{12} \alpha^3 \beta^3 \mu^4 \nu^2 \hbar^2 - \frac{23}{12} T \alpha^3 \beta^3 \mu^4 \nu^2 \hbar^2 + \frac{1}{6} \alpha^3 \beta^3 \mu^3 \nu \hbar^3 \right) U[y, x, x, x] + \\
 & \left(T \alpha^2 \beta^3 \mu^3 \nu^4 \hbar - 3 T^2 \alpha^2 \beta^3 \mu^3 \nu^4 \hbar + 4 T \alpha^2 \beta^3 \mu^2 \nu^3 \hbar^2 \right) U[y, y, a, x] + \\
 & \left(\frac{3}{8} \alpha^3 \beta^3 \mu^4 \nu^4 \hbar - 2 T \alpha^3 \beta^3 \mu^4 \nu^4 \hbar + \frac{21}{8} T^2 \alpha^3 \beta^3 \mu^4 \nu^4 \hbar + \frac{1}{2} \alpha^2 \beta^2 \mu^2 \nu^2 \hbar^2 + \right. \\
 & \quad \left. \frac{7}{4} \alpha^3 \beta^3 \mu^3 \nu^3 \hbar^2 - \frac{27}{4} T \alpha^3 \beta^3 \mu^3 \nu^3 \hbar^2 + \frac{3}{2} \alpha^3 \beta^3 \mu^2 \nu^2 \hbar^3 \right) U[y, y, x, x] + \\
 & \left(\frac{1}{8} \alpha^3 \beta^3 \mu^3 \nu^5 \hbar - \frac{3}{4} T \alpha^3 \beta^3 \mu^3 \nu^5 \hbar + \frac{9}{8} T^2 \alpha^3 \beta^3 \mu^3 \nu^5 \hbar + \frac{5}{12} \alpha^3 \beta^3 \mu^2 \nu^4 \hbar^2 - \frac{23}{12} T \alpha^3 \beta^3 \mu^2 \nu^4 \hbar^2 + \right. \\
 & \quad \left. \frac{1}{6} \alpha^3 \beta^3 \mu \nu^3 \hbar^3 \right) U[y, y, y, x] + T \alpha^2 \beta^3 \mu^3 \nu^3 \hbar^2 U[y, y, a, x, x] + \\
 & \left(\frac{1}{4} \alpha^3 \beta^3 \mu^4 \nu^3 \hbar^2 - \frac{3}{4} T \alpha^3 \beta^3 \mu^4 \nu^3 \hbar^2 + \frac{1}{2} \alpha^3 \beta^3 \mu^3 \nu^2 \hbar^3 \right) U[y, y, x, x, x] + \\
 & \left(\frac{1}{4} \alpha^3 \beta^3 \mu^3 \nu^4 \hbar^2 - \frac{3}{4} T \alpha^3 \beta^3 \mu^3 \nu^4 \hbar^2 + \frac{1}{2} \alpha^3 \beta^3 \mu^2 \nu^3 \hbar^3 \right) U[y, y, y, x, x] +
 \end{aligned}$$

$$\frac{1}{6} \alpha^3 \beta^3 \mu^3 \nu^3 \hbar^3 U[y, y, y, x, x, x]$$

LFF = Simp@Sum[

SeriesCoefficient[Log[1 + x], {x, 0, k}] * NonCommutativeMultiply@@ Table[FF - U[], {k}], {k, 0, \$TBD}]

$$\begin{aligned} & \left(\frac{1}{8} \alpha^2 \beta^2 \mu^2 \nu^2 - \frac{3}{4} T \alpha^2 \beta^2 \mu^2 \nu^2 + \frac{5}{8} T^2 \alpha^2 \beta^2 \mu^2 \nu^2 + \frac{1}{24} \alpha^3 \beta^3 \mu^3 \nu^3 - \frac{17}{24} T \alpha^3 \beta^3 \mu^3 \nu^3 + \frac{31}{24} T^2 \alpha^3 \beta^3 \mu^3 \nu^3 - \right. \\ & \quad \frac{5}{8} T^3 \alpha^3 \beta^3 \mu^3 \nu^3 + \frac{\alpha \beta \mu^2 \nu^2}{4 \hbar} - \frac{T \alpha \beta \mu^2 \nu^2}{\hbar} + \frac{3 T^2 \alpha \beta \mu^2 \nu^2}{4 \hbar} + \frac{\alpha^2 \beta^2 \mu^3 \nu^3}{72 \hbar} - \frac{5 T \alpha^2 \beta^2 \mu^3 \nu^3}{24 \hbar} + \\ & \quad \frac{5 T^2 \alpha^2 \beta^2 \mu^3 \nu^3}{24 \hbar} - \frac{T^3 \alpha^2 \beta^2 \mu^3 \nu^3}{72 \hbar} - \frac{\alpha^3 \beta^3 \mu^4 \nu^4}{48 \hbar} + \frac{T \alpha^3 \beta^3 \mu^4 \nu^4}{6 \hbar} - \frac{3 T^2 \alpha^3 \beta^3 \mu^4 \nu^4}{4 \hbar} + \frac{7 T^3 \alpha^3 \beta^3 \mu^4 \nu^4}{6 \hbar} - \\ & \quad \left. \frac{9 T^4 \alpha^3 \beta^3 \mu^4 \nu^4}{16 \hbar} + \frac{1}{24} \alpha^3 \beta^3 \mu^2 \nu^2 \hbar - \frac{5}{12} T \alpha^3 \beta^3 \mu^2 \nu^2 \hbar + \frac{3}{8} T^2 \alpha^3 \beta^3 \mu^2 \nu^2 \hbar \right) U[] + \\ & \left(2 T \beta \mu \nu + 2 T \alpha \beta^2 \mu^2 \nu^2 - 3 T^2 \alpha \beta^2 \mu^2 \nu^2 + \frac{5}{12} T \alpha^2 \beta^3 \mu^3 \nu^3 - \frac{5}{6} T^2 \alpha^2 \beta^3 \mu^3 \nu^3 + \right. \\ & \quad \left. \frac{1}{12} T^3 \alpha^2 \beta^3 \mu^3 \nu^3 + \frac{3}{2} T \alpha^2 \beta^3 \mu^2 \nu^2 \hbar - \frac{5}{2} T^2 \alpha^2 \beta^3 \mu^2 \nu^2 \hbar \right) U[a] + \\ & \left(\frac{1}{2} \alpha \beta \mu^2 \nu - \frac{3}{2} T \alpha \beta \mu^2 \nu + \frac{1}{6} \alpha^2 \beta^2 \mu^3 \nu^2 - \frac{4}{3} T \alpha^2 \beta^2 \mu^3 \nu^2 + \frac{7}{6} T^2 \alpha^2 \beta^2 \mu^3 \nu^2 - \frac{1}{48} \alpha^3 \beta^3 \mu^4 \nu^3 - \right. \\ & \quad \frac{1}{16} T \alpha^3 \beta^3 \mu^4 \nu^3 - \frac{9}{16} T^2 \alpha^3 \beta^3 \mu^4 \nu^3 + \frac{13}{16} T^3 \alpha^3 \beta^3 \mu^4 \nu^3 + \frac{1}{4} \alpha^2 \beta^2 \mu^2 \nu \hbar - \frac{5}{4} T \alpha^2 \beta^2 \mu^2 \nu \hbar + \\ & \quad \left. \frac{1}{4} \alpha^3 \beta^3 \mu^3 \nu^2 \hbar - 3 T \alpha^3 \beta^3 \mu^3 \nu^2 \hbar + \frac{15}{4} T^2 \alpha^3 \beta^3 \mu^3 \nu^2 \hbar + \frac{1}{12} \alpha^3 \beta^3 \mu^2 \nu \hbar^2 - \frac{3}{4} T \alpha^3 \beta^3 \mu^2 \nu \hbar^2 \right) U[x] + \\ & \left(\frac{1}{2} \alpha \beta \mu \nu^2 - \frac{3}{2} T \alpha \beta \mu \nu^2 + \frac{1}{6} \alpha^2 \beta^2 \mu^2 \nu^3 - \frac{4}{3} T \alpha^2 \beta^2 \mu^2 \nu^3 + \frac{7}{6} T^2 \alpha^2 \beta^2 \mu^2 \nu^3 - \frac{1}{48} \alpha^3 \beta^3 \mu^3 \nu^4 - \right. \\ & \quad \frac{1}{16} T \alpha^3 \beta^3 \mu^3 \nu^4 - \frac{9}{16} T^2 \alpha^3 \beta^3 \mu^3 \nu^4 + \frac{13}{16} T^3 \alpha^3 \beta^3 \mu^3 \nu^4 + \frac{1}{4} \alpha^2 \beta^2 \mu \nu^2 \hbar - \frac{5}{4} T \alpha^2 \beta^2 \mu \nu^2 \hbar + \\ & \quad \left. \frac{1}{4} \alpha^3 \beta^3 \mu^2 \nu^3 \hbar - 3 T \alpha^3 \beta^3 \mu^2 \nu^3 \hbar + \frac{15}{4} T^2 \alpha^3 \beta^3 \mu^2 \nu^3 \hbar + \frac{1}{12} \alpha^3 \beta^3 \mu \nu^2 \hbar^2 - \frac{3}{4} T \alpha^3 \beta^3 \mu \nu^2 \hbar^2 \right) U[y] + \\ & \left(-2 T \beta^2 \mu \nu \hbar - 2 T \alpha \beta^3 \mu^2 \nu^2 \hbar + 6 T^2 \alpha \beta^3 \mu^2 \nu^2 \hbar \right) U[a, a] + \\ & \left(3 T \alpha \beta^2 \mu^2 \nu \hbar + \frac{8}{3} T \alpha^2 \beta^3 \mu^3 \nu^2 \hbar - \frac{14}{3} T^2 \alpha^2 \beta^3 \mu^3 \nu^2 \hbar + \frac{5}{2} T \alpha^2 \beta^3 \mu^2 \nu \hbar^2 \right) U[a, x] + \\ & \left(\frac{1}{6} \alpha^2 \beta^2 \mu^3 \nu \hbar - \frac{7}{6} T \alpha^2 \beta^2 \mu^3 \nu \hbar + \frac{1}{16} \alpha^3 \beta^3 \mu^4 \nu^2 \hbar - T \alpha^3 \beta^3 \mu^4 \nu^2 \hbar + \right. \\ & \quad \left. \frac{15}{16} T^2 \alpha^3 \beta^3 \mu^4 \nu^2 \hbar + \frac{1}{6} \alpha^3 \beta^3 \mu^3 \nu \hbar^2 - 2 T \alpha^3 \beta^3 \mu^3 \nu \hbar^2 \right) U[x, x] + \\ & \left(3 T \alpha \beta^2 \mu \nu^2 \hbar + \frac{8}{3} T \alpha^2 \beta^3 \mu^2 \nu^3 \hbar - \frac{14}{3} T^2 \alpha^2 \beta^3 \mu^2 \nu^3 \hbar + \frac{5}{2} T \alpha^2 \beta^3 \mu \nu^2 \hbar^2 \right) U[y, a] + \\ & \left(\alpha \beta \mu \nu \hbar + \frac{3}{4} \alpha^2 \beta^2 \mu^2 \nu^2 \hbar - \frac{11}{4} T \alpha^2 \beta^2 \mu^2 \nu^2 \hbar + \frac{1}{8} \alpha^3 \beta^3 \mu^3 \nu^3 \hbar - \frac{23}{12} T \alpha^3 \beta^3 \mu^3 \nu^3 \hbar + \right. \\ & \quad \left. \frac{31}{24} T^2 \alpha^3 \beta^3 \mu^3 \nu^3 \hbar + \frac{1}{2} \alpha^2 \beta^2 \mu \nu \hbar^2 + \alpha^3 \beta^3 \mu^2 \nu^2 \hbar^2 - \frac{13}{2} T \alpha^3 \beta^3 \mu^2 \nu^2 \hbar^2 + \frac{1}{6} \alpha^3 \beta^3 \mu \nu \hbar^3 \right) U[y, x] + \\ & \left(\frac{1}{6} \alpha^2 \beta^2 \mu \nu^3 \hbar - \frac{7}{6} T \alpha^2 \beta^2 \mu \nu^3 \hbar + \frac{1}{16} \alpha^3 \beta^3 \mu^2 \nu^4 \hbar - T \alpha^3 \beta^3 \mu^2 \nu^4 \hbar + \right. \\ & \quad \left. \frac{15}{16} T^2 \alpha^3 \beta^3 \mu^2 \nu^4 \hbar + \frac{1}{6} \alpha^3 \beta^3 \mu \nu^3 \hbar^2 - 2 T \alpha^3 \beta^3 \mu \nu^3 \hbar^2 \right) U[y, y] + \end{aligned}$$

$$\begin{aligned}
 & \frac{4}{3} \tau \beta^3 \mu \nu \hbar^2 U[a, a, a] - 3 \tau \alpha \beta^3 \mu^2 \nu \hbar^2 U[a, a, x] + \frac{7}{3} \tau \alpha^2 \beta^3 \mu^3 \nu \hbar^2 U[a, x, x] + \\
 & \left(\frac{1}{24} \alpha^3 \beta^3 \mu^4 \nu \hbar^2 - \frac{5}{8} \tau \alpha^3 \beta^3 \mu^4 \nu \hbar^2 \right) U[x, x, x] - \\
 & 3 \tau \alpha \beta^3 \mu \nu^2 \hbar^2 U[y, a, a] + \frac{11}{2} \tau \alpha^2 \beta^3 \mu^2 \nu^2 \hbar^2 U[y, a, x] + \\
 & \left(\frac{1}{2} \alpha^2 \beta^2 \mu^2 \nu \hbar^2 + \frac{5}{12} \alpha^3 \beta^3 \mu^3 \nu^2 \hbar^2 - \frac{31}{12} \tau \alpha^3 \beta^3 \mu^3 \nu^2 \hbar^2 + \frac{1}{2} \alpha^3 \beta^3 \mu^2 \nu \hbar^3 \right) U[y, x, x] + \\
 & \frac{7}{3} \tau \alpha^2 \beta^3 \mu \nu^3 \hbar^2 U[y, y, a] + \\
 & \left(\frac{1}{2} \alpha^2 \beta^2 \mu \nu^2 \hbar^2 + \frac{5}{12} \alpha^3 \beta^3 \mu^2 \nu^3 \hbar^2 - \frac{31}{12} \tau \alpha^3 \beta^3 \mu^2 \nu^3 \hbar^2 + \frac{1}{2} \alpha^3 \beta^3 \mu \nu^2 \hbar^3 \right) U[y, y, x] + \\
 & \left(\frac{1}{24} \alpha^3 \beta^3 \mu \nu^4 \hbar^2 - \frac{5}{8} \tau \alpha^3 \beta^3 \mu \nu^4 \hbar^2 \right) U[y, y, y] + \\
 & \frac{1}{6} \alpha^3 \beta^3 \mu^3 \nu \hbar^3 U[y, x, x, x] + \\
 & \frac{1}{2} \alpha^3 \beta^3 \mu^2 \nu^2 \hbar^3 U[y, y, x, x] + \frac{1}{6} \alpha^3 \beta^3 \mu \nu^3 \hbar^3 U[y, y, y, x]
 \end{aligned}$$

Cases [FF, _U, ∞]

{U[], U[a], U[x], U[y], U[a, a], U[a, x], U[x, x], U[y, a], U[y, x], U[y, y], U[a, a, a],
 U[a, a, x], U[a, x, x], U[x, x, x], U[y, a, a], U[y, a, x], U[y, x, x], U[y, y, a], U[y, y, x],
 U[y, y, y], U[y, a, a, x], U[y, a, x, x], U[y, x, x, x], U[y, y, a, x], U[y, y, x, x],
 U[y, y, y, x], U[y, y, a, x, x], U[y, y, x, x, x], U[y, y, y, x, x], U[y, y, y, x, x, x]}

Cases [LFF, _U, ∞]

{U[], U[a], U[x], U[y], U[a, a], U[a, x], U[x, x], U[y, a], U[y, x], U[y, y],
 U[a, a, a], U[a, a, x], U[a, x, x], U[x, x, x], U[y, a, a], U[y, a, x], U[y, x, x],
 U[y, y, a], U[y, y, x], U[y, y, y], U[y, x, x, x], U[y, y, x, x], U[y, y, y, x]}