```
$TD = 3; ħ /: ħ^d_. /; d > $TD := 0;

q = e^(ħ α β);

ϕ0[x_] := e^x;

ϕk_[x_] := Module[{t}, Simplify[(e^t - Normal@Series[e^t, {t, 0, k - 1}])/t^k /. t → x]];

K = 3; $TD = 5;
GS1 = μ T (A^-1 ϕ1[ħ μ (q - 1) x] - 1) - μ (A ϕ1[ħ μ (q^-1 - 1) x] - 1) /. {A → e^(-ħ β a)}
```

$$-\mu\left(-1+\frac{e^{-a\beta\hbar}\left(-1+e^{\left(-1+e^{-\alpha\beta\hbar}\right)x\mu\hbar}\right)}{\left(-1+e^{-\alpha\beta\hbar}\right)x\mu\hbar}\right)+T\mu\left(-1+\frac{e^{a\beta\hbar}\left(-1+e^{\left(-1+e^{\alpha\beta\hbar}\right)x\mu\hbar}\right)}{\left(-1+e^{\alpha\beta\hbar}\right)x\mu\hbar}\right)$$

```
K = 3;
GS2 = Collect[Normal@Series[GS1, {β, 0, K}], β, Factor]
```

$$\frac{1}{2}(1+T)\beta\mu\hbar\left(2a+x\alpha\mu\hbar\right)+\frac{1}{12}(-1+T)\beta^2\mu\hbar^2\left(6a^2+6ax\alpha\mu\hbar+3x\alpha^2\mu\hbar+2x^2\alpha^2\mu^2\hbar^2\right)+\frac{1}{24}(1+T)\beta^3\mu\hbar^3\left(4a^3+6a^2x\alpha\mu\hbar+6ax\alpha^2\mu\hbar+2x\alpha^3\mu\hbar+4ax^2\alpha^2\mu^2\hbar^2+4x^2\alpha^3\mu^2\hbar^2+x^3\alpha^3\mu^3\hbar^3\right)$$

```
Series[GS1 - GS2, {ħ, 0, $TD}]
```

$$\frac{1}{24}\left(-a^4\beta^4\mu+a^4T\beta^4\mu\right)\hbar^4+\frac{1}{240}\left(2a^5\beta^5\mu+2a^5T\beta^5\mu-20a^3x\alpha\beta^4\mu^2+20a^3Tx\alpha\beta^4\mu^2-30a^2x\alpha^2\beta^4\mu^2+30a^2Tx\alpha^2\beta^4\mu^2-20ax\alpha^3\beta^4\mu^2+20aTx\alpha^3\beta^4\mu^2-5x\alpha^4\beta^4\mu^2+5Tx\alpha^4\beta^4\mu^2\right)\hbar^5+O[\hbar]^6$$

```
Series[Normal@Series[GS1 - GS2, {ħ, 0, $TD}], {β, 0, K}]
```

$$\frac{1}{48}\left(-2a^4\mu\hbar^4+2a^4T\mu\hbar^4-4a^3x\alpha\mu^2\hbar^5+4a^3Tx\alpha\mu^2\hbar^5-6a^2x\alpha^2\mu^2\hbar^5+6a^2Tx\alpha^2\mu^2\hbar^5-4ax\alpha^3\mu^2\hbar^5+4aTx\alpha^3\mu^2\hbar^5-x\alpha^4\mu^2\hbar^5+Tx\alpha^4\mu^2\hbar^5\right)\beta^4+O[\beta]^5$$

```
Normal@Series[GS1 - GS2, {ħ, 0, $TD}] + O[β]^(K+1)
```

$$O[\beta]^4$$

```
Series[GS1 - GS2, {β, 0, K}]
```

$$O[\beta]^4$$

```
Clear[ħ]
```