

Pensieve header: The Benkart-Witherspoon representation.

$$\hbar = \frac{\sigma - \rho}{\alpha \beta}; \quad q = e^{\hbar \alpha \beta};$$

$$y = \begin{pmatrix} \theta & \theta \\ -e^\rho & \theta \end{pmatrix}; \quad a = \frac{\alpha}{\rho - \sigma} \begin{pmatrix} \rho & \theta \\ \theta & \sigma \end{pmatrix}; \quad x = \frac{e^{-\sigma} - e^{-\rho}}{\hbar} \begin{pmatrix} \theta & 1 \\ \theta & \theta \end{pmatrix}; \quad b = \frac{\beta}{\sigma - \rho} \begin{pmatrix} \sigma & \theta \\ \theta & \rho \end{pmatrix};$$

MatrixForm /@

{A = MatrixExp[-\hbar \beta a], B = MatrixExp[-\hbar \alpha b], t = Simplify[\beta a - \alpha b], T = MatrixExp[\hbar t]}

$$\left\{ \begin{pmatrix} e^\rho & \theta \\ \theta & e^\sigma \end{pmatrix}, \begin{pmatrix} e^{-\sigma} & \theta \\ \theta & e^{-\rho} \end{pmatrix}, \begin{pmatrix} \frac{\alpha \beta (\rho + \sigma)}{\rho - \sigma} & \theta \\ \theta & \frac{\alpha \beta (\rho + \sigma)}{\rho - \sigma} \end{pmatrix}, \begin{pmatrix} e^{-\rho - \sigma} & \theta \\ \theta & e^{-\rho - \sigma} \end{pmatrix} \right\}$$

{a.x - x.a == \alpha x, x.A == q A.x, a.y - y.a == -\alpha y,

b.y - y.b == -\beta y, x.y - q y.x == \left(\begin{pmatrix} 1 & \theta \\ \theta & 1 \end{pmatrix} - T.A.A \right) / \hbar} // Simplify

{True, True, True, True, True}