

$$\mathbf{x} = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix};$$

$$\mathbf{a} = \frac{\alpha}{2} \begin{pmatrix} -1 & 0 \\ 0 & 1 \end{pmatrix}; \mathbf{A} = \text{MatrixExp}[-\hbar \beta \mathbf{a}]; \mathbf{y} = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}; \mathbf{b} = \begin{pmatrix} s & 0 \\ 0 & s \end{pmatrix} - \frac{\beta}{\alpha} \mathbf{a}; \mathbf{B} = \text{MatrixExp}[-\hbar \alpha \mathbf{b}];$$

$$\{\mathbf{a} \cdot \mathbf{x} - \mathbf{x} \cdot \mathbf{a} == -\alpha \mathbf{x}, \mathbf{a} \cdot \mathbf{y} - \mathbf{y} \cdot \mathbf{a} == \alpha \mathbf{y}\}$$

{True, True}

$$\{\mathbf{b} \cdot \mathbf{y} - \mathbf{y} \cdot \mathbf{b} == -\beta \mathbf{y}, \mathbf{x} \cdot \mathbf{b} - \mathbf{b} \cdot \mathbf{x} == -\beta \mathbf{x}\}$$

{True, True}

$\beta \mathbf{a} + \alpha \mathbf{b}$ // Simplify // MatrixForm

$$\begin{pmatrix} s \alpha & 0 \\ 0 & s \alpha \end{pmatrix}$$

$$\text{eqns} = \text{Thread}[\text{Flatten} /@ (\mathbf{x} \cdot \mathbf{y} - \mathbf{y} \cdot \mathbf{x} == \frac{\mathbf{B} - \mathbf{A}}{\hbar})]$$

$$\{1 == \frac{-e^{\frac{\alpha \beta \hbar}{2}} + e^{-s \alpha \hbar - \frac{\alpha \beta \hbar}{2}}}{\hbar}, \text{True}, \text{True}, -1 == \frac{-e^{-\frac{1}{2} \alpha \beta \hbar} + e^{-s \alpha \hbar + \frac{\alpha \beta \hbar}{2}}}{\hbar}\}$$

Solve[eqns, s]

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eqns /. $\beta \rightarrow 0$

$$\{1 == \frac{-1 + e^{-s \alpha \hbar}}{\hbar}, \text{True}, \text{True}, -1 == \frac{-1 + e^{-s \alpha \hbar}}{\hbar}\}$$

Solve[eqns /. $\beta \rightarrow 0$, \hbar]

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