

$$\mathbf{x} = \begin{pmatrix} 0 & 1 \\ 0 & 0 \end{pmatrix};$$

$$\mathbf{a} = \begin{pmatrix} r & 0 \\ 0 & s \end{pmatrix}; \mathbf{A} = \text{MatrixExp}[\hbar \alpha \mathbf{a}]; \mathbf{y} = \begin{pmatrix} 0 & 0 \\ 1 & 0 \end{pmatrix}; \mathbf{b} = \begin{pmatrix} s & 0 \\ 0 & r \end{pmatrix}; \mathbf{B} = \text{MatrixExp}[\hbar \beta \mathbf{b}];$$

$$\text{eqns1} = \text{Thread}[\text{Flatten} /@ \left( \mathbf{x} \cdot \mathbf{y} - \mathbf{y} \cdot \mathbf{x} == \frac{\mathbf{B} - \mathbf{A}}{e^{\hbar \gamma} - e^{-\hbar \gamma}} \right)]$$

$$\{1 == \frac{-e^{r \alpha \hbar} + e^{s \beta \hbar}}{-e^{-\gamma \hbar} + e^{\gamma \hbar}}, \text{True}, \text{True}, -1 == \frac{-e^{s \alpha \hbar} + e^{r \beta \hbar}}{-e^{-\gamma \hbar} + e^{\gamma \hbar}}\}$$

eqns = eqns1

$$\{1 == \frac{-e^{r \alpha \hbar} + e^{s \beta \hbar}}{-e^{-\gamma \hbar} + e^{\gamma \hbar}}, \text{True}, \text{True}, -1 == \frac{-e^{s \alpha \hbar} + e^{r \beta \hbar}}{-e^{-\gamma \hbar} + e^{\gamma \hbar}}\}$$

SolveAlways[eqns, {r, s, h}]

⋯ SolveAlways: The expression  $e^{\gamma \hbar}$  involves unknowns in more than one argument, so inverse functions cannot be used.

$$\text{SolveAlways}\left[\left\{1 == \frac{-e^{r \alpha \hbar} + e^{s \beta \hbar}}{-e^{-\gamma \hbar} + e^{\gamma \hbar}}, \text{True}, \text{True}, -1 == \frac{-e^{s \alpha \hbar} + e^{r \beta \hbar}}{-e^{-\gamma \hbar} + e^{\gamma \hbar}}\right\}, \{r, s, \hbar\}\right]$$

Solve[eqns2 ∪ eqns3 ∪ eqns4 ∪ eqns5, {r, s, α, β, γ}]

⋯ Solve: Equations may not give solutions for all "solve" variables.

$$\{\{\alpha \rightarrow -r + s, \beta \rightarrow -r + s\}\}$$

{sol} = Solve[eqns, {q, r, s, t}]

⋯ PolynomialGCD: Exponent is out of bounds for function PolynomialGCD.

⋯ PolynomialGCD: Exponent is out of bounds for function PolynomialGCD.

⋯ Solve: Solve was unable to solve the system with inexact coefficients. The answer was obtained by solving a corresponding exact system and numericizing the result.

$$\left\{ \left\{ q \rightarrow \text{ConditionalExpression}\left[ 4.8931 \times 10^{-9} \left( 1.53098 \times 10^7 + 3.63674 \times 10^9 \left( (4.09944 + 3.14159 i) + (0. + 6.28319 i) C[2] \right) \right), (C[1] | C[2]) \in \text{Integers} \right], r \rightarrow \text{ConditionalExpression}\left[ 17.7949 \left( (4.09944 + 3.14159 i) + (0. + 6.28319 i) C[2] \right), (C[1] | C[2]) \in \text{Integers} \right], s \rightarrow \text{ConditionalExpression}\left[ 1.12279 \times 10^{-8} \left( -1.96731 \times 10^7 + (0. + 4.67321 \times 10^9 i) \left( (3.14159 - 4.10365 i) + 6.28319 C[1] \right) \right), (C[1] | C[2]) \in \text{Integers} \right], t \rightarrow \text{ConditionalExpression}\left[ (0. + 52.4701 i) \left( (3.14159 - 4.10365 i) + 6.28319 C[1] \right), (C[1] | C[2]) \in \text{Integers} \right] \right\} \right\}$$

```
NSolve[{True, -1 ==  $\frac{1}{h}(-e^{-0.2420748525683183` r h} + e^{-0.3740462780156224` t h})$ ,
1 ==  $\frac{1}{h}(-e^{-0.2420748525683183` q h} + e^{-0.3740462780156224` s h})$ , q - r == -0.3740462780156224`,
-q + r == 0.3740462780156224`, -s + t == -0.2420748525683183`}, {q, r, s, t}]
```

PolynomialGCD: Exponent is out of bounds for function PolynomialGCD.

NSolve: NSolve was unable to solve the system with inexact coefficients. The answer was obtained by solving a corresponding exact system and numericizing the result.

```
{ {q -> ConditionalExpression[1.61626 x 10^-8
(-2.31427 x 10^7 + 1.00463 x 10^9 ((-2.39034 - 3.14159 i) + (0. + 6.28319 i) C[2])),
(C[1] | C[2]) in Integers],
r -> ConditionalExpression[16.2374 ((-2.39034 - 3.14159 i) + (0. + 6.28319 i) C[2]),
(C[1] | C[2]) in Integers], s -> ConditionalExpression[
1.14847 x 10^-9 (2.1078 x 10^8 + 9.14996 x 10^9 ((-2.41338 + 3.14159 i) + (0. + 6.28319 i) C[1])),
(C[1] | C[2]) in Integers], t -> ConditionalExpression[
10.5085 ((-2.41338 + 3.14159 i) + (0. + 6.28319 i) C[1]), (C[1] | C[2]) in Integers] ] }
```

```
sol /. C[1 | 2] -> 0
```

```
{q -> 73.0242 + 55.9045 i, r -> 72.9493 + 55.9045 i, s -> 215.098 + 164.84 i, t -> 215.319 + 164.84 i}
```

```
eqns /. Equal -> List
```

```
{True, {-1, 3.93066 (-e^0.0561957 r + e^0.0190585 t)}, {1, 3.93066 (-e^0.0561957 q + e^0.0190585 s)},
{q - r, 0.0749124}, {-q + r, -0.0749124}, {-s + t, 0.220886}}
```

```
eqns /. Equal -> List /. (sol /. C[1 | 2] -> 0)
```

```
{True, {-1, -1. - 2.10859 x 10^-13 i}, {1, 1. - 1.05835 x 10^-13 i}, {0.0749124 + 0. i, 0.0749124},
{-0.0749124 + 0. i, -0.0749124}, {0.220886 + 2.84217 x 10^-14 i, 0.220886}}
```