```
{α, β, ħ} = RandomReal[{-1, 1}, 3];
x = {{0, 1}, {0, 0}};
a = {{q, 0}, {0, r}}; A = MatrixExp[-ħ β a]; y = {{0, 0}, {1, 0}}; b = {{s, 0}, {0, t}}; B = MatrixExp[-ħ α b];
eqns1 = Thread[Flatten /@ (x.y - y.x == (B - A)/ħ)]
```

$$\left\{1 == 3.93066\ \left(-e^{0.0561957\,q} + e^{0.0190585\,s}\right),\ \text{True},\ \text{True},\ -1 == 3.93066\ \left(-e^{0.0561957\,r} + e^{0.0190585\,t}\right)\right\}$$

```
eqns2 = Thread[Flatten /@ (a.x - x.a == -α x)]
```

$$\{\text{True},\ q - r == 0.0749124,\ \text{True},\ \text{True}\}$$

```
eqns3 = Thread[Flatten /@ (b.y - y.b == -β y)]
```

$$\{\text{True},\ \text{True},\ -s + t == 0.220886,\ \text{True}\}$$

```
eqns4 = Thread[Flatten /@ (a.y - y.a == α y)]
```

$$\{\text{True},\ \text{True},\ -q + r == -0.0749124,\ \text{True}\}$$

```
eqns5 = Thread[Flatten /@ (x.b - b.x == -β x)]
```

$$\{\text{True},\ -s + t == 0.220886,\ \text{True},\ \text{True}\}$$

```
eqns = eqns1 ⋃ eqns2 ⋃ eqns3 ⋃ eqns4 ⋃ eqns5
```

$$\left\{\text{True},\ -1 == 3.93066\ \left(-e^{0.0561957\,r} + e^{0.0190585\,t}\right),\ 1 == 3.93066\ \left(-e^{0.0561957\,q} + e^{0.0190585\,s}\right),\right.$$
$$\left.q - r == 0.0749124,\ -q + r == -0.0749124,\ -s + t == 0.220886\right\}$$

```
Reduce[eqns, {q, r, s, t}]
```

Reduce: Reduce was unable to solve the system with inexact coefficients. The answer was obtained by solving a corresponding exact system and numericizing the result.

$$(C[1] \mid C[2]) \in \text{Integers}\ \&\&\ q == 17.7949\ \left(\left(4.10365 + 3.14159\,i\right) + \left(0. + 6.28319\,i\right) C[2]\right)\ \&\&$$
$$r == -0.0749124 + q\ \&\&\ s == \left(0. + 52.4701\,i\right)\ \left(\left(3.14159 - 4.09944\,i\right) + 6.28319\,C[1]\right)\ \&\&$$
$$t == 1.12279 \times 10^{-8}\ \left(1.96731 \times 10^{7} + 8.90641 \times 10^{7}\,s\right)$$

```
Solve[eqns2 ⋃ eqns3 ⋃ eqns4 ⋃ eqns5, {q, r, s, t}]
```

Solve: Equations may not give solutions for all "solve" variables.

$$\{\{r \to -0.0749124 + 1.\,q,\ t \to 0.220886 + 1.\,s\}\}$$

```
{sol} = Solve[eqns, {q, r, s, t}]
```

... PolynomialGCD: Exponent is out of bounds for function PolynomialGCD.

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... Solve: Solve was unable to solve the system with inexact coefficients. The answer was obtained by solving a corresponding exact system and numericizing the result.

$$\{\{q \to \text{ConditionalExpression}\big[4.8931\times10^{-9}\left(1.53098\times10^{7}+3.63674\times10^{9}\left(\left(4.09944+3.14159\,i\right)+\left(0.+6.28319\,i\right)C[2]\right)\right),\left(C[1]\mid C[2]\right)\in\text{Integers}\big],\ r \to \text{ConditionalExpression}\big[17.7949\left(\left(4.09944+3.14159\,i\right)+\left(0.+6.28319\,i\right)C[2]\right),\left(C[1]\mid C[2]\right)\in\text{Integers}\big],\ s \to \text{ConditionalExpression}\big[1.12279\times10^{-8}\left(-1.96731\times10^{7}+\left(0.+4.67321\times10^{9}\,i\right)\left(\left(3.14159-4.10365\,i\right)+6.28319\,C[1]\right)\right),\left(C[1]\mid C[2]\right)\in\text{Integers}\big],\ t \to \text{ConditionalExpression}\big[\left(0.+52.4701\,i\right)\left(\left(3.14159-4.10365\,i\right)+6.28319\,C[1]\right),\left(C[1]\mid C[2]\right)\in\text{Integers}\big]\}\}$$

```
NSolve[{True, -1 == 1/ħ (-ℯ^(-0.2420748525683183` r ħ) + ℯ^(-0.3740462780156224` t ħ)),
  1 == 1/ħ (-ℯ^(-0.2420748525683183` q ħ) + ℯ^(-0.3740462780156224` s ħ)), q - r == -0.3740462780156224`,
  -q + r == 0.3740462780156224`, -s + t == -0.2420748525683183`}, {q, r, s, t}]
```

... PolynomialGCD: Exponent is out of bounds for function PolynomialGCD.

... NSolve: NSolve was unable to solve the system with inexact coefficients. The answer was obtained by solving a corresponding exact system and numericizing the result.

$$\{\{q \to \text{ConditionalExpression}\big[1.61626\times10^{-8}\left(-2.31427\times10^{7}+1.00463\times10^{9}\left(\left(-2.39034-3.14159\,i\right)+\left(0.+6.28319\,i\right)C[2]\right)\right),\left(C[1]\mid C[2]\right)\in\text{Integers}\big],\ r \to \text{ConditionalExpression}\big[16.2374\left(\left(-2.39034-3.14159\,i\right)+\left(0.+6.28319\,i\right)C[2]\right),\left(C[1]\mid C[2]\right)\in\text{Integers}\big],\ s \to \text{ConditionalExpression}\big[1.14847\times10^{-9}\left(2.1078\times10^{8}+9.14996\times10^{9}\left(\left(-2.41338+3.14159\,i\right)+\left(0.+6.28319\,i\right)C[1]\right)\right),\left(C[1]\mid C[2]\right)\in\text{Integers}\big],\ t \to \text{ConditionalExpression}\big[10.5085\left(\left(-2.41338+3.14159\,i\right)+\left(0.+6.28319\,i\right)C[1]\right),\left(C[1]\mid C[2]\right)\in\text{Integers}\big]\}\}$$

```
sol /. C[1 | 2] → 0
```

$$\{q \to 73.0242+55.9045\,i,\ r \to 72.9493+55.9045\,i,\ s \to 215.098+164.84\,i,\ t \to 215.319+164.84\,i\}$$

```
eqns /. Equal → List
```

$$\{\text{True},\ \{-1,\ 3.93066\left(-e^{0.0561957\,r}+e^{0.0190585\,t}\right)\},\ \{1,\ 3.93066\left(-e^{0.0561957\,q}+e^{0.0190585\,s}\right)\},\ \{q-r,\ 0.0749124\},\ \{-q+r,\ -0.0749124\},\ \{-s+t,\ 0.220886\}\}$$

```
eqns /. Equal → List /. (sol /. C[1 | 2] → 0)
```

$$\{\text{True},\ \{-1,\ -1.-2.10859\times10^{-13}\,i\},\ \{1,\ 1.-1.05835\times10^{-13}\,i\},\ \{0.0749124+0.\,i,\ 0.0749124\},\ \{-0.0749124+0.\,i,\ -0.0749124\},\ \{0.220886+2.84217\times10^{-14}\,i,\ 0.220886\}\}$$